Attention Cycles

Joel P. Flynn[*] Karthik A. Sastry[†]

Yale Princeton

September 1, 2023

Abstract

We study how the extent of bounded rationality evolves with the state of the economy. We introduce a business-cycle model in which firms face a cognitive cost of making precise decisions and decide how much to "pay attention" to specific states. We analytically characterize equilibrium with non-parametric, state-dependent stochastic choice and nonlinear aggregation and dynamics. Firms have greater incentives to direct attention toward states of lower aggregate consumption because they are owned by risk-averse households. This mechanism generates counter-cyclical attention, pro-cyclical mistakes, and an endogenous attention wedge that depresses aggregate productivity when attention is low. We test and validate these macroeconomic predictions and our proposed microeconomic mechanism using novel measures of US public firms' input-choice mistakes and a textual proxy for their attention toward macroeconomic conditions. When calibrated to match our measurements, attention cycles generate a significant fraction of the observed stochastic volatility of output growth and shock propagation asymmetries.

1   Introduction

Firms often make decisions which, *ex post*, are inconsistent with profit maximization. An influential explanation is that firms' managers face constraints on their attention and decisionmaking capacity that introduce "mistakes" (Simon, 1947). But, although this view relaxes perfect optimization, it still preserves incentives at its core—even if plans are imperfect, an optimizing manager will design them to minimize mistakes in the microeconomic and macroeconomic circumstances in which mistakes are most costly. By implication, the state of the economy may be a central determinant of decisionmakers' apparent "bounded rationality." Moreover, the macroeconomic consequences of bounded rationality may depend on the state of the economy.

We apply this incentives-based view of bounded rationality to study the co-determination of business cycles and *attention cycles*, aggregate fluctuations in cognitive effort and mistakes. Theoretically, we develop a general-equilibrium model in which heterogeneous firms choose non-parametric, state-dependent production plans and face a cognitive control cost of choosing more precise plans. We analytically derive the equilibrium relationship between cognitive effort, mistakes, aggregate output, and aggregate productivity. Because firms are owned by risk-averse investors, incentives for precise profit maximization are higher when aggregate consumption is low. This *risk-pricing channel* generates a prediction that, during downturns, firms exert more cognitive effort and make smaller mistakes relative to what is *ex post* optimal. Endogenous attention, in turn, induces asymmetric propagation of positive and negative macroeconomic shocks.

To test the model empirically, we introduce strategies to measure precision in "what firms do," an exact measure of mistakes as defined by our model, and attention to the macroeconomy in "what firms say," a suggestive proxy for cognitive effort. At the macro level, firms make smaller input-choice mistakes and speak more about the macroeconomy in downturns. At the micro level, input-choice mistakes reduce firms' stock returns, this market punishment is significantly harsher during downturns, and firms that make smaller mistakes use more attentive language. These findings validate our interpretation of measured mistakes as capturing *ex post* misoptimization that is more severe when firms are less attentive. Moreover, the results confirm the model's link between microeconomic incentives and macroeconomic phenomena: firms make smaller mistakes exactly when controlling mistakes is most valuable.

Quantitatively, we calibrate the model to match our empirical evidence and study how attention cycles affect macroeconomic dynamics. We find that counter-cyclical attention induces both larger output responses to negative (as opposed to positive) productivity shocks and counter-cyclical volatility of output growth. The magnitude of these effects is quantitatively significant—for example, endogenous attention in our model generates 20% of the spike in output growth volatility that Jurado et al. (2015) measure over the Great Recession. Our results suggest that the attention cycle may contribute significantly to the observed asymmetric and state-dependent propagation of macroeconomic shocks.

Model. We build on a standard Neoclassical real business cycle model with heterogeneous firms. A representative household owns, supplies labor to, and consumes the output of a continuum of monopolistically competitive firms. These firms differ in their productivity. An aggregate shock shifts

the cross-firm productivity distribution to generate a business cycle.

We model firms' state-dependent attention by subjecting firms to a cognitive *control cost* of choosing any non-parametric stochastic choice rule, *i.e.*, a mapping from all payoff-relevant microeconomic and macroeconomic states (productivity, demand, and input prices) to a distribution of production. These control costs are greater in a given state when decisionmaking is more precise in that state. Firms therefore choose costly "attention," or cognitive effort, to optimally trade off these costs with the benefit of increasing expected, stochastically discounted profits in every state of the world. Our approach of directly modelling stochastic choice allows for equilibrium analysis of rich decision frictions in a way that is robust to the specific (and generally untestable) microfoundation for why precise decisionmaking is hard, like costly information acquisition, behavioral biases, or trembling hands in implementation (see Morris and Yang, 2022, and Flynn and Sastry, 2023, for a discussion). Despite this generality, we show that equilibrium analysis in our setting is well-posed and tractable. We prove equilibrium existence, uniqueness, and monotonicity of output in the productivity distribution (in the sense of first-order stochastic dominance).

Attention Cycles and Their Consequences. In partial equilibrium, our model predicts that bounded rationality is lower when stakes are higher. That is, firms pay more attention, measured by the extent of their cognitive effort, and make smaller mistakes, measured by the variance of their actions around the *ex post* ideal point, when the benefit of paying attention is high. We show that the correct metric for these benefits is the curvature of firms' objective functions in their own production. This curvature is the product of two terms: the curvature of firms' dollar profits ("dollar losses from mistakes") and the stochastic discount factor ("utility losses per dollar loss"). We refer to each of these effects on attention choice as the *profit-curvature* and *risk-pricing* channels, respectively.

We next identify the determinants of each channel in our model. The profit-curvature channel depends on the price elasticity of demand that firms face and the elasticity of wages to real output. While this channel has an ambiguous sign in general, we argue that in realistic parameterizations it pushes toward high attention when aggregate output and firm productivity are low. The risk-pricing channel depends on the representative household's (*i.e.*, the firm owner's) risk aversion. This channel always pushes toward high attention when aggregate consumption is low, and it is retained in *any* model in which the stochastic discount factor that prices firms decreases in aggregate consumption.

We next combine these partial-equilibrium insights with our characterization of equilibrium to describe the *general-equilibrium* properties of "attention cycles." We show that, if household relative risk aversion exceeds an empirically modest lower bound of one plus the elasticity of real wages to output and wages are sufficiently rigid, then equilibrium cognitive effort decreases in output and the average size of agents' mistakes increases in output. Intuitively, equilibrium market incentives push firms toward paying more attention to decisions and precisely maximizing profits when the economy is doing poorly.

We finally study the macroeconomic consequences of attention cycles. We show that output is the product of the counterfactual output under no cognitive friction with an *attention wedge* that is less than one. The wedge arises because inattentive, stochastic choices translate into dispersion of value marginal products across firms ("misallocation") and reduced aggregate total factor productivity

(TFP). Due to counter-cyclical attention, the wedge widens when the economy is booming and firms optimize less precisely. Thus, all else equal, misallocation is endogenously higher in booms than recessions. Starkly, equilibrium *aggregate* TFP can even be non-monotone in the distribution of underlying *microeconomic* productivity. Moreover, the translation of negative shocks into increased attention leads to asymmetric, state-dependent shock propagation and endogenous stochastic volatility.

Measurement: Misoptimization and Attention. The model makes micro and macro predictions for both choice mistakes ("misoptimization") and cognitive effort ("attention"). To test these predictions, we first construct a panel dataset of both variables among firms.

We measure misoptimization, as defined in the model, by extracting the residual component of firm hiring conditional on productivity. We use data on public firms' production and input choices from Compustat from 1986-2018. Using conventional methods, we estimate firm-level TFP and estimate empirical policy functions for labor choice conditional on TFP, firm fixed effects, and sector-by-time fixed effects. In our model, the empirical policy function estimates the counterfactual "unconstrained optimal" input choice of firms and the residuals estimate the misoptimizations. To test if our results are driven by unmodeled features of firm behavior or by measurement error, we replicate all our empirical results for alternative constructions of misoptimizations based on richer models of firm behavior (e.g., with adjustment costs, forward-looking investments, and financial constraints) and alternative measurement (e.g., sector-specific policy functions, time-varying responsiveness to TFP, nonlinear effects of TFP, time-varying production functions, and alternative TFP estimation). We finally note that this method does not impose that misoptimizations, by our empirical definition, harm firm payoffs. This will motivate a simple but important test of our model interpretation.

Attention, unlike misoptimization, has no direct analog in observable firm choices. We therefore construct an informative proxy for attention based on the language that firms use to describe their choices in their regulatory filings (forms 10-Q/K) and conference calls with investment analysts. Our premise is that, when management uses more language to describe the relationship of their decisions with a specific topic, they are revealing greater cognitive effort toward adapting decisions to those factors. Our model predicts that attention toward *all* factors increases in downturns; but, for measurement, it is necessary to focus on factors commonly faced by all firms. We therefore quantify firms' attention toward *macroeconomic conditions*. We measure a firm's attention toward the macroeconomy as the similarity of its language to that of macroeconomics references (introductory college-level textbooks), using a natural-language-processing technique that builds on the work of Hassan et al. (2019). The attention measure complements the "model-based" misoptimization measure by being constructed in a more "model-free" way.

Testing the Theory. We first find that *both* positive and negative misoptimizations (i.e., "over"- and "under"-hiring) have significant negative effects on firms' contemporaneous stock returns and profitability. This rules out a story in which misoptimizations purely represent measurement error, misspecification, or wedges in the sense of Chari et al. (2007) and Restuccia and Rogerson (2008). Moreover, misoptimizations do not predict higher future productivity, stock returns, or profitability, ruling out the possibility that we have purely picked up forward-looking adjustments with short-run

costs and long-run benefits. These results validate our interpretation of misoptimizations as mistakes and not as other unmodeled wedges.

We next find that firms' misoptimizations are pro-cyclical, consistent with our theoretical prediction. On average, a five percentage point increase in unemployment predicts a 53% decrease in cross-sectional misoptimization variance. Misoptimization is pro-cyclical under every aforementioned alternative construction of misoptimizations (including constructions that account for time-varying financial frictions and adjustment costs), under alternative aggregation that adjusts for compositional bias, for materials and capital, and on the entire Compustat sample back to 1950. This finding enriches the literature's observation, replicable in our data, that the variance of firm productivity spikes in recessions (e.g., Kehrig, 2015; Bloom et al., 2018). Our new fact implies that choice dispersion *conditional on* productivity decreases exactly when productivity itself becomes more volatile. This can help explain why dispersion in investment rates (Bachmann and Bayer, 2014) and implied stock-return volatility (Dew-Becker and Giglio, 2020) are not counter-cyclical, as these quantities blend fundamentals with mistakes in firms' response to them.

We next test the model's main microeconomic mechanism—that incentives toward paying attention drive fluctuations in apparent bounded rationality. We do this by testing the model's most direct microeconomic prediction—that markets punish mistakes more in downturns. We find, consistent with the theory, that the negative effect of misoptimizations on stock returns significantly strengthens when aggregate markets are doing poorly. For example, in the stock market trough of 2008, a misoptimization was more than 6 times more costly for returns than it would have been in a year with 10% aggregate returns. By contrast, the effect of misoptimizations on firm *profitability* is close to constant over the cycle. Interpreted via the model, this rules out a story driven primarily by changing *dollar* profit curvature, and points toward a story driven by the risk-pricing channel and financial incentives.

We finally test the model's predictions for attention. At the aggregate level, we find that macroeconomic attention in language is counter-cyclical. At the firm level, we find that macroeconomic attention is associated with smaller misoptimizations. These findings, together, are consistent with our interpretation that attention in language corresponds to increased effort in restraining mistakes. Our findings are inconsistent with two alternative interpretations: that firms use the macroeconomy as a scapegoat for poor performance or merely report their exposure to downside risks. The former is contradicted by our finding that attentive firms make smaller mistakes, and the latter by a finding that attention is counter-cyclical even in sectors with acyclical production.

Quantification. We finally combine our empirical findings with the model to assess the quantitative implications of attention cycles. We calibrate the scale of the cognitive friction and risk aversion to match the level and cyclicality of misoptimizations in the data. We use external calibrations for the substitutability between goods, the cyclicality of real wages, and the evolution of productivity. This approach lets the data speak directly toward our model's main mechanism, the interaction between cognitive frictions and risk-pricing incentives.

We find that negative productivity shocks have larger effects on aggregate output than equal-sized positive ones; all shocks have larger effects in low states; and output volatility is highest in low states. In the model, negative productivity shocks have a 7% larger effect on aggregate output

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(5% larger on employment) than positive shocks of the same size. This is 25% of the asymmetry estimated by Ilut et al. (2018) using industry-level data on productivity and employment. Similarly, in the model, a shock that replicates a "Great-Recession-sized" 5% peak-to-trough reduction in output generates also an 11% increase in the conditional volatility of output. This is 19% of the increase in uncertainty about output growth between the trough of the Great Recession and the preceding peak as measured by Jurado et al. (2015). Our model therefore explains a significant fraction of observed non-linearities and state-dependence of macroeconomic dynamics as consequences of state-dependent attention. Thus, attention cycles generate fast crashes and slow recoveries.

Related Literature. Our study's primary contribution is to theoretically justify and empirically validate the hypothesis that agents' inattention, or their extent of bounded rationality, systematically varies with the business cycle because of changing incentives. The idea that attention lines up with incentives is obtained in classic single-agent models of rational inattention (e.g., Sims, 2003; Gabaix, 2014). By contrast, most macroeconomic models with cognitive frictions (e.g., Woodford, 2003; Van Nieuwerburgh and Veldkamp, 2006; Maćkowiak and Wiederholt, 2009; Angeletos and La'O, 2010; Gabaix, 2020) do not study how optimal state-dependent attention would vary over the business cycle. Three exceptions are Mäkinen and Ohl (2015), Benhabib et al. (2016), and subsequent work by Chiang (2023).[1] In contrast to all three studies, we emphasize a distinct risk-pricing mechanism, develop novel measures of firms' attention and misoptimization, and use these measures to directly test and quantify both the macroeconomic and microeconomic predictions of the theory.

An additional contribution is to show, theoretically and empirically, how endogenous attention contributes to microeconomic and macroeconomic volatility. This expands upon previous work about the static relationship among exogenous cognitive frictions, micro volatility, and misallocation (David et al., 2016; David and Venkateswaran, 2019; Ma et al., 2020; Barrero, 2022). A key lesson from our analysis is that endogenous volatility in choices conditional on fundamentals may have an opposite cyclicality to exogenous volatility in fundamentals, helping reconcile the finding of counter-cyclical TFP variance (Kehrig, 2015; Bloom et al., 2018) with more ambiguous evidence for the volatility of other variables (e.g., Bachmann and Bayer, 2014; Dew-Becker and Giglio, 2020). We also predict and empirically validate that firms respond to shocks with lower *variance* in downturns compared to booms, supplementing existing evidence about higher responsiveness in *means* (Berger and Vavra, 2019). Finally, we propose state-dependent attention as a new mechanism that generates macroeconomic volatility as an endogenous response to macroeconomic shocks.

Methodologically, we show how to use a control-cost model to tractably and analytically study state-dependent attention in general equilibrium while allowing for rich choice patterns. The closest comparison in this respect is a literature that studies state-dependent attention under mutual-information costs (Sims, 2003). In this literature, studies have numerically solved decision problems in linear-quadratic-Gaussian environments (Afrouzi and Yang, 2021; Miao et al., 2022) and in more general environments (Matějka, 2016; Stevens, 2020; Turen, 2023). In contrast to these studies, our

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[1]In Mäkinen and Ohl (2015), decreasing returns to scale lead firms to demand more information when aggregate productivity is low. Chiang (2023) has considered a complementary mechanism in the context of a model where firms acquire Gaussian signals about the state. Benhabib et al. (2016) predict lower information demand in recessions, a prediction at odds with our empirical findings.

control-cost model allows for exact analytical solutions to firms' globally optimal choice patterns. This is crucial in the theory to obtain an analytical characterization of equilibrium and in our empirical work to obtain a direct and realistic mapping from the model's predictions to observed stochastic choices. Costain and Nakov (2015, 2019) also apply the control-cost model but study different questions, regarding price dynamics, and do not provide comparable analytical results. Our approach is similar in spirit to that of Ilut and Valchev (2023), who model cognitive constraints as costly learning of policy rules. These authors focus on different applications and also use numerical analysis of aggregative equilibrium.

Our textual measure of attention relates to independent work by Song and Stern (2023), who develop a similar text-based measure of firms' attention to macroeconomic news and use this measure to test a different hypothesis. Specifically, Song and Stern (2023) show that more attentive firms, according to their text-based measure, gain more market value from expansionary monetary policy shocks and lose less value from contractionary shocks. They interpret how this heterogeneous responsiveness can drive monetary non-neutrality. Together, the two papers validate that textual measures can capture aspects of "attention" that are consistent with economic models. These findings also relate to a literature documenting that firms' language contains relevant information about risk exposures (Hassan et al., 2019) and sentiment (Loughran and McDonald, 2011; Flynn and Sastry, 2022).

Overview. The rest of the paper proceeds as follows. In Section 2, we introduce our model. In Section 3, we present our theoretical results. In Section 4, we describe the data and how we construct our empirical measures of misoptimizations and attention. In Section 5, we test the model's six main predictions. In Section 6, we calibrate our model and analyze the impact of attention cycles on macroeconomic dynamics. Section 7 concludes.

2 Model

We first describe our model, a Neoclassical real business cycle model with a stochastic choice friction for intermediate goods firms that captures inattention and mistake making.

2.1 Consumers and Final Goods

Time periods are indexed by $t \in \mathbb{N}$. A representative household has constant relative risk-aversion preferences over final-good consumption C_t and labor L_t. Their payoffs are:

$$\mathcal{U}(\{C_{t+j}, L_{t+j}\}_{j=0}^{\infty}) = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^{1-\gamma}}{1-\gamma} - v(L_{t+j}) \right) \tag{1}$$

where $\beta \in [0,1)$ is the discount factor, v is an increasing and convex labor disutility, and $\gamma > 0$ is the coefficient of relative risk aversion. The aggregate final good is produced from a continuum of intermediate goods, indexed by $i \in [0,1]$, by a representative, perfectly competitive firm using a

constant-elasticity-of-substitution production function

$$X_t = X(\{x_{it}\}_{i \in [0,1]}) = \left(\int_{[0,1]} x_{it}^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{2}$$

where $\epsilon > 1$ is the elasticity of substitution. This firm buys inputs at prices $\{q_{it}\}_{i \in [0,1]}$ and sells its output at a normalized price of one. The household owns equity in firms that produce intermediate goods, receiving profits $\{\pi_{it}\}_{i \in [0,1]}$.

Wages are determined by the following wage rule:

$$w_t = \bar{w} \cdot \left(\frac{X_t}{\bar{X}} \right)^{\chi} \tag{3}$$

where $\bar{w} > 0$ and $\bar{X} > 0$ are constants, and $\chi \geq 0$ measures the extent of real wage rigidity. Households supply labor to meet firms' labor demand at the wages from Equation 3. Describing labor-market dynamics via a wage rule is for technical simplicity as it allows us to study the economy via a scalar fixed-point equation.[2] Moreover, it allows our model to parsimoniously match the empirical acyclicality of real wages (Solon et al., 1994; Grigsby et al., 2021). In Appendix B, we micro-found this wage rule when households have Greenwood et al. (1988) preferences and markets clear in the standard fashion. We show in Appendix E.1 that the quantitative results of Section 6 are robust to considering wages set in this manner.

2.2 Intermediate Goods Firms and Productivity Shocks

Each intermediate goods firm i is a monopoly producer of its own variety and faces a demand curve $d(x_{it}, X_t) = X_t^{\frac{1}{\epsilon}} x_{it}^{-\frac{1}{\epsilon}}$ from the final goods producer. They hire a labor quantity L_{it}, pay wage w_t per worker, and produce with the following linear technology:

$$x_{it} = \theta_{it} L_{it} \tag{4}$$

where θ_{it} is a firm-level shifter of productivity, which lies in a set $\Theta \subset \mathbb{R}_+$.

We parameterize the stochastic process for productivity in the following way that allows for both rich cross-firm heterogeneity and "aggregate productivity" shocks. We let $\theta_t \in \Theta$ be an aggregate productivity variable. For each firm i, (θ_{it}, θ_t) follows a "triangular" first-order Markov process: the transition density can be factored as $h(\theta_{it}, \theta_t \mid \theta_{i,t-1}, \theta_{t-1}) = h_{agg}(\theta_t \mid \theta_{t-1}) h_{idio}(\theta_{it} \mid \theta_{i,t-1}, \theta_t, \theta_{t-1})$. In words, aggregate productivity follows a first-order Markov process by itself and firm-level productivity depends flexibly on its own lag and the current and lagged aggregate state. This model allows for aggregate productivity shocks to affect all moments of the idiosyncratic productivity distribution. For example, it is consistent with the model empirically estimated by Bloom et al. (2018) wherein aggregate shocks affect both the mean of firm level productivity and its idiosyncratic variance. This allows our model to be consistent with realistic productivity dynamics and to incorporate time-varying

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[2]Blanchard and Galí (2010) and Alves et al. (2020) use similar wage-rule formulations.

microeconomic and macroeconomic uncertainty.

Throughout our analysis, we will reference two aggregate summaries of the productivity distribution. The first is the cross-sectional distribution of θ_{it} at time t, $G_t \in \Delta(\Theta)$. The second is the statistic,

$$\hat{\theta}(G) = \left(\int_\Theta \theta_i^{\epsilon-1} \, \mathrm{d}G(\theta_i) \right)^{\frac{1}{\epsilon-1}} \tag{5}$$

which is a quasi-arithmetic mean of the productivity distribution. Note that, because the integrand inside is an increasing function, $\hat{\theta}(G)$ is monotone in distributions G and G' ranked by first-order stochastic dominance. In this sense, $\hat{\theta}(G') \geq \hat{\theta}(G)$ is a weaker notion of higher productivity under G' than G than first-order stochastic dominance.

We proceed in the rest of our main analysis under a "Real Business Cycle" case in which productivity shocks drive micro and macro fluctuations. But we will emphasize when presenting results why the "supply-side" view is inessential for our main conclusions.

2.3 Modelling Attention via Costly Control

We now describe our model for costly, state-dependent attention for intermediate goods firms. Due to cognitive constraints, these agents struggle to perfectly adapt their production choice to the microeconomic and macroeconomic state without making mistakes. We model this by having them choose stochastic choice rules, or mappings from states to a distribution of production outcomes, subject to a cognitive cost. Both the choice set and the cost are flexible enough to allow for unrestricted, state-dependent attention, as we formalize below.

Decision State and Choice Variable. We first define the firm-level *decision state* $z_{it} = (\theta_{it}, X_t, w_t) \in \mathcal{Z}$ as the concatenation of all decision-relevant variables that the firm takes as given.[3] Firms believe that z_{it} follows a first-order Markov process with transition density $f(z_{it}|z_{i,t-1})$. As we will soon clarify, this conjecture combines the first-order Markov process for (θ_{it}, θ_t) with a conjecture for how aggregates depend on the productivity distribution. We denote the corresponding set of possible transition densities by $\mathcal{F}$. At time t, each firm i observes the history of states, $\{z_{is}\}_{s<t}$, but not the contemporaneous value z_{it}.

The firm's choice variable is a stochastic choice rule $p : \mathcal{Z} \to \Delta(\mathcal{X})$ in set $\mathcal{P}$, or a mapping from states of the world to *distributions of actions* described by probability density function $p(\cdot \mid z_{it})$ when the decision state is z_{it}. A firm using rule p commits to delivering the realized quantity $x_{it} \in \mathcal{X}$ to the market and hiring sufficient labor in production. Helpfully, in the problem that we study, it will be fully equivalent to interpret firms' choices as committing to hire L_{it} workers and producing $x_{it} = \theta_{it} L_{it}$.

Costly Control. We model the cognitive cost of attention via a cost functional $c : \mathcal{P} \times \Lambda \times \mathcal{Z} \times \mathcal{F} \to \mathbb{R}$ which returns how costly any given stochastic choice rule is to implement in units of utility. The cost can depend on a firm-specific type $\lambda_i \in \Lambda \subseteq \mathbb{R}_+$, by assumption independent from the decision state

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[3]As will become clear, $\mathcal{Z} = \Theta \times \mathcal{X} \times \mathcal{W}$, where $\mathcal{X}$ is feasible set of production, and $\mathcal{W}$ is the image of $\mathcal{X}$ via the wage rule (3).

and distributed in the cross-section as $L \in \Delta(\Lambda)$, and the previous value of the decision state $z_{i,t-1}$, which under the Markov assumption summarizes the transition probabilities for z_{it}.

The basic idea that we wish to embody is that playing actions that are more *precise* is more costly. To make this tension clear, and also to make the analysis tractable, we specialize to the following cost functional that equals the negative expected entropy of the action distribution multiplied by $\lambda_i > 0$:

$$c(p, \lambda_i, z_{i,t-1}, f) = \lambda_i \int_{\mathcal{Z}} \int_{\mathcal{X}} p(x \mid z_{it}) \log(p(x \mid z_{it})) \, \mathrm{d}x \, f(z_{it} \mid z_{i,t-1}) \, \mathrm{d}z_{it} \qquad (6)$$

These costs embody the idea that it is difficult for firms to avoid making mistakes (see Morris and Yang, 2022, and Flynn and Sastry, 2023, for a discussion). In particular, these costs are consistent with the experimentally verified fact that the size and character of choice mistakes respond to incentives (Woodford, 2020), the central premise of our analysis.

Comparison to Other Models of Inattention. This notwithstanding, there are other costs that could embody the notion of incentives-driven attention. Two prominent examples are unrestricted costly information acquisition (Caplin and Dean, 2015), including the mutual information model of Sims (2003), and behavioral sparse maximization (Gabaix, 2014).

In contrast to these alternative models, however, our model jointly has three properties that are important for our analysis. First, our model of costly control admits an analytical solution for the globally optimal policy, despite the model's state-dependence. In particular, no comparable results exist in the literature on rational inattention with mutual information costs.[4] Second, our model allows for tractable and well-posed analysis of equilibrium and its comparative statics. The literature on equilibrium results with general information acquisition and/or sparsity-based decision frictions does not describe comparative statics (e.g., Angeletos and Sastry, 2023; Gabaix, 2019). Third, our model allows for rich, state-dependent, and stochastic mistakes. This will prove a more realistic prediction in our setting than, for example, the mutual information model's prediction of sparse actions (Matějka, 2016; Caplin et al., 2018; Jung et al., 2019; Stevens, 2020) or the sparsity model's prediction of deterministic mistakes. We will use these predictions as the cornerstone for empirical tests of the theory in Sections 4 and 5.

A key cost relative to these alternative models, however, is that our predicted optimal policy functions will not include a force of "anchoring" toward commonly played actions and therefore under-reacting to shocks. To gauge the importance of missing this force, we discuss and analyze extensions of the model with unrestricted and parametric information acquisition in Section 3.5 and Appendices D.2 and D.3.

2.4 The Firm's Problem

Intermediate goods firms are owned by the representative household and maximize the product of their dollar profit, which we write as $\pi(x_{it}, z_{it})$, and the household's marginal utility, which we write

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[4]Even when approximated as a *state-dependent* quadratic function of the action, our objective is not "linear-quadratic" in the sense studied by Sims (2010), Maćkowiak et al. (2018), Miao et al. (2022), or Afrouzi and Yang (2021). Therefore, these papers' results characterizing necessary and sufficient conditions for optima do not apply in our setting.

as $M(z_{it})$. We define "risk-adjusted profits" as the product of these terms:

$$\Pi(x_{it}, z_{it}) = \pi(x_{it}, z_{it}) \cdot M(z_{it}) \tag{7}$$

Under our assumed structure for the firms' cost and revenue structure and the household's utility function, the profit function and marginal utility are respectively

$$\pi(x_{it}, z_{it}) = x_{it} \left(x_{it}^{-\frac{1}{\epsilon}} X_t^{\frac{1}{\epsilon}} - \frac{w_t}{\theta_{it}} \right) \qquad\qquad M(z_{it}) = X_t^{-\gamma} \tag{8}$$

We next note that the firm's dynamic choice problem reduces to a series of one-shot problems. This follows because $z_{i,t-1}$ is an observed sufficient statistic for the history of states. Therefore the firm's beliefs in future periods do not depend on its decisions in the current period.

The firm's choice at time t therefore solves the following problem of maximizing expected risk-adjusted profits net of cognitive costs:

$$\max_{p \in \mathcal{P}} \left\{ \int_{\mathcal{Z}} \int_{\mathcal{X}} \Pi(x, z_{it}) \, p(x \mid z_{it}) \, \mathrm{d}x \; f(z_{it} \mid z_{i,t-1}) \, \mathrm{d}z_{it} - c\left(p, \lambda_i, z_{i,t-1}, f\right) \right\} \tag{9}$$

Firms can flexibly and non-parametrically choose a production plan for each decision state realization z_{it}. For example, firms can choose how the mean and variance of their production vary with idiosyncratic productivity, aggregate output, and wages. Planning precisely, however, has a cognitive cost, which is proportional to entropy. The firm's optimal choice of "attention" or cognitive effort in each state trades off higher expected utility from precise planning with the increased costs of so doing.

Units for Costs. We finally note that, consistent with our premise that decision frictions arise because of *cognitive* limitations, the cost c is in utility units. An alternative model, motivated for instance by a story of physical costs within the firm (e.g., hiring planners or consultants), might denominate the costs in units of the numeraire good. In this case, we would alter the cost function in Equation 6 to take the expectation over states z_{it} weighted by the stochastic discount factor. In Section 3.5, we show that this dollar-cost model (under any stochastic discount factor) is outcome-equivalent to our utility-cost model with risk-neutral owners ($\gamma = 0$), and is therefore nested in the main analysis. We further discuss how to interpret our main results in this special case.

2.5 Linear-Quadratic Approximation and Equilibrium

To tractably study equilibrium, we simplify the intermediate goods firms' objective and the final goods firm's production with quadratic approximations. Both approximations are derived in Appendix A.7.

Toward simplifying the intermediate goods firms' objective, we first define an intermediate firm's *ex post* optimal production level, $x^*(z_i) := \arg\max_{x \in \mathcal{X}} \Pi(x, z_i)$. We next define $\bar{\Pi}(z_i)$ as risk-adjusted profits evaluated at $(x^*(z_i), z_i)$ and $\Pi_{xx}(z_i)$ as the function's second derivative in x evaluated at the same point. The latter measures the *state-dependent cost* of misoptimizations relative to $x^*(z_i)$ and

will be central to our analysis. The objective of the intermediate goods firm is, to the second order:

$$\tilde{\Pi}(x, z_i) := \bar{\Pi}(z_i) + \frac{1}{2}\Pi_{xx}(z_i)(x - x^*(z_i))^2 \tag{10}$$

The first-order term of this approximation drops out due to the envelope theorem: there are no first-order costs of deviating from $x^*(z_i)$. So that this approximate payoff is globally defined, we will also apply the simplifying assumption that $\mathcal{X} = \mathbb{R}$.

Next, we approximate the final goods firm's production function (2) around the *ex post* optimal production levels $x^*(z_i)$ to the second order. We first define the aggregate of the *ex post* optimal production levels as $X^* = \left(\int_0^1 x^*(z_i)^{1-\frac{1}{\epsilon}}\, \mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}}$. We then write the approximate production function as

$$X = X^* - \frac{1}{2\epsilon}\int_0^1 \frac{(x_i - x^*(z_i))^2}{(X^*)^{-\frac{1}{\epsilon}}(x^*(z_i))^{1+\frac{1}{\epsilon}}}\, \mathrm{d}i \tag{11}$$

We now define a rational expectations equilibrium, up to these approximations, in which all agents optimize, all markets clear, and expectations are consistent. It is convenient for us to state this definition in the following, compact way in terms of the stochastic choices of intermediate goods firms p and the transition density f:

Definition 1 (Equilibrium). *An equilibrium is a stochastic choice rule $p \in \mathcal{P}$ and a transition density $f \in \mathcal{F}$ such that:*

1. *Intermediate goods firms' stochastic choice rules p solve program (9) given f, with $\tilde{\Pi}$, defined in (10), replacing Π.*
2. *The transition density f is consistent with p in the sense that: the marginal distribution of firm-level productivity is given by G; aggregate output is given by the aggregator (11) evaluated in the cross-sectional distribution of production implied by p and G; and the wage is derived from the wage rule (3) evaluated at aggregate output.*

Part 1 explicitly requires that intermediate goods firms optimize; but, in this construction, it is also automatic that all markets clear and all other agents (households and final goods firms) optimize. Part 2 of this definition embeds rational expectations by requiring that firms' subjective prior about endogenous variables is correct.

3   Attention Cycles and Their Consequences

We now present our main theoretical results, in four parts. First, we characterize firms' production and attention choices in partial equilibrium. State-dependent attention is shaped by two critical mechanisms, a *profit-curvature channel* related to the dollar cost of misoptimization and a *risk-pricing channel* related to the utility translation of these costs. Second, we derive two primitive conditions on the substitutability of goods, the rigidity of real wages, and risk aversion under which attention is counter-cyclical in general equilibrium. We argue these conditions are *ex ante* reasonable based on existing macro-financial evidence. Third, we characterize equilibrium output in the economy as the

product of output in the full-attention, frictionless benchmark and an attention wedge that depresses output and widens when productivity is high. This attention wedge reflects the cyclical nature of misallocation in the economy: when productivity is lower, firms optimize more precisely and productivity is closer to that in the frictionless benchmark. Finally, we show how endogenous variation in misallocation drives asymmetric and state-dependent shock propagation and endogenous stochastic volatility of output growth.

We conclude the section by summarizing six key predictions of the theory at the micro and macro levels. These will form the basis of our empirical tests of the model in Sections 4 and 5 and our strategy for calibrating the model in Section 6.

3.1 Attention and Misoptimization in Partial Equilibrium

We begin by describing the firms' choices in partial equilibrium:

Proposition 1 (Firms' Optimal Stochastic Choice Rules)**.** *The random production of a type-λ_i firm, conditional on realized decision state $z_i = (\theta_i, X, w)$, can be written as*

$$x_i = x^*(z_i) + \sqrt{\frac{\lambda_i}{|\pi_{xx}(z_i)| \, M(z_i)}} \cdot v_i \tag{12}$$

where $x^*(z_i)$ is the unconstrained optimal action, $|\pi_{xx}(z_i)|$ is the magnitude of curvature for the firms' profit function, $M(z_i)$ is the firm owner's marginal utility, and v_i is an idiosyncratic, standard Normal random variable.

Proof. See Appendix A.1. $\qquad \square$

Mathematically, this result follows from three observations. First, entropy costs (Equation 6) make the firm's problem *linearly separable* across state realizations z_{it}.[5] This implies that firms' optimal policies are independent of the prior distribution f. Second, the firms' payoff functions are (up to approximation) quadratic in the action conditional on each state. Third, under entropy costs and quadratic payoffs, the optimal non-parametric action distribution is Gaussian with the properties identified in Proposition 1. While superficially similar to existing results in quadratic-Gaussian rational inattention models (see, e.g., Sims, 2003), this *analytical* characterization has two important differences from all existing results with unrestricted information acquisition of which we are aware. First, the cross-sectional action distribution is a normal-mixture distribution within each aggregate state. Second, and more importantly for our analysis, the variance of the cross-sectional action distribution depends on the exogenous and endogenous aggregate states.

The Profit-Curvature and Risk-Pricing Channels. Economically, Proposition 1 says that firms center their action around the full-attention optimum $x^*(z_i)$ but, due to costly control, make an idiosyncratic misoptimization. The variance of the misoptimization increases if the marginal cost of precision increases (higher λ_i), and decreases if either of two components of the marginal benefits

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[5]This property is shared by any state-separable cost, as defined by Flynn and Sastry (2023), but not by any known model of costly information acquisition, like the mutual information model (Sims, 2003).

of precision increases. In this way, two channels of incentives determine agents' apparent bounded rationality, or failure to perfectly optimize from an *ex post* perspective.

The first component of this marginal benefit is the state-dependent curvature of the firms' dollar profit function, $|\pi_{xx}(z_i)|$, which translates small misoptimizations into their dollar cost near the optimal production level. How $|\pi_{xx}(z_i)|$ moves as a function of the aggregate business cycle depends on the demand and cost curves that firms face. We will refer to the effect of these incentives on misoptimization as the *profit-curvature channel*.

The second component of this marginal benefit is the firm owner's marginal utility, $M(z_i)$. This translates dollar losses into utility losses, which can be directly compared to the utility cost of cognition. As every firm is owned by the representative household in our model, the relevant marginal utility can be written only as a function of output X (i.e., aggregate consumption). When this household is risk-averse, this marginal utility is a decreasing function of output. Thus, the representative household is "hungrier" for any firm's dollar profits, in utility terms, when the aggregate economy is doing poorly. Because of this, they are less tolerant of misoptimizations when output is low. We will refer to the effect of these incentives on misoptimization as the *risk-pricing channel*.

Comparative Statics: What Drives Attention and Misoptimization? The previous argument used only the structure of the cost functional and the assumption of a "Neoclassical firm" owned by a representative household. We can use the specific structure of our model to re-state the comparative statics in the discussion above in terms of firms' productivity θ_i and aggregate output X, after imposing the stochastic discount factor, demand curves, and cost curves that our model implies. In particular, the curvature terms of interest can be written as:[6]

$$|\pi_{xx}(z_i)| := v_\pi(\epsilon, \chi, \bar{X}, \bar{w}) \cdot \theta_i^{-1-\epsilon} X^{\chi(1+\epsilon)-1} \qquad M(z_i) = X^{-\gamma} \qquad (13)$$

where $v_\pi(\epsilon, \chi, \bar{X}, \bar{w}) > 0$. We observe also that the variance of production conditional on the realized decision state, or $\mathbb{E}[(x_i - x^*(z_i))^2 \mid z_i]$, is a summary statistic for both misoptimization and "attention" measured by realized cognitive costs, which are decreasing in this variance. We summarize the comparative statics of this conditional variance under the assumed payoff structure in the following result:

Corollary 1 (Comparative Statics for Mistakes)**.** *Consider a type-λ_i firm in state $z_i = (\theta_i, X, w(X))$. The extent of misoptimization, $\mathbb{E}[(x_i - x^*(z_i))^2 \mid z_i]$, increases in θ_i. Moreover, $\mathbb{E}[(x_i - x^*(z_i))^2 \mid z_i]$ strictly increases in X if and only if $\gamma > \chi(1 + \epsilon) - 1$.*

Proof. Immediate from combining Proposition 1 with Equation 13. □

The (absolute) curvature of the profit function always increases in marginal costs, and therefore decreases in θ_i. The monotonicity of curvature in aggregate output depends jointly on the cyclicality of wages, which contributes a term with exponent $\chi(1 + \epsilon)$; the aggregate demand externality, which contributes a term with exponent -1; and marginal utility, which contributes an exponent $-\gamma$. In

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[6]The first expression and the associated constant are derived in Appendix A.7.

particular, an economy with sufficiently cyclical wages would have misoptimization decrease in aggregate output due to the profit-curvature channel, while an economy with sufficient risk aversion or sufficiently cyclical marginal utility would have misoptimization increase in aggregate output due to the risk-pricing channel. We will discuss the interpretation of this parameter condition "horse race" in the context of our general-equilibrium result in the next subsection.

3.2 Attention and Misoptimization Cycles in Equilibrium

We now translate the partial equilibrium behavior of the firm into general equilibrium. We first provide conditions under which equilibrium analysis is well-posed and output is a uniquely determined, monotone function of the underlying productivity state:

Proposition 2 (Existence, Uniqueness, and Monotonicity). *For any $\chi > 0$, an equilibrium in the sense of Definition 1 exists. If $\chi\epsilon < 1$ and $\gamma > \chi+1$, there is a unique equilibrium. In this equilibrium, output depends on the productivity distribution G only through the sufficient statistic $\hat{\theta}(G) \in \Theta$ (Equation 5) and can be expressed via a function $X : \Theta \to \mathbb{R}$ that is strictly positive and strictly increasing in that sufficient statistic.*

Proof. See Appendix A.2. □

To establish these properties, we derive a representation of equilibrium as a fixed point for aggregate output X as a function of the aggregate sufficient statistic $\hat{\theta}(G)$. To establish uniqueness and monotonicity, we derive conditions under which the fixed-point equation is a contraction map that depends positively on $\hat{\theta}$. The condition $\chi\epsilon < 1$ ensures that firms' production plans are on average an increasing function of aggregate output, by bounding wage pressure relative to the aggregate demand externality. The condition $\gamma > \chi + 1$ bounds the variance of actions around this optimum and ensures that, even in the presence of endogenous dispersion, there is positive but bounded complementarity.

The latter condition $\gamma > \chi + 1$ is both conservative in the model, as it ensures uniqueness and monotonicity for any possible distribution of λ_i, and highly plausible in practice. The elasticity of detrended real wages to GDP in US data since 1987 is 0.095, and micro-level studies find similarly severe wage rigidity (Solon et al., 1994; Grigsby et al., 2021).[7] Moreover, to be consistent with the restrictions $\chi\epsilon < 1$ (upward-sloping best responses) and $\epsilon > 1$ (substitutable goods), χ cannot exceed one. The corresponding conditions $\gamma > 1.095$ or $\gamma > 2$ are likely both slack by an order of magnitude, given abundant evidence in financial economics about the high cyclicality of the stochastic discount factor (e.g., Hansen and Jagannathan, 1991). As γ *only* governs the properties of the stochastic discount factor in our model, this is the relevant evidence for determining its appropriate value.

We now give conditions under which the economy exhibits aggregate misoptimization and attention cycles. We say that firms "misoptimize less" in a state if, averaging over idiosyncratic states (θ_i, λ_i), they have a lower expected mean-squared error around the *ex post* optimal action $x^*(z_i)$. We say that all firms "pay more attention" in a state if, averaging over idiosyncratic states (θ_i, λ_i), they pay a greater attention cost conditional on that state being realized. Formally:

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[7]This calculation uses quarterly-frequency, seasonally-adjusted data on real GDP and median, CPI-adjusted wages of all full-time employed wage and salary workers. Both series are linearly detrended.

Definition 2 (Aggregate Misoptimization and Attention). *Fix an equilibrium law of motion $X : \Theta \to \mathbb{R}$. Firms' aggregate misoptimization is their average mean-squared-error around the ex post optimal action, or*

$$m(G) := \mathbb{E}_{\theta_i, \lambda_i, x_i} \left[(x_i - x^*(z_i(\hat{\theta}(G))))^2 \right] \tag{14}$$

where $z_i(\theta) = (\theta_i, X(\theta), w(X(\theta)))$, $p^*(\cdot \mid z_i(\theta); \lambda_i)$ is the uniquely optimal plan of a type-λ_i firm contingent on realized state z_i, and the expectation is taken over $\theta_i \sim G$, $\lambda_i \sim L$, and $x_i \sim p^*(\cdot \mid z_i(\theta); \lambda_i)$. Firms' aggregate attention is their average realized cognitive cost

$$a(G) := \mathbb{E}_{\theta_i, \lambda_i, x_i} \left[\lambda_i \, p^*(x_i \mid z_i(\theta); \lambda_i) \log p^*(x_i \mid z_i(\theta); \lambda_i) \right] \tag{15}$$

We now show that, under the same assumptions as Proposition 2, the model features counter-cyclical attention and pro-cyclical misoptimization:

Proposition 3 (Attention Cycles). *Assume $\chi\epsilon < 1$ and $\gamma > \chi + 1$ and consider productivity distributions G and G'. If $\hat{\theta}(G) \geq \hat{\theta}(G')$, then aggregate output X and aggregate misoptimization m are larger under G. If additionally $G \succsim_{FOSD} G'$, then aggregate attention a is lower under G.*

Proof. See Appendix A.3. ☐

In the proof of the first part of this result, we verify that the stated conditions are sufficient for output X to be monotone increasing in $\hat{\theta}(G)$, via Proposition 2; observe that the same conditions are sufficient for misoptimization variance to be monotone increasing in θ_i and X, via Corollary 1; and translate these into predictions for cross-firm averages. In the second part of the result, we use a very similar argument to show that attention in payoff units is monotone decreasing under the stronger assumption that G and G' are rankable via first-order stochastic dominance.

Economically, Proposition 3 says that any calibration of the model that is consistent with existing evidence about wage rigidity and the stochastic discount factor predicts that firms should pay more attention to their decisions and make smaller misoptimizations, conditional on stochastic fundamentals, in downturns. Importantly, these conditions are *sufficient* but not *necessary*. For example, misoptimization would also be pro-cyclical (and attention counter-cyclical) if the (looser) condition in Corollary 1 held and if, by assumption, output were monotone increasing in productivity. We use this observation in Section 3.5 to study when attention cycles occur even with risk-neutral firm owners (or, equivalently, dollar-denominated control costs).

3.3 Misoptimization, Output, and Productivity

Having provided model conditions that generate attention and misoptimization cycles, we now study the effects of these phenomena on output and production. Despite unrestricted heterogeneity in the cross-sectional distributions of productivity and attention costs, there are scalar sufficient statistics in equilibrium for each distribution. As described earlier, the cross-sectional distribution of productivity is summarized by the sufficient statistic $\hat{\theta}(G)$. We therefore, without loss of generality, write $\theta_t = \hat{\theta}(G_t)$ for the remainder of the analysis. The cross-sectional distribution of attention costs is summarized by the mean $\lambda := \mathbb{E}_L[\lambda_i]$.

We define $\log X(\log\theta)$ as a mapping from the log productivity sufficient statistic to log output in the economy, holding fixed all other parameters. The following result describes output in log units as the sum of that which would be obtained absent inattention and an *attention wedge* $\log W(\log\theta)$:

Proposition 4 (Equilibrium Output Characterization). *Equilibrium output follows:*

$$\log X(\log\theta) = X_0 + \chi^{-1}\log\theta + \log W(\log\theta) \tag{16}$$

where $\theta = \hat{\theta}(G)$ is a sufficient statistic for the productivity distribution, X_0 is a constant, and $\log W(\log\theta) \leq 0$, with equality if and only if $\lambda = 0$. When $\chi\epsilon < 1$, $\gamma > \chi + 1$, and $\lambda > 0$, the wedge has the following properties:

1. $\frac{\partial\log W}{\partial\lambda} < 0$, *or the wedge widens with the average cost of attention.*
2. $\frac{\partial\log W}{\partial\log\theta} < 0$, *or the wedge widens as the state increases.*

Proof. See Appendix A.4. □

Absent inattention, output is log-linear in aggregate productivity. With inattention and under our stated conditions from Propositions 2 and 3, output is depressed by the presence of mistakes. The magnitude of this force increases in the extent of cognitive costs λ and in aggregate productivity θ. Both results have a partial-equilibrium and general-equilibrium component. In partial equilibrium, both increasing λ and increasing θ make firms play more dispersed actions, as shown in Proposition 1, and this dispersion has a cost to output when the aggregate production function is concave. In general equilibrium, we iterate this logic until convergence; our comparative statics results verify that this fixed-point operation converges on a lower value of output.

To better understand this wedge, we can re-cast it in terms of labor productivity:

Corollary 2 (The Productivity Wedge). *Equilibrium labor productivity $A := \frac{X}{L}$ follows:*

$$\log A(\log\theta) = \log\theta + \chi\epsilon\log W(\log\theta) \tag{17}$$

Proof. See Appendix A.5. □

The productivity wedge representation allows for three useful parallels between our paper's mechanism and classic arguments in the macroeconomics literature. The first relates to a literature on the cleansing effect of recessions following Caballero and Hammour (1994). Our mechanism is like an attentional, *intensive margin* version of the same effect: conditional on a given firm operating, it is more focused on making precise choices in recessions, and this on average reduces the wedge and raises aggregate labor productivity.

The second relates to an empirical literature studying the dynamics of labor productivity in the US, and in particular highlighting its unstable and often negative cyclicality (e.g., Galí and Gambetti, 2009; Barnichon, 2010; Galí and Van Rens, 2021). Our model accommodates a non-monotone relationship between aggregate labor productivity and aggregate output, due to the competing forces of increased

productivity with increased misallocation. Our quantitative analysis of Section 6 will generate such a non-monotone relationship.

The third relates to the literature on the aggregate effects of resource misallocation across firms, pioneered by Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), and related to informational and cognitive frictions by David et al. (2016), David and Venkateswaran (2019), Ma et al. (2020), and Barrero (2022). The mechanism whereby dispersion in firm-level value marginal products depresses aggregate productivity is shared with these analyses. What is new is our prediction about the cyclicality of this force, driven by changing incentives for (in)attention, and the implications of that cyclicality for business cycles.

3.4 Shock Propagation and Volatility

The fact that agents are differentially attentive to shocks across states of the world makes the economy differentially sensitive to shocks. This observation is formalized in the following Corollary, which describes the aggregate output dynamics predicted by the model in an illustrative case in which the productivity aggregate follows a homoskedastic AR(1) process. The result shows how the model generates endogenous stochastic volatility, state-dependent propagation of shocks, and asymmetric shock propagation, features which are all absent in the benchmark with fully attentive firms or $\lambda = 0$:

Corollary 3 (Endogenous Stochastic Volatility, State-Dependent Propagation, and Asymmetric Propagation). *Suppose that aggregate productivity follows the process* $\log \theta_t = \rho_\theta \log \theta_{t-1} + (1 - \rho_\theta) \log \bar{\theta} + \nu_t$ *where* $Var[\nu_t] = \sigma_\theta^2$. *The model generates the following properties:*

1. *Endogenous stochastic volatility: to a first-order approximation in* ν_t, *the variance of output conditional on last period's productivity* θ_{t-1} *is given by*

$$\mathbb{V}[\log X_t | \theta_{t-1}] = \left(\chi^{-1} + \left. \frac{\partial \log W(\log \theta)}{\partial \log \theta} \right|_{\theta = \theta_{t-1}} \right)^2 \sigma_\theta^2 \tag{18}$$

2. *State-dependent shock propagation: the impact on output from a small shock* ν_t *in state* θ_{t-1} *is given by:*

$$\left. \frac{\partial \log X(\log \theta)}{\partial \log \theta} \right|_{\theta = \theta_{t-1}} = \chi^{-1} + \left. \frac{\partial \log W(\log \theta)}{\partial \log \theta} \right|_{\theta = \theta_{t-1}} \tag{19}$$

3. *Asymmetric shock propagation: the impact of a shock to second-order in* ν_t *in state* θ_{t-1} *is given by*

$$\left. \frac{\partial \log X(\log \theta)}{\partial \log \theta} \right|_{\theta = \theta_{t-1}} = \chi^{-1} + \left. \frac{\partial \log W(\log \theta)}{\partial \log \theta} \right|_{\theta = \theta_{t-1}} + \left(\left. \frac{\partial^2 \log W(\log \theta)}{\partial (\log \theta)^2} \right|_{\theta = \theta_{t-1}} \right) \nu_t \tag{20}$$

where the sign of the wedge's second derivative determines the direction of asymmetry.

Proof. See Appendix A.6. □

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To understand this result, note that the sensitivity of the economy to shocks is simply the sum of the frictionless economy's response to shocks, which is χ^{-1}, and the response of the attention wedge to shocks. This latter response is negative in the case studied by Proposition 4, so the economy is less responsive to shocks than the full attention benchmark. This dampened response is a familiar prediction in the literature with cognitively constrained agents (e.g., Sims, 1998, 2003; Gabaix, 2014). Here, the result arises not from individual-level underreaction but instead from the interaction of individual *mistakes* with concave aggregation. Moreover, this dampening from the attention wedge can depend on the state. This is a direct consequence of modeling state-dependent attention and accommodating attention cycles. This fact generates the asymmetry, state-dependence, and stochastic volatility indicated in Corollary 3.

The ultimate macroeconomic implications of this result hinge on the concavity or convexity of the attention wedge in the state. When the attention wedge is concave (respectively, convex), the economy generates greater (smaller) volatility in low states, a larger (smaller) impact of shocks in low states, and features larger (smaller) impact of negative versus positive shocks from any initial state. Owing to the intuition that when θ is very small, it is as if the economy is operating in its "full pass-through" RBC core, the natural case appears to be a concave wedge. While we cannot establish theoretically that the wedge is globally concave, it cannot be globally convex and therefore must be concave in at least some part of the parameter space. Our quantitative analysis in Section 6 will feature a concave wedge. In that section we will review the implications of this finding.

3.5 Extensions

Decision Costs in Dollar Units. Motivated by the nature of cognitive costs, we denominated the cost of precise planning in utility units. An alternative choice is to specify these costs in units of final output or numeraire "dollars." This is natural if overcoming decision frictions requires investing in inputs, like employees tasked with planning. Since relative risk aversion enters our analysis only in translating dollar costs to utility costs, the assumption of dollar-denominated cognitive costs is nested by setting $\gamma = 0$. Proposition 1 and Corollary 1, concerning misoptimization in partial equilibrium hold as written. The latter's condition for misoptimization to strictly increase in X is that $1 > \chi(1 + \epsilon)$. Under the assumption that output is monotone in productivity, the model can feature equilibrium attention cycles under the same condition. This demonstrates that the profit-curvature channel by itself can induce attention cycles if wages are sufficiently rigid, a condition likely to hold based on our earlier discussion.

However, our empirical and quantitative analysis will support a strong role for the risk-pricing channel. First, we will find weak evidence of differential sensitivity of dollar profits to misoptimizations over the business cycle (Section 5.3). Second, we will find strong procyclicality of misoptimization variance. Thus, to rationalize these facts, our quantitative model requires a strong risk-pricing channel ($\gamma = 11.5$), rejecting the dollar-cost model nested with $\gamma = 0$ (Section 6.1).

Wage-Rule Shocks. Our main analysis studies supply shocks that moved the productivity distribution. Consider a case also with stochastic $\bar{w}$, the level term in the wage rule. Shocks to $\bar{w}$ can be interpreted as a *labor wedge shock* or *demand shock* with the appropriate micro-foundation. Inspection

of the formulae in Proposition 1 reveals that $\bar{w}$ is equivalent to an aggregate shifter of firms' revenue productivity. Propositions 2 and 3 provide conditions for output to be monotone *decreasing* in $\bar{w}$ and for counter-cyclical attention and pro-cyclical misoptimization in an economy driven by both supply shocks (to the productivity distribution) and demand shocks (to the labor wedge). Proposition 4 and Corollary 3 hold as written, with $\theta\bar{w}^{-1}$ replacing θ. Thus, our core conclusions about attention cycles and their effects on business cycles hold in a model with both supply and demand shocks.

Costly Information Acquisition. As referenced earlier, our model of costly control and non-parametric choice differs from models of unrestricted costly information acquisition in several respects. Costly control allows us to tractably study several partial and general equilibrium phenomena. Nonetheless, we sacrifice the model's ability to capture individual-level underreaction (while we do generate aggregate underreaction). Motivated by this, we study the robustness of our results to information-acquisition costs by examining our model with Gaussian signal extraction with costly precision (Appendix D.2) and mutual-information costs as studied by Sims (2003) (Appendix D.3). In both cases we provide conditions under which a greater cost of making mistakes from either the risk-pricing or profit-curvature channel leads to firms making smaller mistakes. We do, however, note that application of unrestricted rational inattention to the model we study is not analytically possible using any state-of-the-art techniques (e.g., Afrouzi and Yang, 2021; Miao et al., 2022) as our firms' have non-Gaussian priors and non-quadratic payoff functions, and aggregation is non-linear.

Multiple Inputs and Classical Labor Supply. In Appendix B, we extend the model to allow for intermediate inputs, separate capital owners and laborers, and market-clearing wages rather than a wage rule. The first two features will be useful in mapping the model to the data in Section 5, while the third enables a more Neoclassical micro-foundation. We show under general conditions how the main results derived in this section, regarding the cyclicality of attention and misoptimization and the effects on output, continue to hold so long as the extent of the cognitive friction is not too large. Together, these extensions demonstrate the stability of our main model insights to a richer macroeconomic environment.

3.6 Microeconomic and Macroeconomic Predictions

We now distill our theoretical findings into six testable predictions, which apply at both the microeconomic and the macroeconomic levels. These predictions will motivate our measurement strategy in Section 4 and our empirical analysis in Section 5.

The first is that firms make costly misoptimizations:

Prediction 1. *Firms make misoptimizations (under- and over-production relative to the ex post optimal level) that reduce profits.*

In our model, this prediction is a consequence of costly cognition (Proposition 1). Because the prediction concerns observable variables, we can take it directly to the data. This prediction is *inconsistent* with models in which deviations from optimal production arise from wedges in the sense of Chari et al. (2007) or Restuccia and Rogerson (2008), as under-production would be associated with lost profits

but over-production would be associated with increased profits. Thus, this prediction directly tests our misoptimization-based interpretation of the data.

The second prediction concerns how misoptimizations move over the cycle:

Prediction 2. *Firms make larger misoptimizations in booms and smaller misoptimizations in downturns.*

This prediction arises from firms' counter-cyclical incentives to rein in mistakes, given assumptions on risk aversion, wage rigidity, and substitutability that we argued are natural (Proposition 3). A similar prediction does *not* obtain under alternative, non-cognitive theories of stochastic wedges—for example, the natural prediction with financial frictions is that their intensity and cross-firm heterogeneity increases, rather than decreases, in downturns. We will revisit this comparison to other models in the empirical analysis.

The third prediction identifies the mechanism underlying attention cycles:

Prediction 3. *Firm valuations are more sensitive to misoptimizations in downturns than in booms.*

In our costly attention model, firms rein in misoptimizations exactly when it is most valuable to do so. This microeconomic prediction for the incentives that arise from macroeconomic conditions lies at the core of our model's mechanism for endogenous, pro-cyclical misoptimization.

The fourth prediction zeros in on the distinction between the profit-curvature and risk-pricing channels in driving Prediction 3:

Prediction 4. *Firm profits, in dollars, may be more or less sensitive to misoptimizations in downturns versus booms.*

In the language of Section 3.1, Prediction 3 signs the *the total effect of the risk-pricing and profit-curvature channels* while Prediction 4 says that the *profit-curvature channel in isolation* has an ambiguous sign. Therefore, insofar as we confront the model with data on both financial valuation (e.g., stock prices) and profitability, we can distinguish the relative strength of these mechanisms.

The fifth prediction concerns the cyclicality of aggregate attention:

Prediction 5. *Firms pay more attention to decisions in downturns and less in booms.*

This prediction, like Prediction 2, derives from the arguments in Proposition 3. But, since cognitive effort is by definition unobserved, it will be more challenging to directly test than its equivalent for misoptimization. In the next section, we will describe a method to measure a proxy for firms' attention using textual analysis.

The sixth prediction concerns the relationship between misoptimization and attention:

Prediction 6. *Firms that pay more attention to decisions make smaller misoptimizations.*

This prediction allows us to microeconomically validate the decision-relevance of measured attention and to further validate the attention interpretation of measured misoptimization.

4   Measuring Misoptimization and Attention

Toward testing the model's main microeconomic and macroeconomic predictions, we introduce new empirical measures of firms' "misoptimizations" and "attention." Measuring a misoptimization requires a counterfactual of what a firm *should* have done as opposed to what it did in fact do. Our model provides a structural method by which we can recover both firms' underlying unconstrained-optimal policy functions and their mistakes. From an econometric perspective, this requires only a standard estimation of firms' productivity and policy functions. We proxy for attention, which is of course unobservable, by constructing a text-based measure: firms' discussion of macroeconomic topics in regulatory filings and earnings conference calls.

4.1   Measuring Misoptimizations

Data.   Our dataset for public firms' production and input choices is Compustat Annual Fundamentals. Compustat compiles information from firms' financial statements on sales, employment, variable input expenses, and capital measured via net and gross values of plants, property and equipment (PPE). It also provides a historically consistent industry classification, based on firms' main operational NAICS codes. We organize firms into 44 industries, which are defined at the NAICS 2-digit level but for Manufacturing (31-33) and Information (51), which we split into the 3-digit level. We drop financial firms and utilities due to their markedly different production functions and profit structures.

We use a sample period from 1986 to 2018, to focus on the modern macroeconomic regime after the Volcker disinflation. We view this relative stability in the macroeconomic environment as important for measuring externally valid patterns; nonetheless, we also consider as an extension a strategy that applies to the entire Compustat sample since 1950. We apply standard filters to remove firms that are based outside the US, are insufficiently large, are likely to have been involved in a merger or acquisition, or do not report one or more data points. These filters yield a final sample of 68,198 firm-year observations, or about 2,200 per year. Appendix C.1 describes the procedure in detail.

From Theory to Estimation.   We start by translating our characterization of firms' optimal stochastic choice (Proposition 1) into an estimable structural relationship. Letting $\sigma(\theta_{it}, X_t, w_t) = \sqrt{\frac{\lambda_i}{|\pi_{xx}(\theta_{it}, X_t, w_t)| M(X_t)}}$ denote the firms' optimally chosen volatility of mistakes and v_{it} denote idiosyncratic, unit variance, and zero-mean firm trembles, we observe that Proposition 1 implies:

$$
\begin{aligned}
\log L_{it} &= \log L^*(\theta_{it}, X_t, w_t) + \log\left(1 + \frac{\sigma(\theta_{it}, X_t, w_t)}{L^*(\theta_{it}, X_t, w_t)} v_{it}\right) \\
&= (\epsilon - 1)\log\theta_{it} - \epsilon \log w_t + \log X_t + \log\left(1 + \frac{\sigma(\theta_{it}, X_t, w_t)}{L^*(\theta_{it}, X_t, w_t)} v_{it}\right) \\
&\approx (\epsilon - 1)\log\theta_{it} - \epsilon \log w_t + \log X_t + \underbrace{\frac{\sigma(\theta_{it}, X_t, w_t)}{L^*(\theta_{it}, X_t, w_t)} v_{it}}_{m_{it}}
\end{aligned}
\tag{21}
$$

where the first line uses Proposition 1 (expressed in terms of labor choice, and in logs); the second line uses the expression for $L^*(\theta_{it}, X_t, w_t) = \frac{x^*(\theta_{it}, X_t, w_t)}{\theta_{it}}$ in our model; and the third approximates

$\log(1 + x) \approx x$ for small x and defines the residual m_{it}. In Appendix C.3, we show that this policy function is also obtained in a generalization of our baseline model that features multiple flexible inputs. In words, Equation 21 says that misoptimizations can be measured as residual variation in firms' choices conditional on productivity, factor prices, and demand. This is the basic blueprint for our measurement strategy.

In practice, to guard against the potential for misspecification of this relationship, we estimate a more flexible model of the following form

$$\log L_{it} = \eta_i + \chi_{j(i),t} + \beta \log \theta_{it} + m_{it} \tag{22}$$

where $j(i)$ is the industry of firm i, β is an unrestricted coefficient, η_i and $\chi_{j(i),t}$ are respectively fixed effects at the firm- and industry-by-time levels, and m_{it} follows an AR(1) process with persistence $\rho \in (0, 1)$ and innovation variance $(1 - \rho^2)\tilde{\sigma}^2(\theta_{it}, X_t, w_t)$:

$$m_{it} = \rho m_{i,t-1} + \left(\sqrt{1 - \rho^2}\right) u_{it} \tag{23}$$

where u_{it} is a zero-mean innovation with variance $\tilde{\sigma}^2(\theta_{it}, X_t, w_t)$. If $\tilde{\sigma}^2(\theta_{it}, X_t, w_t)$ is sufficiently persistent, then it is approximately the variance of m_{it}. Our baseline model is nested when $\eta_i = 0$, $\chi_{j(i),t} = -\epsilon \log w_t + \log X_t$ for all i and $j(i)$, $\beta = \epsilon - 1$, and $\rho = 0$. In the Appendix, we formally describe how the more general model nests two economically important features from which we abstracted in the theoretical analysis for simplicity. The first is time-varying differences in demand and factor prices across industries and time-invariant differences across firms, respectively captured in the industry-by-time and firm fixed effects of Equation 22 (Appendix C.3). The second is persistent mistakes, which can arise in a variant model which accommodates cognitive inertia (Appendix D.1).

Estimation Procedure. To empirically estimate our model, we proceed to estimate productivity and firm policy functions using standard techniques. First, we estimate productivity using a cost-shares approach, as in Foster et al. (2001), Foster et al. (2008), Bloom et al. (2018), and Ilut et al. (2018). We describe the full details in Appendix C and summarize the main points below. We define productivity $\log \theta_{it}$ as the residual of an industry-specific, constant-returns-to-scale, Cobb-Douglas production function over labor L_{it}, materials M_{it}, and capital K_{it}:

$$\log x_{it} = \log \theta_{it} + \alpha_{L,j(i)} \log L_{it} + \alpha_{M,j(i)} \log M_{it} + \alpha_{K,j(i)} \log K_{it} \tag{24}$$

where $\alpha_{L,j(i)} + \alpha_{M,j(i)} + \alpha_{K,j(i)} = 1$.[8] We measure labor expenditures as the product of reported employees and a sector-specific wage calculated from the US County Business Patterns; materials expenditures as the sum of variable costs and administrative expenses net of depreciation and labor expenditures; and the capital stock as the initial gross level of plant, property, and equipment plus net investment. We use industry-specific ratios of labor and materials expenditures to total sales to

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[8]The Cobb-Douglas assumption is a convenient and common step to enable production function estimation via input-cost shares (e.g., Foster et al., 2001, 2008; Bloom et al., 2018). Moreover, a number of studies including Basu and Fernald (1997), Foster et al. (2008), and Flynn et al. (2019) argue that constant returns to scale in physical terms is a reasonable approximation for large, US-based firms.

estimate the *revenue* elasticities of labor and materials, and translate these into *output* elasticities (i.e., $\alpha_{L,j(i)}$ and $\alpha_{M,j(i)}$) given an assumption for the elasticity of demand of $\epsilon = 4$. This assumption equivalently imposes an average profit share of 0.75 or an average markup of 4/3. We measure the output elasticity of capital as $1 - \alpha_{L,j(i)} - \alpha_{M,j(i)}$, using constant returns to scale. We combine these estimates with our production functions and demand curves, and partial out industry-by-time fixed effects to capture factor price and demand variation, to recover an estimate for firm-level productivity $\log \theta_{it}$. We finally observe that the remaining variation in $\log \theta_{it}$ *could* blend both firm-level productivity and demand shocks, as observed by Foster et al. (2008) for any context without access to separate data on both prices and quantities. But since both types of shocks behave identically in our mapping from theory to data, this poses no issue for the interpretation of our estimated policy functions or misoptimizations.

Next, we estimate firm policy functions as linear regressions, as in Ilut et al. (2018) or Decker et al. (2020). In our case, this involves estimating the system of Equations 22 and 23. We first estimate Equation 22 via ordinary least squares (OLS) and obtain a preliminary estimate $\hat{m}_{it}^0$ of the residual. We next estimate Equation 23 using $\hat{m}_{it}^0$ to obtain an estimate $\hat{\rho}$ of the residual persistence (in our main procedure, 0.70). We next estimate via OLS the "quasi-differenced" equation for labor choice:[9]

$$\log L_{it} - \hat{\rho} \log L_{i,t-1} = \eta_i + \chi_{j(i),t} + \beta_0 \log \hat{\theta}_{it} + \beta_1 \log \hat{\theta}_{i,t-1} + \nu_{it} \tag{25}$$

with residual $\nu_{it} = m_{it} - \hat{\rho} m_{i,t-1}$. When $\hat{\rho} = \rho$, using Equation 23, $\nu_{it} = \left(\sqrt{1-\rho^2} \right) u_{it}$. We therefore obtain an estimate of u_{it} as $\hat{u}_{it} = \frac{\hat{\nu}_{it}}{\sqrt{1-\hat{\rho}^2}}$.

We finally define aggregate "Misoptimization Dispersion" as an estimate of the cross-sectional variance of m_{it} with weights s_{it}^* proportional to firms' *predicted* sales based on fundamentals:[10]

$$\text{MisoptimizationDispersion}_t = \frac{\sum_{i \in \mathcal{I}_t} s_{it}^* \cdot \hat{u}_{it}^2}{\sum_{i \in \mathcal{I}_t} s_{it}^*} \tag{26}$$

The weights are the appropriate ones for mapping average misoptimization to misalloation in the theory and will aid in our subsequent structural interpretation of our findings. In a nutshell, these weights give higher influence to larger, more productive firms while not "double-counting" misoptimizations in both input choice and total production. In Appendix A.7.3, we show that pro-cyclical MisoptimizationDispersion by our empirical definition is sufficient for pro-cyclical misoptimization as defined in Definition 2, and thus constitutes a more stringent test of that model prediction.

Robustness to Alternative Methods. Of course, this procedure can only be interpreted as recovering mistakes given the model that we have written. Any additional unmodeled, idiosyncratic firm-level wedges à la Restuccia and Rogerson (2008) would also be captured as a mistake.[11] How-

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[9]Appendix Table A8 contains our estimates of Equations 23 and 25 under the baseline procedure outlined in this section, along with several alternative choices used in robustness checks. In all estimations, we drop the top and bottom 1% tails of the TFP distribution to limit the effects of outliers. Results are quantitatively similar without this trimming.

[10]In particular, the weights are the exponentiated fitted values $\exp(\hat{\beta} \log \hat{\theta}_{it})$ from the following regression equation: $\log \text{Sales}_{it} = \beta \log \hat{\theta}_{it} + \eta_i + \chi_{j(i),t} + \epsilon_{it}$.

[11]Time-varying wedges at the industry or aggregate level would be absorbed in our model's fixed effect, and firm-level correlated distortions that are driven by productivity (Restuccia and Rogerson, 2008) would be included in our

ever, Prediction 1 allows us to distinguish idiosyncratic wedges from misoptimizations: residuals from wedges should have a uniformly positive effect on firms' profitability and stock returns. By contrast, residuals from misoptimization should have a hump-shaped effect on firm performance. This notwithstanding, to explicitly account for other misspecifications, we employ a large number of alternative strategies.

First, we construct measures that are robust to the presence of misspecification arising from adjustment costs (e.g., as in Hopenhayn and Rogerson, 1993) and financial frictions (e.g., as in Ottonello and Winberry, 2020). To capture adjustment costs, we estimate a variant policy function that adds the first lag of labor choice to (22):

$$\log L_{it} = \eta_i + \chi_{j(i),t} + \beta \log \theta_{it} + \tau \log L_{i,t-1} + m_{it} \tag{27}$$

If Equation 27 were the true model of input choice, and we had instead estimated our main model Equation 22, then variation in $\tau \log L_{i,t-1}$ not spanned by current productivity (or the absorbed effects) would be part of the misoptimization m_{it}. Some of this variation would be truly spanned by previous productivity shocks and be misrepresented as a cognitive error, while the remainder of the variation would be a "bad control" spanned by persistent lagged cognitive errors. To capture financial frictions, we add a direct control for leverage (total debt over total assets), as constructed by Ottonello and Winberry (2020), and its interaction with TFP:

$$\log L_{it} = \eta_i + \chi_{j(i),t} + \beta \log \theta_{it} + \tau \mathrm{Lev}_{it} + \phi \cdot (\mathrm{Lev}_{it} \times \log \theta_{it}) + m_{it} \tag{28}$$

This specification allows for more levered firms to face, effectively, a "TFP adjustment" (direct effect) and have different responsiveness to TFP shocks (interactive effect). If Equation 28 were the true model of input choice, and we had instead estimated the main model Equation 22, then both the direct effect and changing responsiveness to TFP would be erroneously attributed to the cognitive friction.

Second, for robustness to the cost-shares-based productivity estimation strategy, we also estimate productivity using the method of Olley and Pakes (1996). We find that the two measures are very similar to one another, with a regression coefficient close to one and a R^2 of 0.98 after including sector-by-time and firm fixed effects (Table A13).

Finally, we perform our analysis under a host of further policy functions that (i) are industry-specific, to combat against different market conditions and/or measurement error in TFP; (ii) have time-varying coefficients on TFP, for instance to capture trends or cycles in business dynamism (Decker et al., 2020); or (iii) allow for TFP to affect the policy function non-linearly, to capture asymmetric hiring and firing rules as found by Ilut et al. (2018). We also consider time-varying production functions estimated with time-varying cost shares, to capture technological trends (e.g., automation) and, to first order, cyclical changes in input shares. And, in a final robustness check, we estimate production functions and policy functions in a pre-sample, while estimating misoptimization in a post-sample, to guard against mechanical over-fitting.

productivity estimates. Thus, neither would affect our estimated misoptimizations.

4.2 Measuring (Macroeconomic) Attention

We now describe our strategy for measuring a proxy for firms' attention. There are two key challenges to this. First, attention, unlike misoptimization, has no direct theoretical analog in observable firm choices (except, trivially, through misoptimization itself). We instead use data on how firms describe their decisions in words and treat these as a window into the contingencies for which firms plan. Second, our model predicts that attention toward *all* relevant factors increases in downturns. But, for the purpose of measurement, it is necessary to focus on factors that are commonly faced by all firms. We will therefore focus on measuring attention to *macroeconomic* conditions, which are obviously faced to some extent by all firms.

Data. Our main data source is the full text of the quarterly 10-Q and annual 10-K reports submitted by all US public firms to the Securities and Exchange Commission (SEC). We use data from 1995 to 2018 in our main analysis.[12] Our total sample consists of 479,403 individual documents, or about 5,000 per quarter, which we index by their date of filing.

We also apply our measurement technique to a supplemental data source of 150,000 quarterly sales and earnings conference calls from 2003 to 2014. We replicate all subsequent results on these data as well. We relegate some additional details about our data construction for the conference calls to Appendix F.1.

Methodology. The key challenge for identifying attention toward macroeconomic conditions is to differentiate characteristic language of macroeconomics from the intrinsically economic and financial vocabulary of standard firm activities (e.g., "credit" and "costs"). To address this, we apply a simple natural language processing technique that identifies specific documents as "attentive to the macroeconomy" if their word choice is both different from the standard word choice in regulatory filings and similar to the word choice of macroeconomics-focused references. Following the method introduced by Hassan et al. (2019) to study firm discussion of political risks, we use introductory college-level textbooks as those references: *Macroeconomics* and *Principles of Macroeconomics* by N. Gregory Mankiw and *Macroeconomics: Principles and Policy* by William J. Baumol and Alan S. Blinder.[13] This choice balances our considerations of keeping the relevant macroeconomic vocabulary mostly non-technical (e.g., "unemployment" instead of "tightness"), but still specific (e.g., "inflation" instead of "price").

To operationalize this method, we first define tf$(w)_{it}$ as the *term frequency* for a word w in the filing of firm i at time t, measured as the proportion of total English-language words; and df(w) as the *document frequency* of a given word w among all observed regulatory filings, measured as a proportion of total documents that use the word at least once. We define the "term frequency inverse document frequency," or tf-idf, as:

$$\text{tf-idf}(w)_{it} := \text{tf}(w)_{it} \cdot \log\left(\frac{1}{\text{df}(w)}\right) \tag{29}$$

The log functional form is a heuristic in natural language processing for scaling the relative importance

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[12]The relevant digitized documents are hosted by the Security and Exchange Commission's EDGAR (Electronic Data Gathering, Analysis, and Retrieval), which began operation in 1994. We choose 1995 as a starting point at which a nearly comprehensive sample of firms' reports are available in the system.

[13]We use electronic copies of the 7th, 3rd, and 12th editions of these books, respectively.

of each term. It is bounded below by 0 when a word appears in all documents and smoothly penalizes words that appear in more documents and are therefore less indicative of the topic of interest.

For each word that appears in the 10-Q and 10-K corpus, we calculate the tf-idf using term frequencies in each of the three textbooks and (inverse) document frequencies among regulatory filings. We rank the top 200 words by this metric in each textbook and take the intersection among the three books to obtain a final set of 89 words.[14] Appendix Figure A5 prints these words in alphabetical order, and plots their time-series frequency. Many of the words relate to common macro indicators ("unemployment","inflation"); some to the topic or profession itself ("macroeconomics","economist"); and some to policy ("Fed," "multiplier"). There are also "false positive" words that are related to pedagogy, like "question" and "equation." To allow the method to be fully devoid of direct researcher manipulation, we do not remove such words from the main analysis. We then use our set of macroeconomic words, denoted by $\mathcal{W}_M$, to calculate our firm-by-time measures of attention as the sum of the (idf-weighted) macroeconomic word frequency:

$$\text{MacroAttention}_{it} = \sum_{w \in \mathcal{W}_M} \text{tf-idf}(w)_{it} \tag{30}$$

We generate an aggregate measure MacroAttention$_t$ by averaging MacroAttention$_{it}$ across firms. In our aggregate results, we remove seasonal trends as quarter-of-the-year means.

5   Testing the Model's Predictions

We now test the model's six main predictions (Predictions 1-6) and report six corresponding findings. Together, these findings are consistent with our macroeconomic predictions and proposed microeconomic mechanism of incentives-driven attention.

5.1   Fact 1: Mistakes in Both Directions Reduce Returns and Profitability

We first test that our measured "misoptimizations" are, as predicted by the theory, bad for firm performance (Prediction 1). We proxy for firm performance with two measures. The first is firms' log stock returns, R_{it}. The second is firms' "profitability" π_{it}, or earnings before interest and taxes (EBIT) divided by the last year's total variable costs.[15] We examine the non-parametric binned scatter relationship of each of these measures $X_{it} \in \{R_{it}, \pi_{it}\}$ with measured misoptimization (residuals) $\hat{u}_{it}$, net of industry-by-time fixed effects $\chi_{j(i),t}$:

$$X_{it} = f(\hat{u}_{it}) + \chi_{j(i),t} + \epsilon_{it} \tag{31}$$

where f is a piecewise-constant function defined over ventiles of $\hat{u}_{it}$.

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[14]Taking the intersection helps guard against the idiosyncratic language of certain books. For instance, in *Principles of Macroeconomics* by N. Gregory Mankiw, a parable about supply and demand for "ice cream" is used often enough to make "ice" and "cream" high tf-idf words in our procedure.

[15]Variable costs, by our definition, are cost of goods sold (COGS) plus administrative expenses (XSGA) net of depreciation (DP). Normalization by lagged costs, rather than current costs, limits mechanical denominator bias related to the current period's mistakes. Results are similar when normalizing by total sales or costs in the current or previous period.

Figure 1: The Negative Relationship Between Misoptimization and Firm Performance
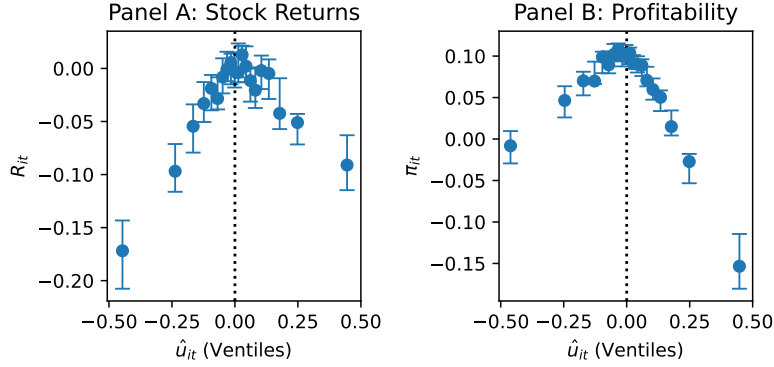
Notes: Both panels are binned scatterplots. The outcome variables are log stock returns (Panel A) and profitability (Panel B). Dots represent means of the corresponding outcome conditional on ventiles of the x-axis variable, $\hat{u}_{it}$, and industry-by-time fixed effects. Error bars are 95% confidence intervals based on the method of Cattaneo et al. (2019), with two-way clustering by firm and year. The construction of $\hat{u}_{it}$ is described in Section 4.1.

Misoptimizations in both directions (i.e., under- or over-hiring labor) hurt firm performance, measured in each way (Figure 1). This "hump-shaped" result is consistent with our interpretation of residuals as misoptimizations. It is inconsistent with the interpretation of residuals as wedges, which would predict a positive, monotone relationship, or pure noise, which would predict a flat relationship. In Appendix Table A1, we show that the result of a negative relationship between $\hat{u}_{it}^2$ and $\{R_{it}, \pi_{it}\}$ is highly statistically significant and robust to different levels of fixed effects.

The negative effects of misoptimizations on firm performance are also persistent. To measure the dynamic relationship between misoptimization and performance, we estimate projection regressions of productivity growth, profitability, and returns on $\hat{u}_{it}^2$, net of industry-by-time fixed effects:

$$X_{i,t+k} = \beta_{X,k} \cdot \hat{u}_{it}^2 + \chi_{j(i),t} + \epsilon_{it} \tag{32}$$

for $X_{i,t+k} \in \{\Delta \log \hat{\theta}_{i,t+k}, \pi_{i,t+k}, R_{i,t+k}\}$ and $k \in \{0, 1, 2\}$. We find no evidence of a quantitatively large effect on current and future TFP growth (Appendix Table A2). We also find strong evidence of persistent negative effects on profitability and stock returns. These dynamic results rule out the possibility that the observed negative effect of $\hat{u}_{it}^2$ operates through a channel related to productivity, or through dynamic trade-offs between poor performance today and improved future performance.

5.2 Fact 2: Misoptimizations Rise in Booms, Fall in Downturns

We now study the model's macroeconomic prediction for the behavior of aggregate misoptimizations (Prediction 2). Figure 2 plots aggregate Misoptimization Dispersion, measured at the annual frequency, against the unemployment rate and (detrended) end-of-year S&P 500 price. Misoptimization Dispersion rises when the real economy and financial markets are doing well (e.g., the late 1990s), falls during recessionary or financial crisis periods (e.g., 1990, 2001, and 2008), and is approximately as

Figure 2: Misoptimization Dispersion is Pro-Cyclical
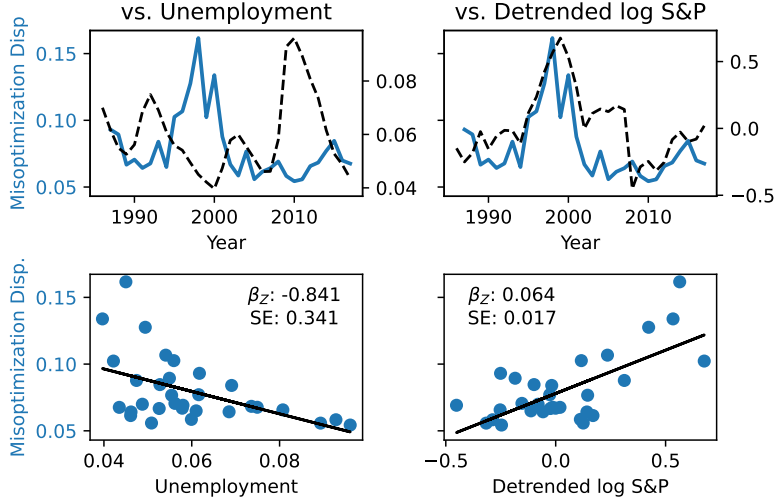
Notes: The top two panels plot Misoptimization Dispersion (blue line, left axis) along with, respectively, unemployment and the linearly detrended S&P 500 price (black dashed lines, right axis). The bottom two panels are scatterplots of Misoptimization Dispersion versus the corresponding macroeconomic aggregate. The black solid line is the linear regression fit. The standard errors are HAC robust based on a Bartlett kernel with a three-year bandwidth.

persistent as the overall business cycle. Appendix Figure A1 shows that the same pro-cyclical pattern is apparent in other measures of dispersion, the mean of $|\hat{u}_{it}|$ and inter-quartile range of $\hat{u}_{it}$.

We benchmark the slope of this relationship with the business cycle by estimating a linear regression of Misoptimization Dispersion on each macroeconomic variable:

$$\text{MisoptimizationDispersion}_t = \alpha + \beta_Z \cdot Z_t + \epsilon_t \qquad (33)$$

for $Z_t \in \{\text{Unemployment}_t/100, \log \text{SPDetrend}_t\}$, over our 31 annual observations. We estimate a slope of -0.841 (SE: 0.341, $p = 0.02$) with respect to unemployment and 0.064 (SE: 0.017, $p = 0.001$) with respect to the detrended S&P 500. The respective correlations are -0.493 and 0.689. The regression on unemployment implies that a five percentage point swing in unemployment is associated with an increase of Misoptimization Dispersion by 0.042 log points, or 53% of its sample mean value. These findings are consistent with the model prediction for attention and misoptimization in Proposition 3.

Robustness to Incorporating Other Frictions. To investigate the robustness of this finding to plausible misspecifications of firms' policy functions, we re-visit Fact 2 under the different constructions for "mistakes" introduced in Section 4.1.

We first consider frictional adjustment dynamics, corresponding to the policy function using Equation 27, which incorporates the hiring effects of adjustment dynamics.[16] We find that the resulting

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[16]The residuals from Equation 27 have negligible autocorrelation (AR(1) coefficient 0.016, as reported in Table A8), and imply a lower misoptimization variance than our baseline measure (a mean of 0.036 across years, compared to 0.080 for the baseline).

measure of Misoptimization Dispersion remains strongly pro-cyclical. The regression coefficient of this version of Misoptimization Dispersion on unemployment is -0.439 (SE: 0.196), attenuated relative to our baseline estimate (-0.841) but statistically indistinguishable from it. Moreover, the new measure has an almost identical correlation with unemployment (-0.468) as our baseline does (-0.493), suggesting that the aforementioned attenuation comes from the smaller scale of the alternatively measured mistakes and not from a diminished cyclicality in normalized units. The inability of adjustment dynamics to explain Fact 2 is also consistent with our earlier finding that misoptimizations had *persistent* negative effects on firm stock returns and profitability (Table A2), instead of the mean-reverting effects which would be natural if firms traded off poor performance today to reduce costly adjustments in the future (e.g., by hoarding labor in downturns).

We next consider financial frictions, corresponding to the policy function in Equation 28. When Misoptimization Dispersion is recalculated with this leverage-augmented policy function, its regression coefficient on unemployment is -0.841 (SE: 0.341), almost identical to that of our main measure. This suggests that time-varying financial frictions do not explain our results. This result is also consistent with the conventional wisdom that the severity and cross-firm heterogeneity of financial frictions increases in downturns rather than in booms, which pushes in the *opposite* direction of both our prediction and finding.

In Appendix Table A3, we show the stability of our pro-cyclical misoptimization finding under the aforementioned variants, under six other alternative constructions of policy and/or production functions described in Section 4.1, and variant strategies that control for linear and quadratic time trends and restrict attention to the manufacturing sector.

Robustness to Compositional Adjustment. Previous research suggests that US public firms have become more volatile in sales growth, employment growth, and stock returns over time, and that these trends can be statistically explained by cohort-of-entry (Davis et al., 2006) and/or firm-specific (Brown and Kapadia, 2007) effects. To guard against the possibility that these compositional trends spuriously drive our findings, we calculate a variant of misoptimization based on the component of squared misoptimizations $\hat{u}_{it}^2$ that is residualized from firm fixed effects. The new measure of Misoptimization Dispersion has correlation 0.91 with our original version. This measure's regression slopes against unemployment and the detrended level of the S&P 500 are -0.520 (SE: 0.206) and 0.040 (SE: 0.009), of comparable magnitude to our baseline estimates.

Robustness to Studied Time Period. Our main analysis uses data from 1986 to 2018. Our choice has two motivations. First, the compositional changes and associated drift in firm-level volatility among public firms are more pronounced earlier in the sample (Davis et al., 2006). Second, macroeconomic policy and therefore dynamics differ markedly before and after the Volcker disinflation. Nevertheless, we can extend our analysis to measure Misoptimization Dispersion for the entire Compustat sample from 1950 to 2018.[17] To allay the concerns about composition, we use the compositional adjustment strategy described above. We plot our resulting Misoptimization Dispersion series in Figure A2. We continue to find strong evidence of pro-cyclicality. Replicating the regressions

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[17]One minor difference in the calculation is that, due to lack of sectoral wage data, we calculate TFP with two factors, materials (inclusive of labor) and capital.

Table 1: Markets Punish Misoptimization Harder in Low-Return States

	(1)	(2)	(3)	(4)
	Outcome: R_{it}			
$\hat{u}_{it}^2$	-0.268	-0.262	-0.097	-0.087
	(0.025)	(0.023)	(0.034)	(0.033)
$\hat{u}_{it}^2 \times R_t$	0.376	0.376	0.443	0.431
	(0.123)	(0.124)	(0.171)	(0.167)
Sector x Time FE	✓	✓	✓	✓
Firm FE			✓	✓
TFP Control		✓		✓
N	41,578	41,578	41,206	41,206
R^2	0.239	0.261	0.385	0.403

Notes: R_{it} is the firm-level log stock return. $\hat{u}_{it}^2$ is the squared firm-level misoptimization residual, constructed using the methods described in Section 4.1. R_t is the log return of the S&P 500. Standard errors are double-clustered at the firm and year level.

against the unemployment rate and detrended S&P 500 price, we obtain coefficients of -0.571 (SE: 0.197) and 0.052 (SE: 0.014). Both estimates are within one standard error of our baseline estimates.

Misoptimization in Other Input Choices. We focused on misoptimization in labor choice because it is the only input or output measured in quantity units in our dataset. However, the model implied that we could measure misoptimization in any input choice. As a robustness exercise, we recalculate misoptimizations and Misoptimization Dispersion using total variable cost expenditures (i.e., materials plus labor) and investment rates (i.e., log growth rates of the capital stock) as the choice variable. We plot the resulting versions of Misoptimization Dispersion in Appendix Figure A3. We find broadly similar patterns, particularly in the spike of the mid-1990s and falls in the 2002 and 2009-10 downturns. The result with investment rates echoes findings by Eisfeldt and Rampini (2006) and Bachmann and Bayer (2014) that investment rates are more dispersed in booms than in downturns.

5.3 Fact 3: Returns Respond More to Misoptimizations During Downturns

We have established that firms make smaller misoptimizations in downturns. We now investigate the extent to which this is explained by the general-equilibrium incentives that our model implies: that firms have higher financial incentives to avoid misoptimization in downturns (Prediction 3).

Specifically, we test whether misoptimizations have a state-dependent effect on stock returns—heuristically, whether the relationship between misoptimizations and stock returns (Figure 1) steepens in bad states. Specifically, we relate a firm's log stock return R_{it} with the firm's squared misoptimiza-

Figure 3: Time-Varying Punishments of Misoptimizations
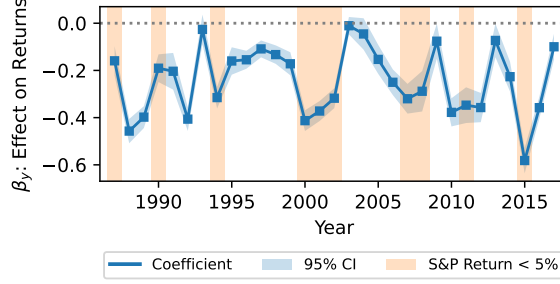
Notes: This Figure shows the estimates of β_y from Equation 35. The blue squares are the point estimates and the blue shaded bands are 95% confidence intervals, based on standard errors clustered by firm and year. Years in which the S&P 500 return was less than 5% are shaded orange.

tion innovation over that year interacted with the log aggregate (S&P 500) stock return R_t:[18]

$$R_{it} = \beta \cdot \hat{u}_{it}^2 + \phi \cdot \left(\hat{u}_{it}^2 \times R_t \right) + \chi_{j(i),t} + \Gamma' X_{it} + \epsilon_{it} \tag{34}$$

Sector-by-time fixed effects partial out industry trends. The vector of control variables X_{it} can include firm fixed effects and the growth rate of firm-level TFP, to partial out other important determinants of returns. As we have shown, the marginal effect of misoptimization on the stock price is negative. The hypothesis that the market punishes misoptimization more severely in times of distress, or low R_t, is captured by $\phi > 0$. In the expanded model of Appendix B, we directly derive the regression equation and the prediction $\phi \geq 0$, with equality only if investors are risk-neutral (there is no risk-pricing channel) and profit sensitivity to mistakes is state-independent (there is no profit-curvature channel).

We find that $\phi > 0$: misoptimization is priced more severely in states of low aggregate returns (Table 1).[19] Our estimates in column (3), in particular, suggest that mistakes have a zero price if the S&P return is 22%, close to its value in the late 1990s or the height of the dot com bubble. By contrast, in the trough of 2008 ($R_t = -0.52$), the model implies that pricing is 6.2 times more severe than in the "usual" states of $R_t \approx 0.10$.

In Figure 3, we show estimates from the following regression which allows for a year-specific coefficient on $\hat{u}_{it}^2$ instead of imposing a parametric interaction with aggregate returns:

$$R_{it} = \sum_y \beta_y \cdot \hat{u}_{it}^2 \cdot \mathbb{I}[t = y] + \chi_{j(i),t} + \epsilon_{it} \tag{35}$$

We shade years in which the S&P 500 return is relatively low ($< 5\%$). Our estimate of $\phi > 0$ in Table 1 corresponds to the fact that, in this more non-parametric model, the plotted coefficients are more negative exactly when the S&P does poorly.

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[18]To be consistent with our earlier analysis, we run this regression using re-scaled mistake innovations $\hat{u}_{it}$. In Appendix Table A4, we replicate the analysis using our (first-stage) estimates of the mistake "level" $\hat{m}_{it}$ and find qualitatively similar results.

[19]Similar results are obtained using mistake "levels" as discussed in Footnote 18 (columns 3-6 of Table A4)

Robustness. In the Appendix, we report two robustness exercises. First, Appendix Table A5 shows the stability of our main finding of $\phi > 0$ to the alternative data-construction approaches highlighted in the previous sections. Second, in Appendix Table A6, we show that our finding of $\phi > 0$ is robust to controlling for other plausible heterogeneities in the effects of misoptimizations on stock returns. In particular, we control for the level and $\hat{u}_{it}^2$-interaction of TFP, lagged stock returns, and financial leverage to control for the observed tendencies for negative-return firms to have higher volatility (the leverage effect) and binding financial constraints; and we control for interactions of $\hat{u}_{it}^2$ with industry and firm fixed effects to model heterogeneous exposure to aggregates.

5.4 Fact 4: The State-Dependent Effects of Misoptimization are Driven by the Pricing of Profits and Not Profits Themselves

Interpreted via our model, the result in Table 1 implies that at least one of the risk-pricing or profit-curvature channels drives changes in the cost of mistakes. To differentiate these explanations, we now explore the extent to which profitability is more sensitive to mistakes during downturns. As emphasized earlier, this prediction isolates the profit-curvature channel (Prediction 4).

To this end, we define profitability π_{it}, as before, as this year's EBIT divided by the last year's total variable costs. We study the state-dependent effects of misoptimizations on profitability using the following regression model that mirrors our previous analysis of stock returns:

$$\pi_{it} = \beta_\pi \cdot \hat{u}_{it}^2 + \phi_\pi \cdot \left(\hat{u}_{it}^2 \times R_t\right) + \chi_{j(i),t} + \gamma_i + \epsilon_{it} \tag{36}$$

In our model, we predict that $\phi_\pi = 0$ if and only if the profit function, in dollars, has state-independent curvature.

We find a positive, small, and statistically insignificant ϕ_π (first column of Table 2). Thus, misoptimizations have a close-to-constant effect on firms' dollar profits up to statistical precision. This suggests that the mechanism for our earlier finding of state-dependent market punishment (Fact 3) relates primarily to the market's greater *reaction* to fixed profit effects of misoptimizations.

We next estimate a sequence of models that explore the joint effects of misoptimizations and profitability on stock returns. We first regress firm stock returns on $\hat{u}_{it}^2$ and π_{it} conditional on firm and sector by time fixed effects. We find that, conditional on profitability, misoptimizations have a severely attenuated, and statistically indistinguishable from zero, effect on stock returns (column 2 of Table 2). This is consistent with the model interpretation that misoptimizations matter for prices by reducing current profits, and not through any other channel.

We next explore whether profitability, more generally, has a larger effect on returns during downturns. This allows us to test whether the market values firm performance more strongly in low-return environments *without* any intermediate structural estimation of production functions or misoptimization. Our estimating equation is the mirror of Equation 34 with profitability in place of misoptimizations:

$$R_{it} = \beta_{\pi \to P} \cdot \pi_{it} + \phi_{\pi \to P} \cdot \left(\pi_{it} \times R_t\right) + \chi_{j(i),t} + \gamma_i + \epsilon_{it} \tag{37}$$

We find that returns respond more to profitability when aggregate returns are low, or $\phi_{\pi \to P} < 0$

Table 2: Misoptimization, Profits, and Pricing

	(1) Outcome: π_{it}	(2)	(3)	(4)
			Outcome: R_{it}	
$\hat{u}_{it}^2$	-0.114	-0.021		
	(0.020)	(0.032)		
$\hat{u}_{it}^2 \times R_t$	0.112			
	(0.089)			
π_{it}		0.400	0.421	0.690
		(0.028)	(0.034)	(0.305)
$\pi_{it} \times R_t$			-0.303	-1.642
			(0.166)	(0.632)
Firm FE	✓	✓	✓	✓
Sector x Time FE	✓	✓	✓	✓
N	50,966	40,879	40,879	40,879
R^2	0.663	0.402	0.402	
First-stage F				17.80

Notes: π_{it} is firm-level profitability and R_{it} is the firm-level log stock return. $\hat{u}_{it}^2$ is the squared firm-level misoptimization residual, constructed using the methods described in Section 4.1. R_t is the log return of the S&P 500. Standard errors are double-clustered at the year and firm level.

(column 3 of Table 2), consistent with our earlier findings and interpretation.

To quantify the pathway from misoptimizations to profitability to stock returns, we estimate an instrumental variables (IV) model of cyclical market responses. Specifically, defining Z_{it} as the vector of regressors $(\pi_{it}, \pi_{it} \times R_t)$ and W_{it} as the vector of instruments $(\hat{u}_{it}^2, \hat{u}_{it}^2 \times R_t)$, we augment the now "second-stage" Equation 37 with a "first-stage" equation

$$Z_{it} = W_{it}'A + \chi_{j(i),t} + \gamma_i + \epsilon_{it} \tag{38}$$

The first stage F-statistic is 17.8, owing to the strong relationship between misoptimizations and profitability. Our IV estimates of $(\beta_{\pi \to P}, \pi_{\pi \to P})$ suggest a greater state-dependence of the market response to misoptimization compared to the market response to other determinants of profits (column 4 of Table 2). We interpret this result, along with the others in this subsection, as empirical validation of the model's microeconomic mechanism for state-dependent misoptimization.

5.5 Fact 5: Macro Attention Rises in Downturns

We now study the aggregate patterns of our Macroeconomic Attention measure (Prediction 5). Figure 4 plots log MacroAttention$_t$ against the US unemployment rate and the linearly detrended log price of the S&P 500. We find that macroeconomic attention persistently rises when the macroeconomy and financial market are distressed.

To measure the cyclicality of MacroAttention, we estimate a linear regression of MacroAttention

Figure 4: Macro Attention is Counter-Cyclical
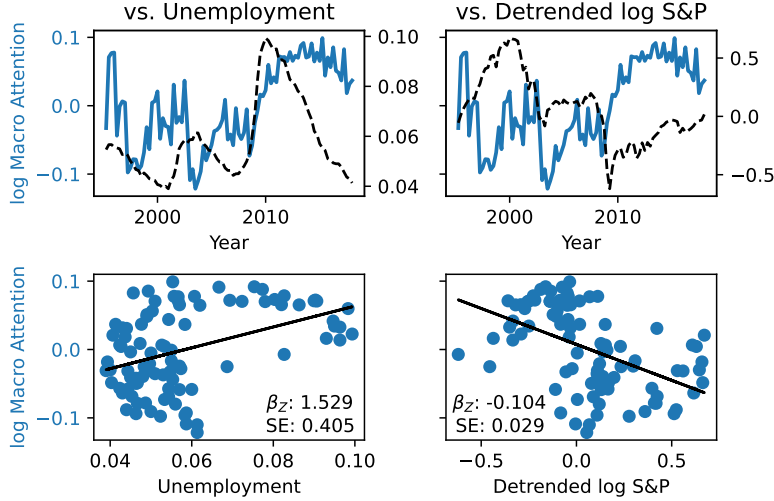
Notes: The top two panels plot log Macro Attention (blue line, left axis) along with, respectively, unemployment and the linearly detrended S&P 500 price (black dashed lines, right axis). The bottom two panels are scatterplots of log Macro Attention versus the corresponding macroeconomic aggregate. The black solid line is the linear regression fit. The standard errors are HAC robust based on a Bartlett kernel with a four-quarter bandwidth.

on each macroeconomic variable:

$$\log \text{MacroAttention}_t = \alpha + \beta_Z \cdot Z_t + \epsilon_t \tag{39}$$

for $Z_t \in \{\text{Unemployment}_t/100, \log \text{SPDetrend}_t\}$. Our coefficient estimates are, respectively, 1.529 (SE: 0.405) and -0.104 (SE: 0.029), with R^2 values of 0.180 and 0.237.

Our interpretation of this fact is that firms' cognitive effort to adapt their decisions to the state of the economy rises in downturns. As mentioned earlier, our model makes this prediction *jointly* for attention toward macroeconomic and microeconomic conditions, but our empirical methods limit us to testing only the former prediction.

Robustness. Appendix F.1 describes how we replicate our procedure using the full text of US public firms' sales and earnings conference calls as an alternative dataset. This produces a similar counter-cyclical pattern over a smaller time period (2004-2013).

Appendix Figure A5 plots the time-series behavior of each word-level component of MacroAttention. In our sample, 61 of the 89 words have a positive correlation with unemployment (Appendix Figure A6). Appendix F.2 describes an alternative procedure for analyzing the 10-Q/Ks which uses the frequency of algorithmically determined word stems rather than full words to build our word set $\mathcal{W}_M$. This yields essentially identical results.

An additional prediction of our framework is that attention is more cyclical in industries with more cyclical productivity due to an intensified profit-curvature channel, but is still cyclical in acyclical industries due to the risk-pricing channel. This prediction contrasts with that of an alternative model in

Table 3: Macro-Attentive Firms Make Smaller Misoptimizations

	(1)	(2)	(3)	(4)
		Outcome: $\hat{u}_{it}^2$		
log MacroAttention$_{it}$	-0.0081	-0.0052	-0.0058	-0.0056
	(0.0028)	(0.0029)	(0.0044)	(0.0038)
Sector x Time FE	✓	✓	✓	✓
Firm FE			✓	✓
TFP, Return Controls		✓		✓
N	28,279	24,392	27,875	23,930
R^2	0.053	0.067	0.383	0.384

Notes: $\hat{u}_{it}^2$ is the squared firm-level misoptimization residual, constructed using the methods described in Section 4.1. log MacroAttention$_{it}$ is the measure of firm-level macroeconomic attention. Standard errors are double-clustered at the firm and year level.

which the cyclicality in macroeconomic attention is purely driven by firms' exposure to macroeconomic shocks. To study this, we compute "output cyclicality" in each of our industries as the correlation between sectoral GDP growth, calculated using quarterly BEA data since 2005 linked appropriately to NAICS-definition sectors, with aggregate nominal GDP growth. In Appendix Figure A7, we plot in the cross-section of industries the relationship between this output cyclicality and the coefficient of sector-level Macro Attention on the US unemployment rate. Of the 42 industries, 36 feature counter-cyclical macroeconomic attention. The extent of counter-cyclicality increases with the industry's output cyclicality, but acyclical industries still have counter-cyclical attention. Thus, counter-cyclical attention does not arise merely as the result of increased exposure to macroeconomic conditions during downturns.

5.6 Fact 6: Macro Attention Predicts Smaller Misoptimizations

We finally investigate the relationship between misoptimization and attention (Prediction 6). We estimate the following regression of $\hat{u}_{it}^2$, the squared innovation of the firm's model-implied misoptimization, on the log of firm-level macro attention:

$$\hat{u}_{it}^2 = \beta_a \cdot \log \text{MacroAttention}_{it} + \chi_{j(i),t} + \Gamma' X_{it} + \epsilon_{it} \tag{40}$$

Absorbed effects at the sector-by-time level partial out all trends, including the cyclical patterns studied earlier. Additional controls X_{it} can include individual fixed effects, to isolate variation at the firm level; and log stock returns and TFP growth, to help further isolate variation in attention unrelated to firm-level fundamentals.

We find that $\beta_a < 0$: higher macroeconomic attention corresponds with smaller misoptimization (Table 3). The main specification in column (1) finds a strongly statistically significant effect ($p < 0.01$). The more controlled specifications in columns (2) to (4) estimate negative, similarly sized effects, but

with less precision.

These findings support our interpretation of macroeconomic attention as a proxy for cognitive effort that reduces mistakes. This sharpens our interpretation of Fact 5 (counter-cyclical attention in the time series). Two alternative hypotheses that would be consistent with Fact 5 in isolation are that managers use the macroeconomy as a convenient scapegoat for bad performance in downturns or that attention toward macroeconomic developments crowds out attention toward microeconomic issues. The former is inconsistent with the negative firm-level relationship between attention and misoptimization—macro-attentive firms do better, not worse, in the cross-section. The second is inconsistent with the same evidence, when combined with the fact that microeconomic risks are relatively larger (and potentially more payoff-relevant) than macroeconomic risks in downturns (Bloom et al., 2018).

Robustness. Appendix Table A7 shows that results are similar using our conference-call measure of attention as well as all considered alternative models and measurement strategies for the misoptimizations.

5.7   Discussion and Relationship with the Literature

In this final subsection, we first discuss how our findings are consistent with those from the literature on uncertainty and productivity over the business cycle. Second, we relate our model's predictions to the cyclicality of forecast and backcast errors and the demand for information.

Dispersion in Fundamentals Versus Misoptimizations. Bloom et al. (2018), using micro-data on the manufacturing sector, estimate that the variance in total factor productivity rises in recessions or periods of negative growth.[20] Our analysis, by contrast, studies variance in input choices *conditional* on productivity, the variance of which may change arbitrarily over time.

To demonstrate the empirical consistency of these sets of findings, we follow Bloom et al. (2018) and estimate a first-order autoregressive model for TFP with firm and sector-by-time fixed effects:

$$\log \theta_{it} = \gamma_i + \chi_{j(i),t} + \rho_\theta \log \theta_{i,t-1} + \epsilon_{it} \tag{41}$$

We estimate "TFP Innovation Variance" as the weighted average, $\mathbb{E}[s_{it}^* \hat{\epsilon}_{it}^2]/\mathbb{E}[s_{it}^*]$.

Appendix Figure A4 plots our estimate of TFP Innovation Variance against the unemployment rate and detrended stock price. TFP Innovation Variance spikes markedly in the 2002 and 2007-09 recessions, as well as in the 1990s boom. Our measure of TFP dispersion has a correlation of 0.39 with the equivalent measure from Bloom et al. (2018) over a common sample, and this correlation increases to 0.47 if we restrict our data to the manufacturing sector.[21] This suggests that our measurement of TFP stochasticity among US public firms is consistent with the measurement of Bloom et al.

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[20]Kehrig (2015) reports that dispersion in levels of TFPR, or the marginal value product of all inputs under a Cobb-Douglas assumption, is counter-cyclical. In Appendix F.3, we study how this object, as well as the related calculation for the value marginal product of labor, behaves in our data.

[21]The common sample for comparing our measure with the one in Bloom et al. (2018) is 1987-2010. The measure from Bloom et al. (2018) that we study is the variance (square of standard deviation) of TFP innovations on the sample of establishments that are in the Bloom et al. (2018) data for 25 years.

(2018) using Census data in the manufacturing sector. As reported by those authors, TFP variance is significantly higher in recessions (coefficient: 0.098, SE: 0.022).

To summarize, in our data, misoptimizations are larger in booms, while TFP is less volatile. Thus, counter-cyclical TFP volatility is fully consistent with our hypothesis and finding that firms optimize more precisely conditional on productivity in downturns.

Forecast Errors and Backcast Errors. Our model makes no specific prediction about the accuracy of firm-level forecasts over the business cycle. To illustrate this, consider a firm's expectation in period t of its production in period $t+k$, for $k > 0$, conditional on the observed history of the decision state z_{it} (firm-level TFP, aggregate output, and aggregate wages). Writing $\mathbb{E}_{it}[\cdot]$ as a shorthand for the firms' expectation conditional on this history and applying Proposition 1, we can write the firm's realized forecast error as

$$x_{i,t+k} - \mathbb{E}_{it}[x_{i,t+k}] = \left(x^*_{i,t+k}(z_{it}) - \mathbb{E}_{it}[x^*_{i,t+k}(z_{i,t+k})]\right) + \sigma_i(z_{i,t+k}) \cdot v_{i,t+k} \tag{42}$$

The first term is the forecast error about the optimal level of production, $x^*_{i,t+k}$, which itself depends on unknown firm-level productivity, aggregate output, and aggregate wages. The second term is the firms' future misoptimization, the product of state-dependent volatility $\sigma_i(z_{i,t+k})$ and the idiosyncratic noise $v_{i,t+k}$. As the noise is uncorrelated with x^*, the cross-sectional dispersion of these errors decomposes into two terms which we label below

$$\underbrace{\mathbb{E}\left[(x_{i,t+k} - \mathbb{E}_{it}[x_{i,t+k}])^2\right]}_{\text{Forecast Error Variance}} = \underbrace{\mathbb{E}\left[\left(x^*_{i,t+k}(z_{i,t+k}) - \mathbb{E}_{it}[x^*_{i,t+k}]\right)^2\right]}_{\text{Fundamental Variance}} + \underbrace{(\sigma_i(z_{i,t+k}))^2}_{\text{Misoptimization Variance}} \tag{43}$$

Fact 2 was that the second term, on average across firms, is high in booms and low in downturns. The finding of both prior work in the literature (Bloom et al., 2018), and our own reconstruction of these findings above, is that the first term is high in downturns and low in booms. Because of these countervailing forces, our main hypotheses about misoptimizations cannot be tested by evaluating the cyclicality of forecast errors.[22] A similar point applies regarding strategies to test the model using the variance of other quantities that "blend" fundamentals and mistakes, like (unconditional) hiring, sales, or stock returns.

By contrast, firm-level *backcasts* of specific state variables, like external macroeconomic conditions, may have a more direct interpretation as "attentiveness." In Appendix G, we show two pieces of evidence consistent with our analysis in the survey of firms in New Zealand conducted by Coibion et al. (2018). First, firms report that they would be significantly more likely to seek out news about the macroeconomy if there were a negative aggregate shock. This is consistent with the cyclicality of the macroeconomic attention cycle. Second, firms that report a higher sensitivity of firm profits to their own choices (in this case, posted prices) demonstrate higher awareness of macroeconomic aggregates. This is consistent with the profit curvature channel.

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[22]See Charoenwong et al. (2021) and Chiang (2023) for an analysis of cyclicality in forecast error variance.

6   Quantifying the Consequences of Attention Cycles

The previous section verified the model's microeconomic and macroeconomic predictions for misoptimization and attention, which govern the model's implications for output and productivity dynamics. In this section, we quantify the macroeconomic consequences of attention cycles by calibrating the model. Our strategy is to match the aggregate level and cyclicality of misoptimization-induced dispersion in production, which control the size and dynamics of the attention wedge, respectively. We find that attention cycles generate: asymmetrically large amplification of negative shocks; greater amplification of shocks when output is low; and endogenously higher volatility when output is low. Each of these features arises from the endogenous dynamics of attention. We show that these findings account for a significant portion of observed business-cycle asymmetries.

6.1   Calibration

In our calibration, as in our theoretical results of Section 3, the aggregate state variable $\theta_t = \left(\mathbb{E}_{G_t}[\theta_{it}^{\epsilon-1}]\right)^{\frac{1}{\epsilon-1}}$ is a one-dimensional sufficient statistic for the productivity distribution. We assume that $\log \theta_t$ follows a zero-mean, Gaussian AR(1) process:

$$\log \theta_t = \rho_\theta \log \theta_{t-1} + \nu_t \tag{44}$$

where $\nu_t \sim^{IID} N(0, \sigma_\theta^2)$. Note that, in this formulation, the shocks ν_t may reflect changes to *any* moment of the productivity distribution that induce changes in the aggregate θ_t, such as a standard shock to average productivity or an "uncertainty shock" to productivity dispersion as studied by Bloom et al. (2018). Since we will study purely macroeconomic phenomena, we can therefore be agnostic about what shocks to the productivity distribution induce the dynamics of Equation 44.

The model has six parameters.[23] We calibrate two to standard values and match four to measured moments, as reviewed in Table 4 and explained below.

The first calibrated parameter is the elasticity of substitution between products. We set $\epsilon = 4$, which implies an optimal average markup of $\frac{\epsilon}{\epsilon-1} = \frac{4}{3}$. This is conservative relative to estimates by De Loecker et al. (2020) (1.60) and Demirer (2020) (1.45) for modern markups for US public firms, and slightly larger than the estimate by Edmond et al. (2018) on the same dataset (1.25). The elasticity of substitution controls the translation of misoptimization into output and productivity, with a lower value translating a fixed misoptimization dispersion into a larger penalty for output and productivity. We next set the persistence of the productivity shock, at the quarterly frequency, to a standard value of $\rho = 0.95$.[24]

The four remaining free parameters are the elasticity of real wages to output χ, the average

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[23]The constants $\bar{w}$ and $\bar{X}$ in the wage rule scale overall production in equilibrium, but are otherwise irrelevant (see Proposition 1). We set $\bar{w}$ and $\bar{X}$ to match the wage and output prevailing in a frictionless market economy with Greenwood et al. (1988) preferences over labor and leisure and elasticity of labor supply $\phi = 1$, evaluated at state $\log \theta = 0$.

[24]We study quarterly dynamics to compare our predictions to standard results about business-cycle asymmetries, although our measurement was annual. As our measurement is based on the long-run, cross-sectional variance of misoptimizations, the frequency of calibration is immaterial.

Table 4: Parameters for Calibration

Fixed Parameters	ϵ	Elasticity of substitution	4
	ρ_θ	Persistence of productivity	0.95
Free Parameters	χ	Elasticity of real wages to output	0.095
	λ	Average weight on entropy penalty	0.406
	γ	Coefficient of relative risk aversion	11.5
	σ_θ^2	Variance of the productivity innovation	4.82×10^{-7}
Matched Moments	β_U	Slope of Misopt. Dispersion on $-\frac{\text{Unemployment}}{100}$	0.841
	$\bar{\sigma}_M^2$	Average level of Misopt. Dispersion	0.080
	χ	Regression of detrended real wages on detrended output	0.095
	σ_Y^2	Variance of quarterly output growth	0.337

Notes: "Fixed Parameters" are externally set. "Free Parameters" are chosen to fit the "Matched Moments," which are calculated from the data and matched exactly by the model.

attention cost λ, the coefficient of relative risk aversion γ, and the volatility of the productivity process σ_θ^2. We set these four parameters in the model to match four simulated moments with exact analogs in the data, as described in Table 4, and fit the model exactly up to machine precision.

Two of these parameters, χ and σ_θ^2, have a relatively simple interpretation. For the first, we match directly an OLS regression of linearly detrended real wages on linearly detrended GDP, at the quarterly frequency over our studied period 1987-2018.[25] Our estimate of 0.095 lines up with recent evidence on the "flattening" of the wage Phillips curve (Galí and Gambetti, 2019). For the second, we match the variance of quarterly real GDP growth over our sample period by scaling the shock variance σ_θ^2.

The key properties of inattention and misallocation are controlled by the remaining two moments. In the model, we calculate the average level of Misoptimization Dispersion and the population regression coefficient of Misoptimization Dispersion on log employment.[26] We match these moments respectively to the time-series average of Misoptimization Dispersion, 0.080, and the (negative) slope of Misoptimization Dispersion in unemployment, 0.841. Intuitively, these moments identify the level of misoptimization and the extent of its cyclicality. In the model, conditional on all other parameters, they identify the average cost of attention λ, itself a sufficient statistic for the cross-sectional distribution, and the coefficient of relative risk aversion γ, which controls the risk-pricing incentive that drives cyclical misoptimization.

Our estimates of γ are thus based entirely on fitting a stochastic discount factor that justifies the observed pattern of misoptimization in our model, rather than incorporating an informed prior from external financial evidence as discussed in Section 3.2. Our finding of $\gamma = 11.5$ (Table 4) is, if anything, slightly conservative relative to the modern asset-pricing literature that estimates, in variations of the consumption capital asset pricing (CCAPM) model, γ of about 15-20 with statistically "unfiltered"

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[25]Our real wage series is the median weekly real earnings for wage and salary workers over the age of 16, as reported by the US BLS.

[26]We calculate the "regression coefficient" in the model using numerical integration over the stationary distribution of states. Appendix A.7.3 shows the exact in-model formula for aggregate Misoptimization Dispersion, calculated with optimal-sales-weights, which depends only on the sufficient statistics (θ, λ) of the cross-sectional type distribution.

Figure 5: Output, Attention Wedge, and Labor Productivity
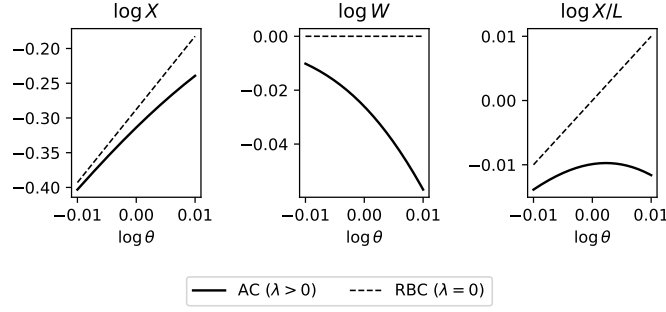
measures of consumption (Savov, 2011; Kroencke, 2017) or long-run variation in consumption growth (Parker and Julliard, 2005). As previously discussed, our finding of $\gamma > 0$ suggests that the risk-pricing channel is necessary to explain misoptimization dynamics and rules out the nested model in which control costs are denominated in dollar (rather than utility) units.

6.2 Output, Productivity, and the Attention Wedge

Figure 5 shows log output, the log attention wedge (as defined in Proposition 4), and log labor productivity in the calibrated model. In each case, we compare to the predictions of an otherwise identical "pure RBC model" with full attentiveness, or $\lambda = 0$, plotted as a dashed line. As implied by Proposition 4 and Corollary 2, inattention reduces output and productivity relative to the fully attentive counterfactual, and this effect grows in higher productivity states. In the mean state, the attention wedge reduces output by 2.6% relative to the fully attentive counterfactual, labor productivity by $\chi\epsilon \times 2.6\% = 1.0\%$, and employment by $(1 - \chi\epsilon) \times 2.6\% = 1.6\%$. In the same state, a firm with the mean productivity loses 3.0% of its profits, on average, due to inattention.

To highlight the importance of studying misoptimization incentives in general equilibrium, we calculate also a "partial equilibrium" attention wedge based on firms' inattentive best-responses to the counterfactual RBC dynamics (Appendix Figure A8). In the mean state, the "partial equilibrium" attention wedge is 1.3% in terms of output, implying that general-equilibrium interactions account for 1/2 of the losses from inattention.

We observe two further properties of output and productivity dynamics which were not immediately clear from the theoretical results and depend on the numerical calibration. First, labor productivity is non-monotone in microeconomic productivity θ (rightmost panel) and hence also in aggregate output. As we mentioned in the discussion following Corollary 2, this property results from the dueling forces of increased microeconomic productivity and increased misallocation from reduced attention.

Second, we find that the attention wedge is concave. Appendix Figure A8 shows this concavity directly and provides also a partial-vs.-general-equilibrium decomposition of the finding. Corollary 3 showed how the concavity or convexity of the attention wedge leads to business cycle asymmetries. The finding of a concave attention wedge implies that, fixing shock sizes, negative shocks have a larger effect on output than positive shocks, and that overall shock responses and volatility are higher in low-output states. We explore these predictions quantitatively in the next subsection.

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Figure 6: Asymmetric Shock Response and Stochastic Volatility
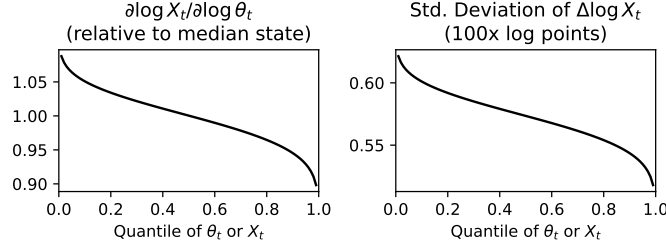
6.3 State-Dependence, Asymmetry, and Stochastic Volatility

Figure 6 plots, as a function of the initial quantile of the stationary distribution for productivity or output, the marginal sensitivity of output to percentage productivity shocks and the standard deviation of output growth. As suggested by the discussion of the concave attention wedge above, both model objects are decreasing functions of the state.

One way to benchmark the extent of asymmetry and state-dependence in shock responses is to consider an "impulse response" thought experiment of a fixed size. Let $\log \hat{\theta}$ be the fundamental shock that induces a 3% change in output from steady state, or solves $\log X(\log \hat{\theta}) - \log X(0) = 0.03$. We compare the effect of this "Positive" shock to the effect of a "Negative" shock from $\log \theta_0 = 0$ to $\log \theta_1 = -\log \hat{\theta}$, and a "Double Dip" shock from $\log \theta_0 = -\log \hat{\theta}$ to $\log \theta_1 = -2 \cdot \log \hat{\theta}$. We find that the negative shock has a 7% larger effect on log output than the positive shock, and the double dip shock has a 14% larger effect on log output than the positive shock. The same results for the response of log employment are 5% and 8%, respectively. Empirically, Ilut et al. (2018) estimate that US industries have on average a 20% larger response to negative aggregate productivity shocks than to positive shocks.[27] In comparison to this benchmark, our model explains 25% of the asymmetry in shock response via the endogenous reallocation of attention. Thus, attention cycles allow models with homoskedastic and symmetrically distributed productivity shocks to generate meaningfully asymmetric macroeconomic dynamics.

To benchmark the extent of stochastic volatility, we observe that a transition from the 90th-percentile to the 10th-percentile productivity state reduces output by 4.7% and increases the conditional standard deviation of output growth by 10.6%. The peak-to-trough fall of output during the Great Recession (e.g., from early 2007 to early 2009) is comparable to this level change. Empirically, Jurado et al. (2015) estimate that the forward-looking volatility of industrial production growth at the three-month horizon increased by 57% during this episode.[28] The endogenous re-allocation of attention in our model can explain about 19% of this movement, without recourse to underlying stochastic volatility in fundamentals.

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[27]This calculation is based on comparing the "data" estimates in columns 6 and 7 of Table 9 in that paper.

[28]This calculation compares the 3-month "macro uncertainty index," as available at https://www.sydneyludvigson.com/macro-and-financial-uncertainty-indexes and based on the methodology of Jurado et al. (2015), from a low point in April 2007 to a high point in October 2008.

6.4 Parameter Robustness and Counterfactual Scenarios

In Appendix E, we provide additional results from our numerical exercise. We first explore the robustness of our main findings to different external calibrations of wage rigidity χ and substitutability ϵ and to introducing classical labor markets using the preferences of Greenwood et al. (1988). We also study the effects of calibrated attention cycles on macroeconomic dynamics under counterfactual scenarios by altering structural parameters without recalibrating the model. To summarize, we find that attention cycles generate greater business-cycle asymmetries in regimes with larger markups, greater wage rigidity, and higher attention costs.

7 Conclusion

This paper studies how attention cycles arise as the consequence of business cycles through their effects on incentives and how attention cycles, in turn, affect business cycle dynamics. Theoretically, we introduce a Neoclassical business cycle model with flexible, state-dependent stochastic choice. We show how firms' choice of state-specific attention is shaped by the incentives embodied in the state-dependent cost of making mistakes. Due to firms' ownership by risk-averse households, the incentives to pay attention and to avoid mistakes are highest when aggregate consumption is low (a *risk-pricing channel*). Attention rises in downturns and falls in booms, leading to asymmetric propagation of macroeconomic shocks. Empirically, we introduce strategies to measure choice misoptimization, which has an exact analog in the model, and attention toward the macroeconomy, which is a more suggestive proxy for attention. Using these measures, we verify the model's macroeconomic hypotheses that attention rises and misoptimization falls in downturns. We also verify the model's microeconomic predictions that misoptimization harms firm valuations more severely in downturns and that attention coincides with lower misoptimization in the cross-section. Calibrating the model to match our evidence on cyclical misoptimization, we uncover a quantitatively important role for attention cycles in driving business-cycle asymmetries.

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Online Appendix
for "Attention Cycles" by Flynn and Sastry

A Omitted Proofs

A.1 Proof of Proposition 1

Proof. Consider a firm of type λ_i, with a payoff $u : \mathcal{X} \times \mathcal{Z} \to \mathbb{R}$ and prior density $\pi \in \Delta(\mathcal{Z})$. The firm's stochastic choice problem can be written as

$$\max_{p \in \mathcal{P}} \int_{\mathcal{X}} \int_{\mathcal{Z}} u(x, z) p(x|z) \, \mathrm{d}x \, \pi(z) \, \mathrm{d}z - \lambda_i \int_{\mathcal{X}} \int_{\mathcal{Z}} p(x|z) \log p(x|z) \, \mathrm{d}x \, \pi(z) \, \mathrm{d}z \tag{45}$$

We can formulate this problem as constrained optimization for choosing $p(x|z)$ pointwise, with constraints embodying non-negativity and the restriction that conditional distributions integrate to one. We can then write a Lagrangian for this problem, giving these constraints multipliers $\kappa(x, z)$ and $\gamma(z)$, respectively:

$$\begin{aligned}
\mathcal{L}(\{p(x|z), \kappa(x, z)\}, \{\gamma(z)\}) &= \int_{\mathcal{Z}} \int_{\mathcal{X}} u(x, z) p(x|z) \, \mathrm{d}x \, \pi(z) \, \mathrm{d}z \\
&\quad - \lambda_i \int_{\mathcal{Z}} \int_{\mathcal{X}} p(x|z) \log p(x|z) \, \mathrm{d}x \, \pi(z) \, \mathrm{d}z \\
&\quad + \int_{\mathcal{Z}} \int_{\mathcal{X}} \kappa(x, z) p(x|z) \, \mathrm{d}x \, \pi(z) \, \mathrm{d}z \\
&\quad + \int_{\mathcal{Z}} \gamma(z) \left(\int_{\mathcal{X}} p(x|z) \, \mathrm{d}x - 1 \right) \pi(z) \, \mathrm{d}z
\end{aligned} \tag{46}$$

The Lagrangian is concave in the collection $\{p(x \mid z)\}$, since the expected utility term and the two constraint terms are linear in these variables, and the control-cost term is convex in these variables. Taking the first-order condition of the Lagrangian with respect to $p(x|z)$ yields the necessary first-order condition

$$u(x, z) - \lambda_i (\log p(x|z) + 1) + \kappa(x, z) + \gamma(z) = 0 \tag{47}$$

Re-arranging this expression and applying the normalization that the density integrates to one, we get the solution

$$p(x|z) = \frac{\exp\left(\lambda_i^{-1} u(x, z)\right)}{\int_{\mathcal{X}} \exp\left(\lambda_i^{-1} u(x', z)\right) \, \mathrm{d}x'} \tag{48}$$

This solution is invariant to the prior distribution $\pi(z)$, and hence can be indexed solely by the *ex post* realized state z.

To solve our firm's problem, we replace u in the above with $\tilde{\Pi}$ and z with z_i. Performing this substitution, and ignoring the normalizing constant, we get

$$p(x|z_i) \propto \exp\left(-\frac{(x - x^*(z_i))^2}{2\lambda_i |\Pi_{zz}(z_i)|^{-1}}\right) \tag{49}$$

Taking $\mathcal{X} = \mathbb{R}$, it is then immediate that $p(x|z_i)$ is a Gaussian random variable with mean $x^*(z_i)$ and variance $\lambda_i |\Pi_{xx}(z_i)|^{-1}$. Observing that $|\Pi_{xx}(z_i)| = |\pi_{xx}(z_i)|M(z_i)$, we can re-write the variance as $\lambda_i(|\pi_{xx}(z_i)|M(z_i))^{-1}$. Finally, we observe that the stochasticity in each firm's action conditional on z_i is independent from z_i and/or any other firm's action. Thus:

$$x_i = x^*(z_i) + \sqrt{\frac{\lambda_i}{|\pi_{xx}(z_i)|M(z_i)}} \cdot v_i, \qquad v_i \sim \text{Normal}(0,1) \tag{50}$$

$\square$

A.2 Proof of Proposition 2

Proof. Throughout this proof, in some abuse of notation, we write $\theta = \hat{\theta}(G)$ (Equation 5) as the scalar summary of the productivity distribution. Our argument below makes, and then verifies, the conjecture that this is a scalar sufficient statistic for the productivity distribution in the definition of equilibrium output.

To prove existence, we first study the problem of a single firm i who is best replying to the conjecture that the law of motion of the aggregate is $X : \Theta \to \mathbb{R}$. In particular, they believe that output is given by $X(\theta)$ in each state θ.

As established by Proposition 1, the firm's best-response is invariant to the firm's prior state $z_{i,t-1}$ and described by the following random variable conditional on each realization of z_{it}:

$$x_{it} = x^*(z_{it}) + \sqrt{\frac{\lambda_i}{|\Pi_{xx}(z_{it})|}} \cdot v_{it}, \qquad v_{it} \sim \text{Normal}(0,1) \tag{51}$$

As derived in Appendix A.7.1, the mean and variance scalings are the following, after substituting in the equilibrium conjecture $X_t = X(\theta_t)$:

$$
\begin{aligned}
x^*(z_{it}) &= v_x(\epsilon, \chi, \bar{w}, \bar{X}) \cdot X(\theta_t)^{1-\chi\epsilon} \theta_{it}^\epsilon \\
|\Pi_{xx}(z_{it})| &= v_\Pi(\epsilon, \chi, \bar{w}, \bar{X}) \cdot X(\theta_t)^{-1-\gamma+\chi(1+\epsilon)} \theta_{it}^{-1-\epsilon}
\end{aligned}
\tag{52}
$$

for constants $v_x, v_\Pi > 0$ given by:

$$
\begin{aligned}
v_x &:= \epsilon^{-\epsilon}(\epsilon-1)^\epsilon \bar{w}^{-\epsilon} \bar{X}^{\chi\epsilon} \\
v_\Pi &:= (\epsilon-1)^{-\epsilon}\epsilon^{\epsilon-1} \bar{w}^{1+\epsilon} \bar{X}^{-\chi(1+\epsilon)}
\end{aligned}
\tag{53}
$$

Conditional on the realization of any state θ, aggregate output must solve the fixed point equation defined by combining the aggregate-good production function (11) and the firms' best responses. Applying the law of iterated expectations, we can re-write the aggregate-good production function as

$$X = X^* - \frac{1}{2\epsilon(X^*)^{-\frac{1}{\epsilon}}} \mathbb{E}_{\theta_i} \left[(x^*(z_i))^{-1-\frac{1}{\epsilon}} \mathbb{E}_{\lambda_i, v_i} \left[(x_i - x^*(z_i))^2 \mid \theta_i, \theta \right] \mid \theta \right] \tag{54}$$

2

where

$$X^* = \left(\mathbb{E}_{\theta_i}[x^*(z_i)^{1-\frac{1}{\epsilon}} \mid \theta] \right)^{\frac{\epsilon}{\epsilon-1}} \tag{55}$$

We now specialize the expressions above using the structure of the best response in Equations 51, 52, and 53. We first compute X^* as

$$X^* = v_x X^{1-\chi\epsilon} \theta^\epsilon \tag{56}$$

where θ is the transformation defined in Equation 5. We next calculate the the second, "variance" term. We start with the "misoptimization variance"

$$\mathbb{E}_{\lambda_i, v_i} \left[(x_i - x^*(z_i))^2 \right] = \frac{\lambda}{v_\Pi} X^{1+\gamma-\chi(1+\epsilon)} \theta_i^{1+\epsilon} \tag{57}$$

and then calculate the full term

$$(X^*)^{\frac{1}{\epsilon}} \mathbb{E}_{\theta_i} \left[(x^*(z_i))^{-1-\frac{1}{\epsilon}} \mathbb{E}_{\lambda_i, v_i} \left[(x_i - x^*(z_i))^2 \right] \right] = \frac{\lambda}{v_\Pi v_x} X^{\gamma-\chi} \theta \tag{58}$$

where we simplify and apply the definition of $\hat{\theta}(G)$ from Equation 5.

Substituting in Equations 56 and 58, we derive that equilibrium output solves:

$$X(\theta) = v_x X(\theta)^{1-\chi\epsilon} \theta^\epsilon - \frac{\lambda}{2\epsilon v_x v_\Pi} X(\theta)^{\gamma-\chi} \theta \tag{59}$$

There is always a trivial equilibrium $X = 0$ arising from our approximations. Toward proving existence and uniqueness of a non-trivial equilibrium, define:

$$g(X, \theta) = a_0 X^{1-\chi\epsilon} \theta^\epsilon - a_1 X^{\gamma-\chi} \theta \tag{60}$$

where $a_0 = v_x > 0$ and $a_1 = \frac{\lambda}{2\epsilon v_x v_\Pi} > 0$. We now compute this function's derivatives in X:

$$\begin{aligned} g_X(X, \theta) &= a_0(1-\chi\epsilon)X^{-\chi\epsilon}\theta^\epsilon - a_1(\gamma-\chi)X^{\gamma-\chi-1}\theta \\ g_{XX}(X, \theta) &= -a_0(1-\chi\epsilon)\chi\epsilon X^{-\chi\epsilon-1}\theta^\epsilon - a_1(\gamma-\chi)(\gamma-\chi-1)X^{\gamma-\chi-2}\theta \end{aligned} \tag{61}$$

If $1 - \chi\epsilon > 0$ and $\gamma > 1 + \chi$, then

$$\lim_{X \to 0} g_X(X, \theta) = +\infty \qquad \lim_{X \to \infty} g_X(X, \theta) = -\infty \tag{62}$$

Moreover, if $\gamma > \chi + 1$ we have that $g_{XX}(X, \theta) < 0$ on $(0, \infty)$. Thus, when $\gamma > \chi + 1$ and $\chi\epsilon < 1$, $g(X, \theta)$ crosses X from above and there exists a unique, positive fixed point for each θ. Iterated for all states $\theta \in \Theta$, this reasoning shows the existence of a unique, positive equilibrium mapping $X : \Theta \to \mathbb{R}_+$.

We now show monotonicity of the fixed point. To this end, we implicitly differentiate the fixed

3

point condition:

$$\frac{\mathrm{d}X(\theta)}{\mathrm{d}\theta} = \left[a_0(1-\chi\epsilon)X(\theta)^{-\chi\epsilon}\theta^\epsilon - a_1(\gamma-\chi)X(\theta)^{\gamma-\chi-1}\theta\right]\frac{\mathrm{d}X(\theta)}{\mathrm{d}\theta}$$
$$+ \left[a_0\epsilon X(\theta)^{1-\chi\epsilon}\theta^{\epsilon-1} - a_1 X(\theta)^{\gamma-\chi}\right] \tag{63}$$

Yielding:

$$\frac{\mathrm{d}X(\theta)}{\mathrm{d}\theta} = \frac{a_0\epsilon X(\theta)^{1-\chi\epsilon}\theta^{\epsilon-1} - a_1 X(\theta)^{\gamma-\chi}}{1 - [a_0(1-\chi\epsilon)X(\theta)^{-\chi\epsilon}\theta^\epsilon - a_1(\gamma-\chi)X(\theta)^{\gamma-\chi-1}\theta]} \tag{64}$$

Multiplying both sides by a factor of $\frac{\theta}{X}$:

$$\frac{\mathrm{d}\log X(\theta)}{\mathrm{d}\log\theta} = \frac{a_0\epsilon X(\theta)^{1-\chi\epsilon}\theta^\epsilon - a_1 X(\theta)^{\gamma-\chi}\theta}{X(\theta) - [a_0(1-\chi\epsilon)X(\theta)^{1-\chi\epsilon}\theta^\epsilon - a_1(\gamma-\chi)X(\theta)^{\gamma-\chi}\theta]} \tag{65}$$

We first show that the numerator is positive

$$a_0\epsilon X(\theta)^{1-\chi\epsilon}\theta^\epsilon - a_1 X(\theta)^{\gamma-\chi}\theta = a_0(\epsilon-1)X(\theta)^{1-\chi\epsilon}\theta^\epsilon + X(\theta)$$
$$> 0 \tag{66}$$

The first equality substitutes in the original fixed-point equation. The second follows from observing that $\epsilon > 1$, $a_0 > 0$, $X(\theta) > 0$, and $\theta > 0$. To show $\frac{\mathrm{d}\log X(\theta)}{\mathrm{d}\log\theta} > 0$ it now suffices to show that the denominator is positive, which follows from $\chi\epsilon < 1$ and $\gamma > \chi + 1 > \chi$:

$$X(\theta) = a_0 X(\theta)^{1-\chi\epsilon}\theta^\epsilon - a_1 X(\theta)^{\gamma-\chi}\theta$$
$$\geq a_0 \underbrace{(1-\chi\epsilon)}_{\chi\epsilon<1} X(\theta)^{1-\chi\epsilon}\theta^\epsilon - a_1 \underbrace{(\gamma-\chi)}_{\gamma-\chi>1} X(\theta)^{\gamma-\chi}\theta \tag{67}$$

This shows that $\frac{\mathrm{d}\log X(\theta)}{\mathrm{d}\log\theta} > 0$ and implies that $X(\theta)$ is an increasing function. □

A.3   Proof of Proposition 3

Proof. We start by proving the monotonicity of misoptimization. Using the result of Proposition 1, and the substitution of $(x^*(z_i), |\Pi_{xx}(z_i)|)$ as in the proof of Proposition 2, we show that the average "misoptimization variance" of actions conditional on (z_i, λ_i) is

$$m(z_i, \lambda_i, \theta) := \mathbb{E}_{v_i}\left[(x_i - x^*(z_i(\theta)))^2 \mid z_i, \lambda_i, \theta\right] = \frac{\lambda_i X(\theta)^{1+\gamma-\chi(1+\epsilon)}\theta_i^{1+\epsilon}}{v_\Pi(\epsilon, \chi, \bar{w}, \bar{X})} \tag{68}$$

where $v_\Pi > 0$ is defined in Equation 53. Using the law of iterated expectations, we can write $m(G) = \mathbb{E}_{\theta_i, \lambda_i}[m(z_i, \lambda_i, \theta)]$. Assessing this outer expectation, we derive

$$m(G) = \frac{\lambda}{v_\Pi(\epsilon, \chi, \bar{w}, \bar{X})} \cdot X(\theta)^{1+\gamma-\chi(1+\epsilon)} \cdot \mathbb{E}_{\theta_i}[\theta_i^{1+\epsilon}]$$
$$= \frac{\lambda}{v_\Pi(\epsilon, \chi, \bar{w}, \bar{X})} \cdot X(\theta)^{1+\gamma-\chi(1+\epsilon)} \cdot \theta^{1+\epsilon} \tag{69}$$

where, in the second line, we use the definition of $\theta = \hat{\theta}(G)$. This expression clearly increases in θ, conditional on X. We next observe that $X(\theta)^{1+\gamma-\chi(1+\epsilon)}$ increases in X if $\gamma > \chi(1+\epsilon) - 1$. This condition is guaranteed by $\gamma > \chi + 1$ and $\chi\epsilon < 1$. Moreover, by Proposition 2, the stated conditions ensure that $X(\theta)$ is an increasing function. Thus, $m(G)$ can be written as $m(\theta)$ and $m(\theta)$ increases in θ. This proves the first statement.

We next prove the second statement about the monotonicity of attention. The entropy of a Gaussian random variable with variance σ^2 is proportional, up to scaling and constants, to $\log(\sigma^2)$. We therefore derive, up to scaling and constants,

$$a(G) = (-1 - \gamma + \chi(1+\epsilon))\log X(\theta) - (1+\epsilon)\mathbb{E}_{\theta_i}[\log\theta_i] \tag{70}$$

This is monotone decreasing in X if $\gamma > \chi(1+\epsilon) - 1$, which is implied by our conditions $\chi\epsilon < 1$ and $\gamma > \chi + 1$. It is monotone decreasing in G if $\mathbb{E}_{\theta_i}[\log\theta_i]$ increases in G. The latter claim is true because $y \mapsto \log y$ is an increasing function. Finally, X is monotone in the FOSD ordering as $G \succsim_{FOSD} G' \implies \hat{\theta}(G) \geq \hat{\theta}(G')$. Therefore, $a(G)$ is monotone decreasing in G in the sense of FOSD. This proves the second part of the statement.

$\square$

A.4   Proof of Proposition 4

Proof. We first derive output in the fully attentive $\lambda = 0$ limit, which we define by some mapping $X_0 : \Theta \to \mathbb{R}$. Recall the fixed-point equation for output from Proposition 2:

$$X(\theta) = a_0 X(\theta)^{1-\chi\epsilon}\theta^\epsilon - a_1 X(\theta)^{\gamma-\chi}\theta \tag{71}$$

When $\lambda = 0$, we have that $a_1 = 0$. Thus,

$$X_0(\theta) = a_0 X_0(\theta)^{1-\chi\epsilon}\theta^\epsilon \tag{72}$$

Or simply:

$$X_0(\theta) = a_0^{\frac{1}{\chi\epsilon}}\theta^{\frac{1}{\chi}} \tag{73}$$

We now define the proportional wedge between equilibrium output and output without the attention friction as:

$$W(\theta; \lambda) := \frac{X(\theta)}{X_0(\theta)} = \frac{X(\theta)}{a_0^{\frac{1}{\chi\epsilon}}\theta^{\frac{1}{\chi}}} \tag{74}$$

Via this definition, we re-write output in the form claimed in the Proposition.

$$\log X(\log\theta) = X_0 + \chi^{-1}\log\theta + \log W(\log\theta) \tag{75}$$

where $X_0 := \frac{1}{\chi\epsilon}\log a_0$.

We next prove that the wedge is positive. To prove this and other properties, we write a fixed-point

equation for $W(\theta)$. Combining the definition of the wedge with Equation 71, we obtain

$$W(\theta) = W(\theta)^{1-\chi\epsilon} - a_1(\lambda)a_0^{\frac{\gamma-\chi-1}{\chi\epsilon}} W(\theta)^{\gamma-\chi}\theta^{\frac{\gamma-1}{\chi}} \tag{76}$$

Based on identical arguments to those in the proof of Proposition 2, the wedge is positive and unique under the exact same conditions that $X(\theta)$ is positive and unique: $\chi\epsilon < 1$ and $\gamma > \chi + 1$. Moreover, $W(\theta)$ crosses the 45 degree line from above. To show that $W(\theta) \leq 1$, it then suffices to show that the right-hand-side of the fixed point equation is less than unity when evaluated at $W(\theta) = 1$. As $a_1, a_0 > 0$, this is immediate. Thus $\log W(\theta) \leq 0$, as claimed. Moreover, given that $\frac{\partial a_1}{\partial \lambda} > 1$, it is immediate to show $\frac{\partial W}{\partial \lambda} < 0$.

Toward the final claim, we show that $\log W(\theta)$ is monotone decreasing in θ. First, we implicitly differentiate the fixed point condition:

$$\begin{aligned}
\frac{dW}{d\theta} = &\left[(1-\chi\epsilon)W(\theta)^{-\chi\epsilon} - a_1 a_0^{\frac{\gamma-\chi-1}{\chi\epsilon}} (\gamma-\chi)W(\theta)^{\gamma-\chi-1}\theta^{\frac{\gamma-1}{\chi}} \right] \frac{dW}{d\theta} \\
&- a_1 a_0^{\frac{\gamma-\chi-1}{\chi\epsilon}} \frac{\gamma-1}{\chi} W(\theta)^{\gamma-\chi}\theta^{\frac{\gamma-\chi-1}{\chi}}
\end{aligned} \tag{77}$$

or:

$$\frac{dW}{d\theta} = \frac{-a_1 a_0^{\frac{\gamma-\chi-1}{\chi\epsilon}} \frac{\gamma-1}{\chi} W(\theta)^{\gamma-\chi}\theta^{\frac{\gamma-\chi-1}{\chi}}}{1 - \left[(1-\chi\epsilon)W(\theta)^{-\chi\epsilon} - a_1 a_0^{\frac{\gamma-\chi-1}{\chi\epsilon}} (\gamma-\chi)W(\theta)^{\gamma-\chi-1}\theta^{\frac{\gamma-1}{\chi}} \right]} \tag{78}$$

which we can rewrite as, after multiplying by $\frac{\theta}{W}$, as

$$\frac{d\log W}{d\log \theta} = \frac{-a_1 a_0^{\frac{\gamma-\chi-1}{\chi\epsilon}} \frac{\gamma-1}{\chi} W(\theta)^{\gamma-\chi}\theta^{\frac{\gamma-1}{\chi}}}{W(\theta) - \left[(1-\chi\epsilon)W(\theta)^{1-\chi\epsilon} - a_1 a_0^{\frac{\gamma-\chi-1}{\chi\epsilon}} (\gamma-\chi)W(\theta)^{\gamma-\chi}\theta^{\frac{\gamma-1}{\chi}} \right]} \tag{79}$$

By positivity of a_1 and a_0 and the assumption that $\gamma > \chi + 1$, the numerator of this expression is negative. To show that the wedge is monotone decreasing, we need to show that the denominator is positive. To this end, we see that:

$$\begin{aligned}
W(\theta) &= W(\theta)^{1-\chi\epsilon} - a_1 a_0^{\frac{\gamma-\chi-1}{\chi\epsilon}} W(\theta)^{\gamma-\chi}\theta^{\frac{\gamma-1}{\chi}} \\
&\geq \underbrace{(1-\chi\epsilon)}_{\chi\epsilon<1} W(\theta)^{1-\chi\epsilon} - a_1 a_0^{\frac{\gamma-\chi-1}{\chi\epsilon}} \underbrace{(\gamma-\chi)}_{\gamma>\chi+1} W(\theta)^{\gamma-\chi}\theta^{\frac{\gamma-1}{\chi}}
\end{aligned} \tag{80}$$

This completes the proof. $\qquad\square$

A.5   Proof of Corollary 2

Proof. The labor demand of any given firm i is given by $L_i = \frac{x_i}{\theta_i}$. Total labor demand in the economy is then given by:

$$L = \int_{[0,1]} L_i \, \mathrm{d}i \tag{81}$$

Using Proposition 1, the definition of $x^*(z_i)$ in the proof of Proposition 2, and the equilibrium law of motion $X(\theta)$, we write the production of each firm as

$$x_i = x^*(z_i) + \tilde{v}_i = v_x X(\theta)^{1-\chi\epsilon} \theta_i^\epsilon + \tilde{v}_i \tag{82}$$

where $\tilde{v}_i$ is the misoptimization scaled by its endogenous standard deviation. Plugging this into the expression for L, we derive

$$L = v_x X(\theta)^{1-\chi\epsilon} \int_0^1 \theta_i^{\epsilon-1} \, \mathrm{d}i \tag{83}$$

Simplifying and applying a law of large numbers, we write this as

$$L_t = L(\theta) = v_x X(\theta)^{1-\chi\epsilon} \theta^{\epsilon-1} \tag{84}$$

where we define, as in the main text, $\theta := \left(\mathbb{E}_{\theta_i}[\theta_i^{\epsilon-1} \mid \theta] \right)^{\frac{1}{\epsilon-1}}$.

Combining the definition $\log A(\theta) = \log X(\theta) - \log L(\theta)$ with Equation 84, we derive

$$\log A(\theta) = -\log v_x + \chi\epsilon \log X(\theta) - (\epsilon - 1) \log \theta \tag{85}$$

Using our representation of aggregate output from Proposition 4, we obtain:

$$\log A(\theta) = (\chi\epsilon X_0 - \log v_x) + \log \theta + \chi\epsilon \log W(\theta) \tag{86}$$

where $W(\cdot)$ inherits all of the properties proved in Proposition 4. We finally observe that $X_0 = \frac{\log v_x}{\chi\epsilon}$, as defined in the proof of Proposition 4, so $\chi\epsilon X_0 - \log v_x = 0$. This completes the proof. $\qquad\square$

A.6   Proof of Corollary 3

Proof. From Proposition 4, the expression for output is

$$\log X(\log \theta) = X_0 + \chi^{-1} \log \theta + \log W(\log \theta) \tag{87}$$

Moreover, if $\lambda = 0$, then $\log W(\log \theta) = 0$.

First, consider the response of output to a small shock ν_t starting from θ_{t-1}. We have immediately that:

$$\left. \frac{\partial \log X(\log \theta)}{\partial \log \theta} \right|_{\theta=\theta_{t-1}} = \chi^{-1} + \left. \frac{\partial \log W(\log \theta)}{\partial \log \theta} \right|_{\theta=\theta_{t-1}} \tag{88}$$

When $\lambda = 0$, then $\log W \equiv 0$ according to Proposition 4. Thus $\frac{\partial \log X(\log \theta)}{\partial \log \theta} \equiv \chi^{-1}$ in all states, if

inattention is removed from the model. If instead $\lambda > 0$ then $\frac{\partial \log W (\log \theta)}{\partial \log \theta}|_\theta$ is a non-linear function of θ which satisfies

$$\frac{\mathrm{d} \log W}{\mathrm{d} \log \theta} = \frac{-a_1 a_0^{\frac{\gamma-\chi-1}{\chi\epsilon}} \frac{\gamma-1}{\chi} W(\theta)^{\gamma-\chi} \theta^{\frac{\gamma-1}{\chi}}}{W(\theta) - \left[(1-\chi\epsilon)W(\theta)^{1-\chi\epsilon} - a_1 a_0^{\frac{\gamma-\chi-1}{\chi\epsilon}}(\gamma-\chi)W(\theta)^{\gamma-\chi}\theta^{\frac{\gamma-1}{\chi}}\right]} \tag{89}$$

Toward showing endogenous stochastic volatility, we approximate $\log X$ to the first order in the shock ν_t:

$$\log X(\nu_t; \theta_{t-1}) \approx \log X(\theta_{t-1}) + \frac{\partial \log X(\log \theta)}{\partial \log \theta}\bigg|_{\theta=\theta_{t-1}} \nu_t \tag{90}$$

By the same logic above, this is state independent if $\lambda = 0$ and $\log W \equiv 0$. Taking the variance of this expression conditional on θ_{t-1} yields:

$$\mathrm{Var}[\log X_t \mid \theta_{t-1}] = \left(\chi^{-1} + \frac{\partial \log W(\log \theta)}{\partial \log \theta}\bigg|_{\theta=\theta_{t-1}}\right)^2 \sigma_\theta^2 \tag{91}$$

Taking a second-order approximation in ν_t and differentiating yields the asymmetric shock propagation equation:

$$\frac{\partial \log X(\log \theta)}{\partial \log \theta}\bigg|_{\theta=\theta_{t-1}} = \chi^{-1} + \frac{\partial \log W(\log \theta)}{\partial \log \theta}\bigg|_{\theta=\theta_{t-1}} + \left(\frac{\partial^2 \log W(\log \theta)}{\partial \log \theta^2}\bigg|_{\theta=\theta_{t-1}}\right)\nu_t \tag{92}$$

Again, this is state independent if $\lambda = 0$ and $\log W \equiv 0$. $\qquad\square$

A.7   Additional Calculations

A.7.1   Quadratic Approximation of Risk-Adjusted Profits

Using the expressions for dollar profits and marginal utility in Equation 8, we can write firms' risk-adjusted profits as the following:

$$\Pi(x, z_i) := X^{-\gamma}\left(x^{1-\frac{1}{\epsilon}}X^{\frac{1}{\epsilon}} - x\frac{w}{\theta_i}\right) \tag{93}$$

where, as throughout, we define the decision state vector $z_i = (\theta_i, X, w)$. The optimal action in the absence of stochastic choice solves the first-order condition

$$\left(1 - \frac{1}{\epsilon}\right)x^{*-\frac{1}{\epsilon}}X^{\frac{1}{\epsilon}} = \frac{w}{\theta_i} \tag{94}$$

which can be re-arranged to define

$$x^*(z_i) = \left(1 - \frac{1}{\epsilon}\right)^\epsilon X\left(\frac{w}{\theta_i}\right)^{-\epsilon} \tag{95}$$

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We now approximate the firm's profit function to second order in x around $x^*(z_i)$:

$$\Pi(x, z_i) = \Pi(x^*(z_i), z_i) + \Pi_x(z_i)(x - x^*(z_i)) + \frac{1}{2}\Pi_{xx}(z_i)(x - x^*(z_i))^2 + O^3(x)$$
$$=: \tilde{\Pi}(x, z_i) + O^3(x) \tag{96}$$

where $\Pi_x(z_i) := \Pi_x(x, z_i)|_{x=x^*(z_i)}$ and $\Pi_{xx}(z_i) := \Pi_{xx}(x, z_i)|_{x=x^*(z_i)}$. By the envelope theorem, $\Pi_x(z_i) = 0$. Thus, our approximation reduces to the quadratic utility function in the Linear-Quadratic equilibrium:

$$\tilde{\Pi}(x, z_i) = \Pi(x^*(z_i), z_i) + \frac{1}{2}\Pi_{xx}(z_i)(x - x^*(z_i))^2 \tag{97}$$

It remains to characterize the intercept and curvature. We first derive the intercept:

$$\Pi(x^*(z_i), z_i) = X^{-\gamma} \left(X \left(\frac{w}{\theta_i} \right)^{1-\epsilon} \right) \left(\left[1 - \frac{1}{\epsilon} \right]^{\epsilon\left(1-\frac{1}{\epsilon}\right)} - \left[1 - \frac{1}{\epsilon} \right]^{\epsilon} \right)$$
$$= X^{-\gamma} \left(X \left(\frac{w}{\theta_i} \right)^{1-\epsilon} \right) \epsilon^{-\epsilon} (\epsilon - 1)^{\epsilon-1} \tag{98}$$

We now characterize the curvature, which is the product of marginal utility with the curvature of the dollar-profit function:

$$\Pi_{xx}(z_i) = X^{-\gamma} \cdot \pi_{xx}(z_i) \tag{99}$$

We calculate, using the form of the profit function from Equation 8, the dollar profit function's second derivative:[29]

$$\pi_{xx}(x^*(z_i), X) = -\frac{1}{\epsilon} \left(1 - \frac{1}{\epsilon} \right) (x^*(z_i))^{-1-\frac{1}{\epsilon}} X^{\frac{1}{\epsilon}}$$
$$= -\frac{1}{\epsilon} \left(1 - \frac{1}{\epsilon} \right) \left(1 - \frac{1}{\epsilon} \right)^{-\left(1+\frac{1}{\epsilon}\right)\epsilon} X^{-1-\frac{1}{\epsilon}} X^{\frac{1}{\epsilon}} \left(\frac{w}{\theta_i} \right)^{\epsilon\left(1+\frac{1}{\epsilon}\right)} \tag{100}$$
$$= -\epsilon^{-1}(\epsilon - 1)^{-\epsilon} X^{-1} \left(\frac{w}{\theta_i} \right)^{1+\epsilon}$$

We substitute in the wage rule, Equation 3, to derive

$$\pi_{xx}(z_i) = -v_\pi(\epsilon, \chi, \bar{X}, \bar{w}) \cdot \theta_i^{-1-\epsilon} X^{\chi(1+\epsilon)-1} \tag{101}$$

as in Equation 13, where the constant is

$$v_\pi(\epsilon, \chi, \bar{X}, \bar{w}) := (\epsilon - 1)^{-\epsilon} \epsilon^{-1} \bar{w}^{1+\epsilon} \bar{X}^{-\chi(1+\epsilon)} > 0 \tag{102}$$

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[29]Because marginal costs are constant, this curvature arises purely from the curvature of the revenue function.

A.7.2 Quadratic Approximation of Final-Goods Technology

We now consider the second-order approximation of the aggregator, which is re-printed below

$$X(\{x_i\}_{i\in[0,1]}) = \left(\int_0^1 x_i^{\frac{\epsilon-1}{\epsilon}} \, di\right)^{\frac{\epsilon}{\epsilon-1}} \tag{103}$$

Technically speaking, we take a quadratic approximation of a discretized version of this aggregator, and then take consider the limit of this approximation. First, we suppose that there are $K \times K' \times K''$ discrete firms. Define the firm-level state for any firm $kk'k''$ as $\omega_{kk'k''} = (\theta_k, \lambda_{k'}, v_{k''})$ with corresponding production level $x(\omega_{kk'k''})$. Define the CES aggregator in this economy as:

$$X_{KK'K''}(\{x_{kk'k''}\}) = \left(\frac{1}{K}\sum_{k=1}^{K}\frac{1}{K'}\sum_{k'=1}^{K'}\frac{1}{K''}\sum_{k''=1}^{K''} x(\omega_{kk'k''})^{1-\frac{1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{104}$$

Second, we take a quadratic approximation of this function around the firm-level optimal production points $x_{kk'k''} = x^*(\theta_k)$:

$$X_{KK'K''} = X_K^* + \frac{1}{K}\sum_{k=1}^{K}\frac{1}{K'}\sum_{k'=1}^{K'}\frac{1}{K''}\sum_{k''=1}^{K''} D_k(x(\omega_{kk'k''}) - x^*(\theta_k))$$

$$+ \frac{1}{K}\sum_{k=1}^{K}\frac{1}{K'}\sum_{k'=1}^{K'}\frac{1}{K''}\sum_{k''=1}^{K''}\frac{1}{K}\sum_{\tilde{k}=1}^{K}\frac{1}{K'}\sum_{\tilde{k}'=1}^{K'}\frac{1}{K''}\sum_{\tilde{k}''=1}^{K''}\frac{1}{2}D_{k\tilde{k}}^2(x(\omega_{kk'k''}) - x^*(\theta_k))(x(\omega_{\tilde{k}\tilde{k}'\tilde{k}''}) - x^*(\theta_{\tilde{k}}))$$

$$\tag{105}$$

where:

$$X_K^* = \left(\frac{1}{K}\sum_{k=1}^{K} x^*(\theta_k)^{1-\frac{1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{106}$$

and:

$$D_k = (X_K^*)^{\frac{1}{\epsilon}} x^*(\theta_k)^{-\frac{1}{\epsilon}} \tag{107}$$

and:

$$D_{k\tilde{k}}^2 = \begin{cases} -KK'K''\frac{1}{\epsilon}x^*(\theta_k)^{-\frac{1}{\epsilon}-1}(X_K^*)^{\frac{1}{\epsilon}} + \frac{1}{\epsilon}\frac{\partial X_K^*}{\partial x_{kk'k''}}(X_K^*)^{\frac{1}{\epsilon}-1}x^*(\theta_k)^{-\frac{1}{\epsilon}} & \text{if } kk'k'' = \tilde{k}\tilde{k}'\tilde{k}'' \\ \frac{1}{\epsilon}(X_K^*)^{\frac{1}{\epsilon}-1}(x^*(\theta_k))^{-\frac{1}{\epsilon}}(x^*(\theta_{\tilde{k}}))^{-\frac{1}{\epsilon}} & \text{if } kk'k'' \neq \tilde{k}\tilde{k}'\tilde{k}''. \end{cases} \tag{108}$$

We now take limits of this approximation in the following order. We first send $K'' \to \infty$. Observe that, for fixed k, k', we have that each k'' firm by Proposition 1 has action distributed as $N(x^*(\theta_k), \sigma_{kk'}^2)$. Thus, as $K'' \to \infty$, by the law of large numbers:

$$\frac{1}{K''}\sum_{k''=1}^{K''} D_k(x(\omega_{kk'k''}) - x^*(\theta_k)) \to^{a.s} 0 \tag{109}$$

Thus, the second term in the quadratic expansion above is zero almost surely in the large firm limit.

We can perform the same exercise for the third term in the quadratic expansion, which we can write as:

$$Q = \frac{1}{K}\sum_{k=1}^{K}\frac{1}{K'}\sum_{k'=1}^{K'}\frac{1}{K}\sum_{\tilde{k}=1}^{K}\frac{1}{K'}\sum_{\tilde{k}'=1}^{K'}\left(\frac{1}{K''^2}\sum_{k''=1}^{K''}\sum_{\tilde{k}''=1}^{K''}\frac{1}{2}D_{k\tilde{k}}^2(x(\omega_{kk'k''})-x^*(\theta_k))(x(\omega_{\tilde{k}\tilde{k}'\tilde{k}''})-x^*(\theta_{\tilde{k}}))\right)$$

$$= \frac{1}{K}\sum_{k=1}^{K}\frac{1}{K'}\sum_{k'=1}^{K'}\frac{1}{K}\sum_{\tilde{k}=1}^{K}\frac{1}{K'}\sum_{\tilde{k}'=1}^{K'}\frac{1}{2}D_{k\tilde{k}}^2\left(\frac{1}{K''^2}\sum_{k''=1}^{K''}\sum_{\tilde{k}''=1}^{K''}(x(\omega_{kk'k''})-x^*(\theta_k))(x(\omega_{\tilde{k}\tilde{k}'\tilde{k}''})-x^*(\theta_{\tilde{k}}))\right)$$

$$(110)$$

Fix $k = \tilde{k}$, $k' = \tilde{k}'$ and consider the summation in brackets. This has two terms. First, for $\tilde{k}'' = k''$, the summand is simply $(x(\omega_{kk'k''})-x^*(\theta_k))^2$. Second, $\tilde{k}'' \neq k''$, the summand is the product of two independent normal random variables with common distribution distribution $N(0, \sigma_{kk'}^2)$. Thus, in the $K'' \to \infty$ limit we have that:

$$\frac{1}{K''^2}\sum_{k''=1}^{K''}\sum_{\tilde{k}''\neq k''}^{K''}(x(\omega_{kk'k''})-x^*(\theta_k))(x(\omega_{kk'\tilde{k}''})-x^*(\theta_k)) \to^{a.s} 0 \tag{111}$$

and:

$$\frac{1}{K''}\sum_{k''=1}^{K''}(x(\omega_{kk'k''})-x^*(\theta_k))(x(\omega_{\tilde{k}k'k''})-x^*(\theta_{\tilde{k}})) \to^{a.s} \sigma_{kk'}^2 \tag{112}$$

Thus, using the observation that $\lim_{K''\to\infty}\frac{\partial X_K^*}{\partial x_{kk'k''}} = 0$, we substitute in D_{kk}^2 to obtain:

$$Q = -\frac{1}{2\epsilon}\frac{1}{K}\sum_{k=1}^{K}\frac{1}{K'}\sum_{k'=1}^{K'}\frac{\sigma_{kk'}^2}{x^*(\theta_k)^{1+\frac{1}{\epsilon}}(X_K^*)^{-\frac{1}{\epsilon}}} \tag{113}$$

We now observe that $\sigma_{kk'}^2 = \frac{\lambda_{k'}}{\lambda}\sigma_k^2$. Thus, taking the $K' \to \infty$ limit we have that:

$$Q \to^{a.s.} -\frac{1}{2\epsilon}\frac{1}{K}\sum_{k=1}^{K}\frac{\lambda\sigma_k^2}{x^*(\theta_k)^{1+\frac{1}{\epsilon}}(X_K^*)^{-\frac{1}{\epsilon}}} \tag{114}$$

Now taking the limit as $K \to \infty$, we can express this as:

$$Q \to^{a.s} -\frac{1}{2\epsilon}\mathbb{E}\left[\frac{\lambda\sigma_k^2}{x^*(\theta_k)^{1+\frac{1}{\epsilon}}(X_K^*)^{-\frac{1}{\epsilon}}}\right] \tag{115}$$

Moreover, in the same limit, by applying the law of large numbers and the continuous mapping theorem we have that:

$$X_K^* \to^{a.s.} \left(\mathbb{E}[x^*(\theta_k)^{1-\frac{1}{\epsilon}}]\right)^{\frac{\epsilon}{\epsilon-1}} \tag{116}$$

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Combining all of the above, we have shown that, in the limit, almost surely:

$$X \approx \left(\mathbb{E}[(x^*(\theta_k))^{1-\frac{1}{\epsilon}}] \right)^{\frac{\epsilon}{\epsilon-1}} - \frac{1}{2\epsilon} \mathbb{E}\left[\frac{\lambda \sigma_k^2}{x^*(\theta_k)^{1+\frac{1}{\epsilon}}(X_K^*)^{-\frac{1}{\epsilon}}} \right] \tag{117}$$

Which we denote by the (somewhat imprecise, but standard) integral form over agents:

$$X = \left(\int_0^1 x^*(z_i)^{1-\frac{1}{\epsilon}} \, \mathrm{d}i \right)^{\frac{\epsilon}{\epsilon-1}} - \frac{1}{2\epsilon} \int_0^1 \frac{(x_i - x^*(z_i))^2}{(X^*)^{-\frac{1}{\epsilon}}(x^*(z_i))^{1+\frac{1}{\epsilon}}} \, \mathrm{d}i \tag{118}$$

A.7.3 Mapping Misoptimization Dispersion to the Model

Here, we explicitly calculate the within-model analog to Misoptimization Dispersion. We show that monotone misoptimization dispersion implies our within-model measure of misoptimization is monotone and therefore confirms the misoptimization cycles prediction.

Recall from Definition 2 the definition of aggregate misoptimization,

$$m(\theta) := \mathbb{E}_{\theta_i, \lambda_i, v_i} \left[(x_i - x^*(z_i(\theta)))^2 \mid \theta \right] \tag{119}$$

which, as derived in the proof of Proposition 3, had expression

$$m(\theta) = \frac{\lambda}{v_\Pi(\epsilon, \chi, \bar{w}, \bar{X})} \cdot X(\theta)^{1+\gamma-\chi(1+\epsilon)} \cdot \mathbb{E}_{\theta_i}[\theta_i^{1+\epsilon} \mid \theta] \tag{120}$$

Misoptimization Dispersion is the optimal-sales-weighted population average of the *normalized* mean-squared error of actions. Let us define this model object as

$$\tilde{m}(\theta) = \mathbb{E}_{\theta_i, \lambda_i, v_i} \left[\hat{s}^*(\theta_i) \left(\frac{(x_i - x^*(z_i(\theta)))}{x^*(z_i)} \right)^2 \mid \theta \right] \tag{121}$$

where $s^*(\theta_i)$ are sales weights evaluated at the optimal production levels. We can use the model's structure to simplify these weights:

$$\hat{s}^*(\theta_i) := \frac{q^*(z_i)x^*(z_i)}{\mathbb{E}_{\theta_i}[q^*(z_i)x^*(z_i)]} = \frac{X^{\frac{1}{\epsilon}}(v_x X^{1-\chi\epsilon}\theta_i^\epsilon)^{1-\frac{1}{\epsilon}}}{\mathbb{E}_{\theta_i}[X^{\frac{1}{\epsilon}}(v_x X^{1-\chi\epsilon}\theta_i^\epsilon)^{1-\frac{1}{\epsilon}}]} = \frac{\theta_i^{\epsilon-1}}{\theta^{\epsilon-1}} \tag{122}$$

where, as throughout, $\theta = \left(\mathbb{E}_{\theta_i}[\theta_i^{\epsilon-1}] \right)^{\frac{1}{\epsilon-1}}$. We can therefore write the expected variance of normalized misoptimizations, conditioning on a specific firm, as

$$\tilde{m}(z_i, \lambda_i, \theta) := \mathbb{E}_{v_i} \left[s^*(\theta_i) \left(\frac{x_i - x^*(z_i(\theta))}{x^*(z_i)} \right)^2 \mid z_i, \lambda_i, \theta \right] = \frac{\lambda_i X(\theta)^{\gamma+\chi(\epsilon-1)-1}\theta^{1-\epsilon}}{v_\Pi v_x^2} \tag{123}$$

where $v_\Pi, v_X > 0$ are defined in Equation 53. It is trivial to integrate over (θ_i, λ_i) to derive

$$\tilde{m}(\theta) = \frac{\lambda X(\theta)^{\gamma+\chi(\epsilon-1)-1}\theta^{1-\epsilon}}{v_\Pi v_x^2} \tag{124}$$

We can relate this to $m(\theta)$ by writing

$$\frac{m(\theta)}{\tilde{m}(\theta)} = v_x^2 \theta^{\epsilon-1} \mathbb{E}_{\theta_i}[\theta_i^{1+\epsilon} \mid \theta] X^{2(1-\chi\epsilon)} \tag{125}$$

See that, given $\epsilon > 1$ and $\chi\epsilon < 1$, this is an increasing function of both θ and X. Therefore, if $\tilde{m}(\theta)$ is monotone increasing in θ in an equilibrium with monotone $X(\theta)$, then $m(\theta)$ is also monotone increasing in θ.

B Extended Model

In this appendix, we formally develop the extension of the baseline model from Section 2 to include multiple inputs and market clearing wages. In the process, we will provide more direct model microfoundations for the wage rule and the stock return regression analysis in Section 5.3.

B.1 Set-up

Time is discrete, and indexed by $t \in \mathbb{N}$. There are three kinds of firms: perfectly competitive materials firms who use labor to produce materials; intermediate goods producers who differ in their productivity and who use labor and materials to produce a monopolistic variety indexed by $i \in [0, 1]$; and final goods firms who produce consumption goods as a constant elasticity of substitution aggregate of intermediate goods. There are two types of households: capitalists who own the firms in the economy, do not work and have constant relative risk aversion (CRRA) preferences over consumption; workers who supply labor, are hand-to-mouth (consuming all of their labor income in each period), and have GHH preferences over consumption and labor. Finally, as in our baseline model, the stochastic choice friction is embedded in the production of intermediate goods: intermediate goods producers perfectly cost-minimize but find it hard to produce the optimal amount.

B.1.1 Firms

Materials are produced by perfectly competitive firms with linear production technology in labor so that aggregate production of materials M_t is given by:

$$M_t = \theta_t^M L_t^M \tag{126}$$

where θ_t^M is the productivity of the materials sector and L_t^M is its labor input.

Intermediate goods producers of variety i are the monopoly producers of that variety. They have firm-specific productivity θ_{it} and use materials m_{it} and labor L_{it} to produce output x_{it} with Cobb-Douglas production technology:

$$x_{it} = \theta_{it} L_{it}^\alpha m_{it}^{1-\alpha} \tag{127}$$

where $\alpha \in (0, 1)$. To the extent that other intermediate goods (e.g. capital) exist and are combined in a CRS Cobb-Douglas production function with labor, this is fully general.

The stochastic process of productivity is exactly as described in Section 2.2. There is an aggregate productivity state $\theta_t \in \Theta$, which follows a first-order Markov process with transition density given by $h(\theta_t \mid \theta_{t-1})$. The cross-sectional productivity distribution is given in state Θ by the mapping $G : \Theta \to \Delta(\Theta)$, where we denote the productivity distribution in any state θ_t by $G_t = G(\theta_t)$ with corresponding density g_t. We assume that the total order on θ_t ranks distributions G_t by first-order stochastic dominance, or $\theta \geq \theta'$ implies $G(\theta) \succsim_{FOSD} G(\theta')$. Finally, materials productivity θ_t^M is determined as an increasing function of the overall productivity state θ_t.

Intermediate goods producers perfectly cost-minimize facing wages w_t and intermediate goods prices p_t^M. That is, for given production level x_{it}, they always choose the cost-minimizing input bundle. We define the firm-level *decision state* $z_{it} = (\theta_{it}, X_t, w_t, p_t^M) \in \mathcal{Z}$ as the concatenation of all decision-relevant variables that the firm takes as given; unlike in the baseline model, this definition includes the materials price. All firms believe that the vector z_{it} follows a first-order Markov process with transition densities described by $f : \mathcal{Z} \to \Delta(\mathcal{Z})$, with $f(z_{it}|z_{i,t-1})$ being the density of z_{it} conditional on last period's state being $z_{i,t-1}$. At time t, each firm i knows the sequence of previous $\{z_{is}\}_{s<t}$ but not the contemporaneous value z_{it}.

Given this firms have risk-adjusted profits given by $\Pi(x_{it}, z_{it})$. They then choose stochastic choice rules to maximize expected profits net of control costs, as captured by the following program which is identical to Equation 9 in the main model, with a different definition of the decision state and profits function:

$$\max_{p \in \mathcal{P}} \int_{\mathcal{Z}} \int_{\mathcal{X}} \Pi(x, z_{it}) \, p(x \mid z_{it}) \, \mathrm{d}x \; f(z_{it} \mid z_{i,t-1}) \, \mathrm{d}z - c\left(p, \lambda_i, z_{i,t-1}, f\right) \tag{128}$$

Intermediate goods firms generate profits in units of consumption goods given by π_{it}. The firms store these consumption goods and pay them out as dividends d_{it} to their owners in the following period:

$$d_{it+1} = \pi_{it} \tag{129}$$

A unit supply of stock in the firm, which confers the right to the dividend stream, is available at price P_{it}.

The outputs of intermediate goods firms are combined to produce consumption goods with a CES production technology. Thus, if the intermediate producers produce $\{x_{it}\}_{i\in[0,1]}$, then the aggregate supply of consumption goods is:

$$X_t = X(\{x_{it}\}_{i\in[0,1]}) = \left(\int_{[0,1]} x_{it}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{130}$$

B.1.2 Households

There are two types of households: capitalists and workers. Capitalists own all firms in the economy and workers are hand-to-mouth. Capitalists have preferences over streams of consumption $\{C_{t+j}\}_{j\in\mathbb{N}}$ given by:

$$\mathcal{U}^C(\{C_{t+j}\}_{j\in\mathbb{N}}) = \mathbb{E}_t \sum_{j=0}^{\infty} \beta_C^j \frac{C_{t+j}^{1-\gamma}}{1-\gamma} \tag{131}$$

where $\beta_C \in [0,1), \gamma \geq 0$. The dynamic budget constraint of capitalists is given by:

$$C_t + A_{t+1} + \int_{[0,1]} P_{it} S_{it+1} \, \mathrm{d}i \leq \int_{[0,1]} d_{it} S_{it} \, \mathrm{d}i + (1+r_t) A_t + \int_{[0,1]} P_{it} S_{it} \, \mathrm{d}i \tag{132}$$

where S_{it} is their stock-holding in intermediate firm i at time t and A_t is their bond-holding at time t.

Workers have preferences over streams of consumption and labor $\{C_{t+j}^W, L_{t+j}\}_{j \in \mathbb{N}}$ given by:

$$\mathcal{U}^W(\{C_{t+j}^W, L_{t+j}\}_{j \in \mathbb{N}}) = \mathbb{E}_t \sum_{j=0}^{\infty} \beta_W^j U \left(C_t^W - \frac{L_{t+j}^{1+\psi}}{1+\psi} \right) \tag{133}$$

where $U' > 0, U'' < 0, \psi > 0, \beta^W \in [0,1)$. Workers are hand-to-mouth and they supply labor L_t at wage w_t, meaning that they consume:

$$C_t^W = w_t L_t \tag{134}$$

B.1.3 Equilibrium

An equilibrium is simply a set of all endogenous variables:

$$\{L_t^M, M_t, p_t^M, p_t^*, w_t, \{x_{it}, L_{it}, m_{it}, \pi_{it}, d_{it}, P_{it}, S_{it}\}_{i \in [0,1]}, X_t, L_t, C_t, A_t\} \tag{135}$$

such that all agents optimize as described above and markets clear given the exogenous process $\{\theta_t, \theta_t^M, \{\theta_{it}\}_{i \in [0,1]}\}_{t \in \mathbb{N}}$.

We will primarily be interested, as in the main text, in linear-quadratic equilibria where Π is approximated around its optimal level and the CES aggregator is approximated as described in Section 2.5.

B.2 Characterizing Equilibrium

We now reduce the description of equilibrium to a scalar fixed-point equation that can equivalently be formulated in terms of total production or capitalist consumption. This simplifies the analysis of the model and allows us to establish some equilibrium properties.

B.2.1 Production by Intermediate Goods Firms

Owing to CES aggregation, intermediate goods firms face the following iso-elastic demand curve:

$$q_{it} = X_t^{\frac{1}{\epsilon}} x_{it}^{-\frac{1}{\epsilon}} \tag{136}$$

They moreover perfectly cost-minimize. As a result, given production level x_{it} their unit input choices solve the following program:

$$\min_{L_{it}, m_{it}} w_t L_{it} + p_t^M m_{it} \quad \text{s.t.} \quad x_{it} = \theta_{it} L_{it}^{\alpha} m_{it}^{1-\alpha} \tag{137}$$

Taking the ratio of the two FOCs and rearranging:

$$m_{it} = \frac{1-\alpha}{\alpha} \frac{w_t}{p_t^M} L_{it} \tag{138}$$

Thus, given x_{it}, the optimal labor and materials choices are given by:

$$
\begin{aligned}
L_{it} &= \frac{1}{\theta_{it}} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{p_t^M}{w_t}\right)^{1-\alpha} x_{it} \\
m_{it} &= \frac{1}{\theta_{it}} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \left(\frac{p_t^M}{w_t}\right)^{-\alpha} x_{it}
\end{aligned}
\tag{139}
$$

It follows that the cost of producing x_{it} is given by:

$$w_t L_{it} + p_t^M m_{it} = \frac{q_t}{\theta_{it}} x_{it} \tag{140}$$

where we define the unit marginal cost up to constant $c_\alpha > 0$:

$$q_t := \left[\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \frac{1}{1-\alpha} \right] w_t^\alpha (p_t^M)^{1-\alpha} = c_\alpha w_t^\alpha (p_t^M)^{1-\alpha} \tag{141}$$

We now turn to solving the firm's stochastic choice problem. From the above, firm dollar profits are given by:

$$\pi_{it} = X_t^{\frac{1}{\epsilon}} x_{it}^{1-\frac{1}{\epsilon}} - \frac{q_t}{\theta_{it}} x_{it} \tag{142}$$

Recall that this is paid out as a dividend at period $t+1$, $d_{it+1} = \pi_{it}$. Note moreover in equilibrium by market clearing that $A_t = 0$ and $S_{it} = 1$ for all $i \in [0,1]$ and $t \in \mathbb{N}$. Thus, $C_{t+1} = \int_{[0,1]} d_{it+1} \, di = \int_{[0,1]} \pi_{it} \, di$. The firm's risk-adjusted profit is then given by:

$$\Pi_{it} = C_{t+1}^{-\gamma} \pi_{it} \tag{143}$$

where the firm takes C_{t+1} as given.

As in the main text, we define the optimal production level $x^*(\Lambda, \theta)$ which solves:

$$x^*(\Lambda, \theta_{it}) := \arg\max_{x \in \mathcal{X}} \Pi(x; \Lambda, \theta_{it}) \tag{144}$$

and $\bar{\Pi}(\Lambda, \theta_{it})$ as the maximized objective. Now let $\Pi_{xx}(\Lambda, \theta_{it})$ denote the second derivative of the profits function in x, evaluated at x^*:

$$\Pi_{xx}(\Lambda, \theta_{it}) := \left. \frac{\partial^2 \Pi}{\partial x^2} \right|_{x^*(\Lambda, \theta_{it}); \Lambda, \theta} \tag{145}$$

The approximate objective of the intermediate goods firm is:

$$\tilde{\Pi}(x; \Lambda, \theta_{it}) := \bar{\Pi}(\Lambda, \theta_{it}) + \frac{1}{2} \Pi_{xx}(\Lambda, \theta_{it})(x - x^*(\Lambda, \theta_{it}))^2 \tag{146}$$

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Under this approximate objective, it follows by a slight algebraic variation of the arguments in Proposition 1 that optimal choices follow:

$$x_{it} \sim N\left(x_{it}^*, \frac{\lambda}{|\Pi_{xx,it}|}\right) \tag{147}$$

where:

$$x_{it}^* = \left(1 - \frac{1}{\epsilon}\right)^{\epsilon} X_t \theta_{it}^{\epsilon} q_t^{-\epsilon}$$
$$|\Pi_{xx,it}| = (\epsilon - 1)^{-\epsilon} \epsilon^{\epsilon-1} C_{t+1}^{-\gamma} X_t^{-1} \theta_{it}^{-1-\epsilon} q_t^{1+\epsilon} \tag{148}$$

These expressions mirror those in the main model, with $C_{t+1}^{-\gamma}$ replacing $X_t^{-\gamma}$ as the marginal utility and q_t replacing w_t as the marginal cost.

B.2.2   Finding Materials Prices and Wages

Materials producers maximize profits:

$$p_t^M \theta_t^M L_t^M - w_t L_t^M \tag{149}$$

Thus, in equilibrium, it follows that:

$$p_t^M = \frac{1}{\theta_t^M} w_t \tag{150}$$

The workers' labor supply condition is given by their Euler equation:

$$w_t = L_t^{\psi} \tag{151}$$

Moreover, we know that aggregate labor is equal to the sum of labor used to produce intermediates and materials:

$$L_t = \int_{[0,1]} L_{it} di + L_t^M = \int_{[0,1]} L_{it} di + \frac{1}{\theta_t^M} \int_{[0,1]} m_{it} di \tag{152}$$

where the second equality follows by market clearing for intermediates as $\int_{[0,1]} m_{it} di = M_t = \theta_t^M L_t^M$. We next substitute our expression of materials demand as a function of labor demand for intermediate goods firms to simplify the labor supply condition further:

$$
\begin{aligned}
L_t &= \int_{[0,1]} L_{it} di + \frac{1}{\theta_t^M} \int_{[0,1]} m_{it} di \\
&= \int_{[0,1]} L_{it} \, di + \frac{1}{\theta_t^M} \int_{[0,1]} \frac{1-\alpha}{\alpha} \frac{w_t}{p_t^M} L_{it} \, di \\
&= \left(1 + \frac{1}{\theta_t^M} \frac{1-\alpha}{\alpha} \frac{w_t}{p_t^M}\right) \int_{[0,1]} L_{it} \, di
\end{aligned}
\tag{153}
$$

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We finally substitute in the fact that the material input is priced at marginal cost to simplify further

$$L_t = \left(1 + \frac{1-\alpha}{\alpha}\right) \int_{[0,1]} L_{it} \mathrm{d}i$$
$$= \frac{1}{\alpha} \int_{[0,1]} L_{it} \mathrm{d}i \tag{154}$$

We now write this in terms of prices and output choices by substituting in, from the intermediate goods firm's cost-minimization, $L_{it} = \frac{1}{\theta_{it}} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{p_t^M}{w_t}\right)^{1-\alpha} x_{it}$. Thus:

$$L_t = \frac{1}{\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{p_t^M}{w_t}\right)^{1-\alpha} \int_{[0,1]} \frac{x_{it}}{\theta_{it}} \mathrm{d}i \tag{155}$$

We can then use our earlier characterization of the solution to the intermediate goods producers' stochastic choice problem to compute:

$$\int_{[0,1]} \frac{x_{it}}{\theta_{it}} \mathrm{d}i = \mathbb{E}\left[\frac{x_{it}}{\theta_{it}}\right]$$
$$= \mathbb{E}\left[\mathbb{E}\left[\frac{x_{it}}{\theta_{it}} | \theta_{it}\right]\right]$$
$$= \mathbb{E}\left[\frac{1}{\theta_{it}} x_{it}^*\right] \tag{156}$$
$$= \mathbb{E}\left[\frac{1}{\theta_{it}} \left(1 - \frac{1}{\epsilon}\right)^{\epsilon} X_t q_t^{-\epsilon} \theta_{it}^{\epsilon}\right]$$
$$= \left(1 - \frac{1}{\epsilon}\right)^{\epsilon} X_t q_t^{-\epsilon} \theta^{\epsilon-1}$$

where we use the definition $\theta = \left(\mathbb{E}_{\theta_i}[\theta_i^{\epsilon-1} \mid \theta]\right)^{\frac{1}{\epsilon-1}}$. By combining the previous two equations, we derive that total labor demand is given by:

$$L_t = \frac{1}{\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{p_t^M}{w_t}\right)^{1-\alpha} \left(1 - \frac{1}{\epsilon}\right)^{\epsilon} X_t q_t^{-\epsilon} \theta^{\epsilon-1} \tag{157}$$

Substituting this into the workers' intratemporal Euler equation, and using Equation 141 to write the marginal cost in terms of materials prices and wages, we obtain:

$$w_t^{\frac{1}{\psi}} = \frac{1}{\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\theta_t^M}\right)^{1-\alpha} \left(1 - \frac{1}{\epsilon}\right)^{\epsilon} \theta_t^{\epsilon-1} c_\alpha^{-\epsilon} X_t \left(w_t \left(\frac{p_t^M}{w_t}\right)^{1-\alpha}\right)^{-\epsilon}$$
$$= \frac{1}{\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} (\theta_t^M)^{(\epsilon-1)(1-\alpha)} \left(1 - \frac{1}{\epsilon}\right)^{\epsilon} \theta^{\epsilon-1} c_\alpha^{-\epsilon} X_t w_t^{-\epsilon} \tag{158}$$

Moreover, we can write the marginal cost for the firm as

$$q_t = c_\alpha w_t^\alpha (p_t^M)^{1-\alpha} = \bar{q}_t X_t^\chi \tag{159}$$

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where we define coefficient $\chi = \frac{\psi}{1+\epsilon\psi}$ and intercept

$$\bar{q}_t = \left[\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\frac{1}{1-\alpha}\right]^{1-\alpha}\left(\frac{1}{\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}(\theta_t^M)^{(\epsilon-1)(1-\alpha)}\left(1-\frac{1}{\epsilon}\right)^{\epsilon}\theta^{\epsilon-1}c_\alpha^{-\epsilon}\right)^{\chi}(\theta_t^M)^{-1+\alpha} \quad (160)$$

Marginal costs, holding fixed productivity, increase in output due to upward-sloping labor supply or convex disutilty of effort. The intercept of this "cost rule" varies as a function of productivity in the intermediate-goods and materials sectors. Observe that Equation 159 is the "fully Neoclassical" analog to our wage rule Equation 3; indeed, when $\alpha = 1$ or there is no materials factor, it reduces to a wage rule

$$w_t = \bar{w}_t X_t^{\alpha} \quad (161)$$

where $\bar{w}_t = \bar{q}_t|_{\alpha=1}$. This verifies our claim in the main text that the wage rule can be (essentially) micro-founded in the simple model. Indeed, in a model with materials or $\alpha < 1$, we obtain exactly the wage rule studied in the main text if $\theta_t^M = \theta_t^{\beta}$ where $\beta = \frac{\chi(\epsilon-1)}{(1-\chi(\epsilon-1))(1-\alpha)} > 0$, thereby canceling out the direct effect of productivity on the intercept of the wage rule.

B.2.3 Finding Equilibrium Output

We have characterized all endogenous objects in period t in terms of output X_t and capitalists' consumption C_{t+1}. It remains only to characterize these variables.

To this end, we approximate as we have throughout:

$$X = \left(\int_0^1 x^*(z_i)^{1-\frac{1}{\epsilon}}\,di\right)^{\frac{\epsilon}{\epsilon-1}} - \frac{1}{2\epsilon}\int_0^1 \frac{(x_i - x^*(z_i))^2}{(X^*)^{-\frac{1}{\epsilon}}(x^*(z_i))^{1+\frac{1}{\epsilon}}}\,di \quad (162)$$

where the mean and variance are taken over the realizations of θ_{it}, conditional on the aggregate state θ_t. Substituting in Equation 148, this provides one equation in terms of (X_t, C_{t+1}). Consider first the computation of X_t^*. Observe that we can write:

$$x_{it}^* = \delta_t X_t^{1-\chi\epsilon}\theta_{it}^{\epsilon} \quad (163)$$

where:

$$\delta_t = \left(1-\frac{1}{\epsilon}\right)^{\epsilon}\left(c_\alpha\bar{w}_t\left(\frac{1}{\theta_t^M}\right)^{1-\alpha}\right)^{-\epsilon} \quad (164)$$

Substituting this into the expression for X_t^*, we obtain:

$$X_t^* = \left(\delta_t^{\frac{\epsilon-1}{\epsilon}}X_t^{\frac{\epsilon-1}{\epsilon}(1-\chi\epsilon)}\theta_t^{\epsilon-1}\right)^{\frac{\epsilon}{\epsilon-1}} = \delta_t X_t^{1-\chi\epsilon}\theta_t^{\epsilon} \quad (165)$$

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Now consider the computation of the dispersion term. See that we can write:

$$
\mathbb{E}\left[\frac{(x_{it} - x^*(z_i))^2}{(X^*)^{-\frac{1}{\epsilon}}(x^*(z_i))^{1+\frac{1}{\epsilon}}}\right] = (X_t^*)^{\frac{1}{\epsilon}}\mathbb{E}\left[x^*(z_{it})^{-1-\frac{1}{\epsilon}}\mathbb{E}[(x_{it} - x^*(z_{it}))^2 | z_{it}]\right]
$$

$$
= (X_t^*)^{\frac{1}{\epsilon}}\mathbb{E}\left[x^*(z_{it})^{-1-\frac{1}{\epsilon}}\frac{\lambda}{|\Pi_{xx,it}|}\right]
$$

(166)

To simplify this, observe that we can write:

$$
\frac{1}{|\Pi_{xx,it}|} = \zeta_t^{-1}C_{t+1}^{\gamma}X_t^{1-\chi(\epsilon+1)}\theta_{it}^{1+\epsilon}
$$

(167)

where:

$$
\zeta_t = (\epsilon-1)^{-\epsilon}\epsilon^{\epsilon-1}\left(c_{\alpha}\bar{w}_t\left(\frac{1}{\theta_t^M}\right)^{1-\alpha}\right)^{1+\epsilon}
$$

(168)

So we may express:

$$
\mathbb{E}\left[\frac{(x_{it} - x^*(z_i))^2}{(X^*)^{-\frac{1}{\epsilon}}(x^*(z_i))^{1+\frac{1}{\epsilon}}}\right] = (X_t^*)^{\frac{1}{\epsilon}}\mathbb{E}\left[\delta_t^{-1-\frac{1}{\epsilon}}X_t^{-(1+\frac{1}{\epsilon})(1-\chi\epsilon)}\theta_{it}^{-1-\epsilon}\lambda\zeta_t^{-1}C_{t+1}^{\gamma}X_t^{1-\chi(\epsilon+1)}\theta_{it}^{1+\epsilon}\right]
$$

$$
= \lambda\zeta_t^{-1}\delta_t^{-1}C_{t+1}^{\gamma}X_t^{-\chi}
$$

(169)

Putting all of the above together we have that:

$$
X_t = \delta_t X_t^{1-\chi\epsilon}\theta_t^{\epsilon} - \frac{\lambda}{2\epsilon}\zeta_t^{-1}\delta_t^{-1}C_{t+1}^{\gamma}X_t^{-\chi}\theta_t
$$

(170)

The final equation we require comes from equating capitalists' consumption with the previous period's dividends, which is implied by market clearing in the securities market and the fact that workers are hand-to-mouth. Thus:

$$
C_{t+1} = \int_{[0,1]}\pi_{it}\,\mathrm{d}i
$$

(171)

Using our running approximation:

$$
\pi_{it} = \pi_{it}(x_{it}^*) + \frac{1}{2}\pi_{xx,it}(x_{it} - x_{it}^*)^2
$$

(172)

we obtain:

$$
\begin{aligned}
C_{t+1} &= \int_{[0,1]} \pi_{it} di \\
&= \mathbb{E}\left[\mathbb{E}\left[\pi_{it}(x_{it}^*) + \frac{1}{2}\pi_{xx,it}(x_{it}^*)(x_{it}-x_{it}^*)^2 \mid x_{it}^*\right] \mid \delta_t, \theta_t\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[\pi_{it}(x_{it}^*) + \frac{1}{2}\pi_{xx,it}(x_{it}^*)\frac{\lambda}{|\Pi_{xx,it}|} \mid x_{it}^*\right] \mid \delta_t, \theta_t\right] \\
&= \mathbb{E}\left[\pi_{it}(x_{it}^*) \mid \delta_t, \theta_t\right] - \frac{\lambda}{2}C_{t+1}^\gamma \\
&= (\epsilon-1)^{-1}\delta_t\theta_t^{\epsilon-1}X_t^{1-\chi(\epsilon-1)} - \frac{\lambda}{2}C_{t+1}^\gamma
\end{aligned}
\tag{173}
$$

We can therefore solve for X_t as a function of C_{t+1}:

$$
X_t = \left(\frac{C_{t+1} + \frac{\lambda}{2}C_{t+1}^\gamma}{(\epsilon-1)^{-1}\delta_t\theta_t^{\epsilon-1}}\right)^{\frac{1}{1-\chi(\epsilon-1)}}
\tag{174}
$$

Plugging this into the scalar fixed point equation for output then boils down the equilibrium of the model to a scalar fixed-point equation for the consumption of capitalists:

$$
\begin{aligned}
\left(\frac{C_{t+1} + \frac{\lambda}{2}C_{t+1}^\gamma}{(\epsilon-1)^{-1}\delta_t\theta_t^{\epsilon-1}}\right)^{\frac{1}{1-\chi(\epsilon-1)}} = {}& \delta_t\left(\frac{C_{t+1} + \frac{\lambda}{2}C_{t+1}^\gamma}{(\epsilon-1)^{-1}\delta_t\theta_t^{\epsilon-1}}\right)^{\frac{1-\chi\epsilon}{1-\chi(\epsilon-1)}}\theta_t^\epsilon \\
&- \frac{\lambda}{2\epsilon}\zeta_t^{-1}\delta_t^{-1}C_{t+1}^\gamma\left(\frac{C_{t+1} + \frac{\lambda}{2}C_{t+1}^\gamma}{(\epsilon-1)^{-1}\delta_t\theta_t^{\epsilon-1}}\right)^{\frac{-\chi}{1-\chi(\epsilon-1)}}\theta_t
\end{aligned}
\tag{175}
$$

The above can be summarized in the following result:

Proposition 5. *Equilibria of the model are characterized by the solutions to Equation 175.*

B.3 Existence, Uniqueness, and Monotonicity of Equilibrium

To establish existence of equilibrium, all we require is that the above equation has a solution. As there is always a trivial equilibrium with $C_{t+1} = 0$, we will focus on when there exists an equilibrium with positive output, when it is unique, and when it is monotone. In this more general setting, we show that so long as cognitive frictions are not too large, these properties apply.

Proposition 6. *Suppose $\chi(\epsilon-1) < 1$. There exists $\bar{\lambda} > 0$ such that there exists a unique equilibrium with positive output whenever $\lambda < \bar{\lambda}$. Moreover, equilibrium output is monotone increasing in aggregate productivity θ.*

Proof. Following Equation 175, define:

$$g_\lambda(C) = \left(\frac{C + \frac{\lambda}{2}C^\gamma}{(\epsilon - 1)^{-1}\delta\theta^{\epsilon-1}}\right)^{\frac{1}{1-\chi(\epsilon-1)}} - \left[\delta\left(\frac{C + \frac{\lambda}{2}C^\gamma}{(\epsilon - 1)^{-1}\delta\theta^{\epsilon-1}}\right)^{\frac{1-\chi\epsilon}{1-\chi(\epsilon-1)}}\theta^\epsilon\right.$$
$$\left. - \frac{\lambda}{2\epsilon}\zeta^{-1}\delta^{-1}C^\gamma\left(\frac{C + \frac{\lambda}{2}C^\gamma}{(\epsilon - 1)^{-1}\delta\theta^{\epsilon-1}}\right)^{\frac{-\chi}{1-\chi(\epsilon-1)}}\theta\right]$$

(176)

Observe that g_λ is continuous in λ in the sup-norm. Thus, if we can show that there is a unique value of $C \in \mathbb{R}_{++}$ such that $g_0(C) = 0$ and $g_0'(C) \neq 0$, then there exists λ such that for all $\lambda < \bar{\lambda}$ there will be a unique $C' \in \mathbb{R}_{++}$ such that $g_\lambda(C') = 0$.

To prove the result, it remains to show that there is a unique value of $C \in \mathbb{R}_{++}$ such that $g_0(C) = 0$ and $g_0'(C) \neq 0$ when $\chi(\epsilon - 1) < 1$. To this end, define $\tilde{C} = \frac{C}{(\epsilon-1)^{-1}\delta\theta^{\epsilon-1}}$ and see that:

$$g_0(\tilde{C}) = \tilde{C}^{\frac{1}{1-\chi(\epsilon-1)}} - b_{\tilde{C}}\tilde{C}^{\frac{1-\chi\epsilon}{1-\chi(\epsilon-1)}}$$

(177)

where $b_{\tilde{C}} = \delta\theta^\epsilon$. We can then compute:

$$g_0'(\tilde{C}) = \frac{1}{1-\chi(\epsilon-1)}\tilde{C}^{\frac{\chi(\epsilon-1)}{1-\chi(\epsilon-1)}} - b_{\tilde{C}}\frac{1-\chi\epsilon}{1-\chi(\epsilon-1)}\tilde{C}^{\frac{-\chi}{1-\chi(\epsilon-1)}}$$
$$g_0''(\tilde{C}) = \frac{\chi(\epsilon-1)}{(1-\chi(\epsilon-1))^2}\tilde{C}^{\frac{\chi(\epsilon-1)}{1-\chi(\epsilon-1)}-1} + b_{\tilde{C}}\frac{\chi(1-\chi\epsilon)}{(1-\chi(\epsilon-1))^2}\tilde{C}^{\frac{-\chi}{1-\chi(\epsilon-1)}-1}$$

(178)

From which we observe the following when $\chi(\epsilon - 1) < 1$:

$$\lim_{\tilde{C}\to 0} g_0'(\tilde{C}) = -\infty \quad \lim_{\tilde{C}\to\infty} g_0'(\tilde{C}) = \infty \quad g_0''(\tilde{C}) > 0 \quad \text{for all } \tilde{C} \in \mathbb{R}_{++}$$

(179)

We now establish monotonicity. If we can show that the unique value of $C \in \mathbb{R}_{++}$ such that $g_0(C) = 0$ and $g_0'(C) \neq 0$ is monotone increasing in θ, then there exists $\bar{\lambda}$ such that for all $\lambda < \bar{\lambda}$ the same will be true of the unique $C' \in \mathbb{R}_{++}$ such that $g_\lambda(C') = 0$.

To this end, see that the solution when $\lambda = 0$ is given by:

$$\ln\tilde{C} = \frac{(1 - \chi(\epsilon - 1))}{\chi\epsilon}\ln b_{\tilde{C}}$$

(180)

We also know that $\ln\tilde{C} = \ln C + \ln(\epsilon - 1) - \ln\delta - (\epsilon - 1)\ln\theta$. Thus, we have that:

$$\ln C = \frac{(1 - \chi(\epsilon - 1))}{\chi\epsilon}(\ln\delta + \epsilon\ln\theta) - \ln(\epsilon - 1) + \ln\delta + (\epsilon - 1)\ln\theta$$

(181)

As $\epsilon > 1$ and $1 > \chi(\epsilon - 1)$ by hypothesis, and δ is increasing in θ, the result follows. $\qquad\square$

B.4 Attention and Misoptimization Cycles in the Extended Model

Having shown that equilibrium output is monotone and increasing in the extended model, we now provide conditions under which the analog of Proposition 3 that establishes monotonicity of attention and mistakes holds in this setting:

Proposition 7. *Assume $\chi\epsilon < 1$ and $1 > \chi(\epsilon+1)$. There exists $\bar{\lambda}$ such that when $\lambda < \bar{\lambda}$, intermediate goods firms pay more attention and misoptimize less in lower-productivity, lower-output states.*

Proof. Recall that:

$$m(z_{it}) = \frac{\lambda_i}{|\Pi_{xx,it}|} = \lambda_i \zeta_t^{-1} C_{t+1}^{\gamma} X_t^{1-\chi(\epsilon+1)} \theta_{it}^{1+\epsilon} \tag{182}$$

Thus, the average extent of misoptimization in aggregate state θ is:

$$m(\theta) = \lambda \zeta(\theta)^{-1} C(\theta)^{\gamma} X(\theta)^{1-\chi(\epsilon+1)} \mathbb{E}[\theta_{it}^{1+\epsilon} \mid \theta] \tag{183}$$

See that $1 > \chi(\epsilon+1)$ implies $1 > \chi(\epsilon-1)$. As we assumed $\chi\epsilon < 1$, Proposition 6 implies that C and X are both increasing in θ. By the assumed FOSD ordering on θ, we have that $\mathbb{E}[\theta_{it}^{1+\epsilon} \mid \theta]$ is monotone increasing in θ. We moreover have that $\xi \propto \delta^{-\frac{1+\epsilon}{\epsilon}}$. Thus, as δ is increasing in θ, we have that ξ^{-1} is increasing in θ. This establishes that $m(\theta)$ is increasing in θ, and therefore that intermediate goods firms misoptimize less in lower productivity and lower output states. By the same arguments as in Proposition 3, it is immediate that the opposite pattern holds for attention. $\square$

B.5 Macroeconomic Dynamics in the Extended Model

We can moreover derive an analogous representation of the impact of inattention on macroeconomic dynamics through an attention wedge that depresses output relative to the fully-attentive benchmark. Formally:

Proposition 8. *Output can be written in the following way:*

$$\log X(\log \theta, \lambda) = \frac{1}{\chi} \log \tilde{\theta} + \log W(\log \theta, \lambda) \tag{184}$$

where $\tilde{\theta} = \theta \delta^{\frac{1}{\epsilon}}$ and $\log W(\log \theta, 0) = 0$ for all $\theta \in \Theta$.

Proof. The representation follows immediately by combining Equations 180 and 174. That the wedge is 0 when $\lambda = 0$ follows immediately from the same equations. $\square$

This formula differs from Proposition 4 only in so far as θ is replaced by $\tilde{\theta}$ which captures the effect of the inclusion of other factors of production and the endogenous labor supply of agents. Note that this result does not establish any properties of the wedge in this case, as the fixed point equation is challenging to manipulate. The nature of the wedge is then a quantitative question. Similarly to the main text, a concave attention wedge implies higher shock responsiveness in low states, greater responsiveness to negative than positive shocks, and volatility of output that is greater in low states.

B.6 Micro-foundation and Interpretation of the Stock Return Regressions

In Section 5.3, we showed that mistakes of the same size by firms lead to more adverse impacts on stock returns when the aggregate stock market return is low. We interpreted this as direct evidence in favor of our mechanism that risk-pricing is a key determinant of attention cycles. The simple model of Section 2 is too stylized to formally map to this regression. However, in the extended model developed in this section, we can derive exactly the regression we run from the theory and show how the estimated regression coefficients map to the risk-pricing channel in the theory.

First, from the Euler equation of capitalists, the equilibrium price of firm i at time t, P_{it} solves:

$$u'(C_t)P_{it} = \mathbb{E}_t[\beta u'(C_{t+1})(P_{it+1} + d_{it+1})] \tag{185}$$

where $d_{it+1} = \pi_{it}$. Thus we may write:

$$u'(\pi_{t-1})P_{it} = \beta u'(\pi_t)\pi_{it} + \beta u'(\pi_t)\mathbb{E}_t[P_{it+1}] \tag{186}$$

where $\pi_t = \int_{[0,1]} \pi_{it}\, di$. It follows that:

$$P_{it} = \beta \frac{u'(\pi_t)}{u'(\pi_{t-1})}\pi_{it} + \beta \frac{u'(\pi_t)}{u'(\pi_{t-1})}\mathbb{E}_t[P_{it+1}] \tag{187}$$

A production mistake $m_{it} \equiv x_{it} - x_{it}^*$ leads to profits (under our quadratic approximation) of:

$$\pi_{it} = \pi_{it}(x_{it}^*) + \pi_{xx,it}m_{it}^2 \tag{188}$$

Thus, the firm's stock price follows:

$$P_{it} = \beta \frac{u'(\pi_t)}{u'(\pi_{t-1})}\left(\pi_{it}(x_{it}^*) + \pi_{xx,it}m_{it}^2\right) + \beta \frac{u'(\pi_t)}{u'(\pi_{t-1})}\mathbb{E}_t[P_{it+1}] \tag{189}$$

Thus:

$$\frac{\partial P_{it}}{\partial m_{it}^2} = \beta \frac{u'(\pi_t)}{u'(\pi_{t-1})}\pi_{xx,it} \tag{190}$$

and:

$$\frac{\partial P_{it}}{\partial \pi_{it}} = \beta \frac{u'(\pi_t)}{u'(\pi_{t-1})} \tag{191}$$

To simplify this further, observe by the Euler equation for trading an equally weighted portfolio of all intermediate goods firms must satisfy, where P_t is the price of this portfolio (the stock market):

$$u'(C_t)P_t = \mathbb{E}_t[\beta u'(C_{t+1})(P_{t+1} + \pi_t)] \tag{192}$$

Or:

$$\beta \frac{u'(\pi_t)}{u'(\pi_{t-1})} = \frac{P_t}{\mathbb{E}_t[P_{t+1}] + D_{t+1}} = \frac{1}{R_t} \tag{193}$$

Which is the inverse aggregate return on equity between period t and period $t+1$, R_t. Therefore:

Proposition 9. *The equilibrium effect of mistakes on stock returns is given by:*

$$\frac{\partial P_{it}}{\partial m_{it}^2} = -\frac{1}{R_t}|\pi_{xx,it}| \tag{194}$$

If a mistake is measured in terms of its impact in profit units, then one obtains the simpler:

$$\frac{\partial P_{it}}{\partial \pi_{it}} = -\frac{1}{R_t} \tag{195}$$

Proof. Given in the text above. □

Of course, it is trivial to reformulate the above comparative statics in terms of firm level returns as P_{it-1} is invariant to innovations in m_{it}.

When equity returns are high, mistakes should (all else equal) have a lower price impact. Mapping this slightly more formally to our exact regression analysis: when we instrument for profits with mistakes, we should obtain a negative and significant coefficient on the interaction between profits and the aggregate stock market return. This is exactly what we find. The OLS regressions of returns on mistakes retain a similar structure but are intermediated by the curvature of dollar profits across firms. These regression models therefore provide a less sharp test of the risk-pricing channel, although empirically they produce entirely consistent results.

Supplementary Materials

for "Attention Cycles" by Flynn and Sastry

C Measuring Productivity and Misoptimization

This appendix describes in greater detail our data construction and empirical methodologies for our firm-level analysis of production and misoptimization. It serves in particular as a companion for Section 4.1 of the paper.

C.1 Sample Selection and Data Construction

Our main dataset is Compustat Annual Fundamentals, which compiles detailed information from public firms' financial statements. Table A9 summarizes the main sample variables that we use and their definitions. Production, in value terms, is defined as reported sales. Employment in Compustat is reported as the number of employees. To calculate a wage bill, we multiply this by the average industry wage calculated from the Census Bureau's County Business Patterns dataset in the same year, as the sector's total national wage bill divided by the number of employees. From 1998 onward, we use the 2- or 3-digit NAICS classification that is consistent with our main analysis. Prior to 1997, and the introduction of NAICS codes in the CBP data, we use 2-digit SIC industries. For materials expenditure, we measure the sum of reported variable costs (`cogs`) and sales and administrative expense (`xsga`) net of depreciation (`dp`) and the aforementioned wage bill. To measure the capital stock, we use a perpetual inventory method as in Ottonello and Winberry (2020) starting with the first reported observation of gross value of plant, property, and equipment and adding net investment or the differences in net value of plant, property, and equipment.[30]

We restrict the sample to firms based in the United States, reporting statistics in US Dollars, and present in the "Industrial" dataset. Within this sample, we apply the following additional filters:

1. Sales, material expenditures, and capital stock are strictly positive;
2. Employees exceed 10;
3. 2-digit NAICS is not 52 (Finance and Insurance) or 22 (Utilities);
4. Acquisitions as a proportion of assets (`aqc` over `at`) does not exceed 0.05.

The first two ensure that all companies meaningfully report all variables of interest for our production function estimation; the second applies a stricter cut-off to eliminate firms that are very small, and lead to outlier estimates of productivity and choices. The third filter eliminates firms in two industries that, respectively, may have highly non-standard production technology and non-standard market structure. The fourth is a simple screening device for large acquisitions which may spuriously show up as large innovations in firm choices and/or productivity. We finally restrict attention to firms operating on a fiscal calendar that ends in December, for more straightforward calculations of aggregate time trends.

---

[30]Note that, because of our later usage of fixed effects and lack of direct calculations using capital "expenditures" evaluated at an imputed rental rate, it is inessential to deflate the value of the capital stock.

We categorize the data into 44 sectors. These are defined at the 2-digit NAICS level, but for the Manufacturing (31-33) and Information (51) sectors, which we classify at the 3-digit level to achieve better balance of sector size. Table A12 lists the sectors along with summary statistics for their relative size, in terms of sales and employment, in cross-sections corresponding to 1990 and 2010, in the full (not selected) Compustat sample. Overall, the full dataset covers between 15-20% of US employment and 60-80% of US output, modulo the clarification that not all Compustat sales necessarily occur in the United States.

C.2 Production Function and Productivity Estimation

Our primary method for estimating production functions, and thereby recovering total factor productivity, is a cost share approach. In brief, we use cost shares for materials and labor to back out production elasticities, and treat the elasticity of capital as the implied "residual" given an assumed markup $\mu > 1$ (in our baseline, $\mu = 4/3$) and constant returns to scale. We validate, in subsection C.3 of this Appendix and in particular Lemma 3, that this method is consistent in sample up to an essentially negligible correction term, due to the underlying logic that input choices are "right on average" even in the presence of mistakes. The exact procedure is the following:

1. For all firms in industry j, calculate the estimated materials and labor shares:

$$
\begin{aligned}
\text{Share}_{M,j'} &= \frac{\sum_{i:j(i)=j'} \sum_t \text{MaterialExpenditure}_{it}}{\sum_{i:j(i)=j'} \sum_t \text{Sales}_{it}} \\
\text{Share}_{L,j'} &= \frac{\sum_{i:j(i)=j'} \sum_t \text{WageBill}_{it}}{\sum_{i:j(i)=j'} \sum_t \text{Sales}_{it}}
\end{aligned}
\tag{196}
$$

2. If $\text{Share}_{M,j'} + \text{Share}_{L,j'} \le \mu^{-1}$, then set

$$
\begin{aligned}
\alpha_{M,j'} &= \mu \cdot \text{Share}_{M,j'} \\
\alpha_{L,j'} &= \mu \cdot \text{Share}_{L,j'} \\
\alpha_{K,j'} &= 1 - \alpha_{M,j'} - \alpha_{L,j'}
\end{aligned}
\tag{197}
$$

3. Otherwise, adjust shares to match the assumed returns to scale, or set

$$
\begin{aligned}
\alpha_{M,j'} &= \frac{\text{Share}_{M,j'}}{\text{Share}_{M,j'} + \text{Share}_{L,j'}} \\
\alpha_{L,j'} &= \frac{\text{Share}_{L,j'}}{\text{Share}_{M,j'} + \text{Share}_{L,j'}} \\
\alpha_{K,j'} &= 0
\end{aligned}
\tag{198}
$$

A more flexible method for production function estimation might allow for the returns to scale and/or markups to vary across sectors. We opt not to make such a method a baseline because uniform returns to scale and markups are consistent with our subsequent empirical calibration of the model. In robustness checks, we have experimented in particular with extracting production function parameters under

different assumed markups and found overall stable results for the empirical behavior of production misoptimizations.

To translate our production function estimates into productivity, we first calculate a "Sales Solow Residual" $\tilde{\theta}_{it}$ of the following form:

$$\log \tilde{\theta}_{it} = \log \text{Sales}_{it} - \frac{1}{\mu} \left(\alpha_{M,j(i)} \cdot \log \text{MatExp}_{it} - \alpha_{L,j(i)} \cdot \log \text{Empl}_{it} - \alpha_{K,j(i)} \cdot \log \text{CapStock}_{it} \right) \quad (199)$$

where we intentionally use the "literal" accounting notation to highlight the fact that not all variables are specified in economically meaningful *quantity* units. To alleviate this issue in a conservative way, we define our final estimate of TFP as the residual of the previous from industry-by-time fixed effects. This procedure, under our presumed model of industry-level variation in factor prices, identifies (log) TFP rescaled by μ^{-1}. The rescaling is immaterial for our analysis. The exact mapping from theory to data is shown in Section C.3.

As a robustness check, we also calculate TFP using the method of Olley and Pakes (1996), applied separately to estimate the production function of each industry.[31] The methodology of Olley and Pakes (1996) aims, in particular, to correct the bias in standard least-squares estimates that under-states the output elasticity to capital, since that input is likely pre-determined and therefore mechanically less responsive to productivity. Since the underlying timing assumptions of Olley and Pakes (1996) are not directly compatible with our specifically assumed structure, we do not prefer it as a main estimate. But we are re-assured by these estimates' "upstream" and "downstream" similarity to our baseline estimates. To the first point, Table A13 shows the results from regressing the two TFP measures on one another in a common sample, including various levels of fixed effects. In each case, the slope is close to one and the within-R^2, or goodness of fit net of fixed effects, exceeds 0.6. To the second point, the relevant columns of Tables A3, A5, and A7 demonstrate how our main aggregate and firm-level results replicate under the alternative measurement scheme, with similar quantitative and qualitative take-aways.

C.3 Theory to Data: Micro-foundations

In this subsection, we outline more exactly the mapping from our model to our production function estimation via cost shares and our log-linear estimating equations.

Firms are monopolists within their unique products. We assume that this demand curve has a constant elasticity of substitution form, so prices q_{it} lie on the demand curve $\log q_{it} = \gamma_i - \frac{1}{\epsilon}(\log x_{it} - \log X_t)$ for some inverse elasticity $\epsilon > 1$ and aggregate output X_t.[32] Finally, firms face sector-specific input prices $(q_{j(i),L,t}, q_{j(i),M,t}, q_{j(i),K,t})$ for the three inputs, respectively.

We model firm (mis-) optimization in the following way that is uniform across inputs. Conditional on any chosen level of production, firms cost minimize over their input bundle conditional on observed

---

[31] In particular, we use the implementation by Yasar et al. (2008) of the `opreg` package in Stata. We use log investment as the proxy variable and year dummies as additional controls. We throw out estimates that imply individual elasticities that are negative or greater than 1, but do not otherwise enforce any returns to scale normalization.

[32] It is straightforward, and consistent with our modeling approach, also to allow substitution within more narrowly defined industries.

input prices. Let $q_{i,T,t}$ denote the associated "Total" input cost per unit of produced output, and $x^*(q_T, X)$ denote the unconditionally profit-maximizing level of production. Firms choose a production level which differs from this level by a *misoptimization* m_{it}:

$$\log x_{it} = \log x^*(q_{i,T,t}, X_t) + m_{it} \tag{200}$$

And the dynamics of the misoptimization, as given in the main text's Equation 23 and re-printed here, are described by an AR(1) process:

$$m_{it} = \rho m_{i,t-1} + \left(\sqrt{1 - \rho^2} \right) u_{it} \tag{201}$$

in which innovations u_{it} are mean zero with variance σ_{it}^2.

First, we characterize the firm's optimal production level x^* as the total input price (which itself will depend on productivity) and aggregate demand:

Lemma 1 (Optimal Output Choice). *The firm's optimal output choice is*

$$\log x^*(\theta, q_T, X) = \epsilon \log \left(1 - \frac{1}{\epsilon} \right) + \epsilon \gamma_i + \log X - \epsilon \log q_T \tag{202}$$

Proof. Immediate from the first-order conditions of the program

$$x^*(\theta, q_T, X) = \arg \max_x \left\{ x \left(e^{\gamma_i} x^{-\frac{1}{\epsilon}} X^{\frac{1}{\epsilon}} - q_T \right) \right\} \tag{203}$$

$\square$

We next characterize the optimal choice of each input, based on cost-minimization and the expression for chosen output as a function of optimal output and the misoptimization (Equation 200):

Lemma 2 (Input Choice). *For any input $Z \in \{L, M, K\}$,*

$$\log Z_{it} = \eta_i + \chi_{j(i),Z,t} + (\epsilon - 1) \log \theta_{it} + m_{it} \tag{204}$$

where

$$\eta_i = \epsilon \gamma_i + \epsilon \log \left(1 - \frac{1}{\epsilon} \right) \tag{205}$$

and

$$\chi_{j(i),Z,t} = \log \alpha_{Z.j(i)} - \log q_{j(i),Z,t} + \log X_t + (1 - \epsilon) \sum_Z \alpha_{Z,j(i)} (\log q_{j(i),Z,t} - \log \alpha_{Z,j(i)}) \tag{206}$$

Proof. In the cost minimization step, for any planned output choice Q, the firm solves

$$\min_{L_{it}, M_{it}, K_{it}} \sum_{z \in \{L,M,K\}} q_{j(i),Z,t} Z_{it} \quad \text{s.t. } \theta_{it} L_{it}^{\alpha_{L,j(i)}} M_{it}^{\alpha_{M,j(i)}} K_{it}^{\alpha_{K,j(i)}} \geq Q \tag{207}$$

Standard first-order methods yield the solution, for each input,

$$\log Z_{it} = \log q_{i,T,t} + \log Q + \log \alpha_{Z,j(i)} - \log q_{j(i),Z,t} \tag{208}$$

where the price index $q_{i,T,t}$, which is also the Lagrange multiplier on the constraint, is

$$\log q_{i,T,t} = \sum_Z \alpha_{Z,j(i)}(\log q_{j(i),Z,t} - \log \alpha_{Z,j(i)}) - \log \theta_{it} \tag{209}$$

The desired expression comes from substituting in Equations 200 and 203 into the above, for Q. $\quad\square$

This calculation validates our log-linear regression model Equation 22. It also has the same loading on the misoptimization m_{it} and log productivity $\log \theta_{it}$ for all inputs Z. Thus, all separate inputs, as well as total production in physical units, inherit the "optimal choice plus error" structure.

We finally describe and validate our method for recovering production function parameters from a cost shares approach. The following result shows how cost shares are recovered at the firm level, if all data were observed without noise:

Lemma 3 (Production Function Estimation). *For any input $Z \in \{L, M, K\}$, and firm i,*

$$\alpha_{j(i),Z} = \left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{q_{j(i),Z,t} Z_{it}}{q_{it} x_{it}} \exp\left(-\frac{m_{it}}{\epsilon}\right) \tag{210}$$

Proof. This can be calculated directly using the results of Lemmas 1 and 2. We work backwards starting from the result and substitute input demand from Equation 208 and calculate.

$$
\begin{aligned}
\alpha_{j(i),Z} &= \left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{q_{j(i),Z,t} Z_{it}}{q_{it} x_{it}} \exp\left(-\frac{m_{it}}{\epsilon}\right) \\
&= \left(1 - \frac{1}{\epsilon}\right)^{-1} \frac{\exp\left(\log q_{i,T,t} + \log x_{it} + \log \alpha_{j(i),Z,t}\right)}{\exp\left(\log q_{it} + \log x_{it}\right)} \exp\left(-\frac{m_{it}}{\epsilon}\right) \\
&= \left(1 - \frac{1}{\epsilon}\right)^{-1} \exp\left(\log q_{i,T,t} - \log q_{it} + \log \alpha_{j(i),Z,t} - \frac{m_{it}}{\epsilon}\right)
\end{aligned} \tag{211}
$$

where the third line cancels out x_{it} in the fraction. We take the demand curve $\log q_{it} = \gamma_i - \frac{1}{\epsilon}(\log x_{it} - \log X_t)$ and observe that, using the expression for the unconstrained optimal output in Equation 202 and the fact that $\log x_{it} = \log x_{it}^* + m_{it}$:

$$
\begin{aligned}
\log q_{it} &= \gamma_i + \frac{1}{\epsilon}\log X_t - \frac{1}{\epsilon}\left(\epsilon \log\left(1 - \frac{1}{\epsilon}\right) + \epsilon\gamma_i + \log X_t - \epsilon \log q_{i,T,t} + \log m_{it}\right) \\
&= -\log\left(1 - \frac{1}{\epsilon}\right) + \log q_{i,T,t} - \frac{m_{it}}{\epsilon}
\end{aligned} \tag{212}
$$

5

We then substitute the above back into Equation 211 to get

$$
\begin{aligned}
\alpha_{j(i),Z} &= \left(1 - \frac{1}{\epsilon}\right)^{-1} \exp\left(\log q_{i,T,t} - \left(-\log\left(1 - \frac{1}{\epsilon}\right) + \log q_{i,T,t} - \frac{m_{it}}{\epsilon}\right) + \log \alpha_{j(i),Z} - \frac{m_{it}}{\epsilon}\right) \\
&= \left(1 - \frac{1}{\epsilon}\right)^{-1} \exp\left((\log q_{i,T,t} - \log q_{i,T,t}) + \log\left(1 - \frac{1}{\epsilon}\right) + \log \alpha_{j(i),Z} + \left(\frac{m_{it}}{\epsilon} - \frac{m_{it}}{\epsilon}\right)\right) \quad (213) \\
&= \alpha_{j(i),Z}
\end{aligned}
$$

as desired. □

In words, this result says that the ratio of expenditures on input Z to total sales, multiplied by the markup and a correction factor related to the mistake, equals the production elasticity. In principle, we could simultaneously estimate the production function and the statistical properties of mistakes to correct for the fact that the term $\exp\left(-\frac{m_{it}}{\epsilon}\right)$ is not zero on average. In practice, our mistakes are zero mean by construction and have a variance of about 0.08 in sample. Using a log-linear calculation, and our standard value of $\epsilon = 4$, this implies an average correction factor of $\exp\left(\frac{1}{2 \cdot 4^2} \cdot 0.08\right) = 1.0025$ which is essentially negligible.

We finally show, in the theory, how the calculation of Equation 199, net of fixed effects, recovers a re-scaling of TFP. In this subsection's language, that calculation is

$$
\log \tilde{\theta}_{it} = \log x_{it} + \log q_{it} - \frac{1}{\mu}\left(\sum_Z \alpha_{Z,j(i)}(\log q_{j(i),Z,t} + \log Z_{it})\right) \quad (214)
$$

Substituting in the demand curve $\log q_{it} = \gamma_i - \frac{1}{\epsilon}(\log x_{it} - \log X_t)$

$$
\log \tilde{\theta}_{it} = \left(1 - \frac{1}{\epsilon}\right)\log x_{it} + \gamma_i + \frac{1}{\epsilon}\log X_t - \frac{1}{\mu}\left(\sum_Z \alpha_{Z,j(i)}(\log q_{j(i),Z,t} + \log Z_{it})\right) \quad (215)
$$

We next observe that $\mu = 1 - \frac{1}{\epsilon}$ and that $\log \theta_{it} = \log x_{it} - \left(\sum_Z \alpha_{Z,j(i)} \log Z_{it}\right)$. Thus,

$$
\log \tilde{\theta}_{it} = \frac{1}{\mu}\log \theta_{it} + \gamma_i + \frac{1}{\epsilon}\log X_t - \frac{1}{\mu}\left(\sum_Z \alpha_{Z,j(i)} \log q_{j(i),Z,t}\right) \quad (216)
$$

Grouping terms into fixed effects, this is

$$
\log \tilde{\theta}_{it} = \frac{1}{\mu}\log \theta_{it} + \tau_{j(i),t} + \gamma_i \quad (217)
$$

where $\tau_{j(i),t} = \frac{1}{\epsilon}\log X_t - \frac{1}{\mu}\left(\sum_Z \alpha_{Z,j(i)} \log q_{j(i),Z,t}\right)$ is the industry-by-time fixed effect capturing aggregate demand and factor prices. and γ_i is the firm fixed effect from the demand curve. Thus, net of fixed effects, we recover $\frac{1}{\mu}\log \theta_{it}$.

D Alternative Specifications of Stochastic Choice

In this section, we extend our basic class of cost function to allow for persistent mistakes as in the empirical analysis. In particular, we micro-found the AR(1) structure of mistakes that we uncovered in the data but abstracted from in the simple model. Further, we show how the core logic of attention cycles carries over to settings with alternative foundations for stochastic choice in terms of information acquisition of two forms: Gaussian signal extraction, and optimal signal processing with mutual information costs. Concretely, we will provide simple sufficient conditions under which an increase in the stakes of making mistakes—the curvature of intermediate goods firms' profits—leads to increased attention and smaller mistakes. In our baseline model, this corresponds to firms making smaller mistakes when market risk-pricing is more severe and leads to an attention cycle.

D.1 Persistent Mistakes

In our empirical work we showed that firms' mistakes are persistent. The basic model we have developed, however, places no restrictions on the auto-correlation of mistakes across time within a firm. In this section, we introduce a more general class of cost functional that allows us to place restrictions on the within-firm correlation of mistakes across time and, in particular, to derive the AR(1) formulation of mistakes that we use in the empirical analysis.

We have so far considered likelihood-separable cost functions $c : \mathcal{P} \times \mathcal{Z} \to \mathbb{R}$ of the form:

$$c(p; \theta_{t-1}) = \int_{\Theta} \int_{\mathcal{X}} \phi(p(x|\theta)) \, \mathrm{d}x \, f(z|z_{t-1}) \, \mathrm{d}z \tag{218}$$

for some convex ϕ that we take to be $\phi(y) = y \log y$. To allow for persistent mistakes we now allow the cost functional to depend on the previous period's mistake v_{t-1} and today's optimal action $c : \mathcal{P} \times \mathcal{Z} \times \mathbb{R} \to \mathbb{R}$ of the form:

$$c(p; z_{t-1}, v_{t-1}) = \int_{\Theta} \int_{\mathcal{X}} \phi(p(x|\theta); v_{t-1}, x^*, x) \, \mathrm{d}x \, f(z|z_{t-1}) \, \mathrm{d}z \tag{219}$$

for ϕ convex in its first argument. In this formulation, the full non-parametric distribution of mistakes now depends on the previous period's mistake and today's optimal action.

To derive the Gaussian AR(1) formulation of mistakes, we now suppose that:

$$\phi(y; m, x^*, x) = \lambda y \log y + \omega y((x - x^*) - m)^2 \tag{220}$$

Concretely, this leads to the following cost functional:

$$
\begin{aligned}
c(p; z_{t-1}, v_{t-1}) = \int_{\Theta} \Bigg[& \lambda \int_{\mathcal{X}} p(x|\theta) \log p(x|\theta) \, \mathrm{d}x \\
& + \omega \int_{\mathcal{X}} ((x - x^*(\theta)) - v_{t-1})^2 p(x|\theta) \, \mathrm{d}x \Bigg] f(z|z_{t-1}) \, \mathrm{d}z
\end{aligned} \tag{221}
$$

which penalizes sharply peaked distributions and those where average mistakes differ greatly from the previous period's mistake. If we moreover suppose that firm risk-adjusted profits are of their quadratic form:

$$\tilde{\Pi}(x, z) := \bar{\Pi}(z) + \frac{1}{2}\Pi_{xx}(z)(x - x^*(z))^2 \tag{222}$$

and we suppose that firms solve the problem:

$$\max_{p \in \mathcal{P}} \int_{\Theta} \int_{\mathcal{X}} \Pi(x, z)\, p(x \mid \theta)\, dx\, f(z|z_{t-1})\, dz - c\left(p; z_{t-1}, v_{t-1}\right) \tag{223}$$

Solving this problem yields the AR(1) structure for mistakes, with Gaussian innovations.

Proposition 10. *The optimal stochastic choice pattern is given by:*

$$x = x^*(z) + v \tag{224}$$

where:

$$v = \rho v_{t-1} + u \tag{225}$$

and

$$\rho = \rho(z) = \frac{\omega}{\frac{1}{2}|\Pi_{xx}(z)| + \omega} \tag{226}$$

and

$$u \sim N\left(0, \frac{2\lambda}{\frac{1}{2}|\Pi_{xx}(z)| + \omega}\right) \tag{227}$$

Proof. We will denote $x^*(z)$ by γ and $\frac{1}{2}|\Pi_{xx}(z)|$ by β to simplify notation. We observe that the FOC characterizing optimal stochastic choice is given by:

$$-\beta(x - \gamma)^2 - \lambda\left[1 + \log p(x|z)\right] - \omega(x - \gamma - m)^2 + \mu(z) + \kappa(x, z) = 0 \tag{228}$$

where $\mu(z)$ is the Lagrange multiplier on the constraint that $p(x|z)$ integrates to unity and $\kappa(x, z)$ is the Lagrange multiplier on the non-negativity constraint that $p(x|z) \geq 0$. We can then observe that this has solution:

$$p(x|z) = \frac{\exp\left(-\tilde{\beta}(x - \tilde{z})^2\right)}{\int_{\mathcal{X}} \exp\left(-\tilde{\beta}(x' - \tilde{z})^2\right) dx'} \tag{229}$$

where $\tilde{\beta} = \frac{\beta + \omega}{\lambda}$ and $\tilde{\gamma} = \gamma + \frac{\omega}{\beta + \omega}v_{t-1}$. It follows that:

$$x|z \sim N\left(\gamma + \frac{\omega}{\beta + \omega}v_{t-1}, \frac{2\lambda}{\beta + \omega}\right) \tag{230}$$

Putting this in more explicit terms, and substituting for γ and β, we obtain the desired representation. $\qquad \square$

D.2 Transformed Gaussian Signal Extraction

In this section, we analyze attention cycles in a setting with Gaussian signal extraction and show how the basic logic of our main model carries over to this setting. For notational simplicity, we describe this alternative model under the assumption that there is a common, scalar state variable θ, which represents each firm's productivity.

D.2.1 Set-up

When the state of the world is θ, the previous state is θ_{-1}, agents have priors $\pi_{\theta_{-1}} \in \Delta(\Theta)$, and the equilibrium level of output is $X(\theta, \theta_{-1})$, intermediates goods firms have payoffs given by:

$$\tilde{\Pi}(x, X(\theta, \theta_{-1}), \theta) = \alpha(X(\theta, \theta_{-1}), \theta) - \beta(X(\theta, \theta_{-1}), \theta)(x - \gamma(X(\theta, \theta_{-1}), \theta))^2 \tag{231}$$

where we will write $\beta(\theta, \theta_{-1}) = \beta(X(\theta, \theta_{-1}), \theta)$ and similarly for α and γ, and as we micro-found via a second-order approximation of their true profit functions around the unconditionally optimal level of production in the main text.

Suppose moreover that agents receive a private Gaussian signal regarding their stakes-adjusted optimal action given by:

$$s_i = \frac{\beta(\theta, \theta_{-1})}{\int_\Theta \beta(\theta, \theta_{-1}) d\pi(\theta|\theta_{-1})} \gamma(\theta, \theta_{-1}) + \frac{1}{\tau(\theta_{-1})} \varepsilon_i \tag{232}$$

where ε_i is a $N(0,1)$ variable that is independent across agents and time periods; $\tau(\theta_{-1})$ is the (soon-to-be endogenized) square-root precision; and the agents' prior $\pi_{\theta_{-1}}$ is such that:

$$\frac{\beta(\theta, \theta_{-1})}{\int_\Theta \beta(\theta, \theta_{-1}) d\pi(\theta|\theta_{-1})} \gamma(\theta, \theta_{-1}) \sim N\left(\mu(\theta_{-1}), \sigma^2(\theta_{-1})\right) \tag{233}$$

This model incorporates the tractability of linear signal extraction into our non-quadratic tracking problem.

Conditional on such a signal s, the best reply of any firm is equal to the conditional expectation of the stakes-adjusted optimal action:

$$x(s) = \mathbb{E}_{\pi_{\theta_{-1}}} \left[\frac{\beta(\theta, \theta_{-1})}{\int_\Theta \beta(\theta, \theta_{-1}) d\pi(\theta|\theta_{-1})} \gamma(\theta, \theta_{-1})|s \right]$$
$$= \lambda(\theta_{-1})s + (1 - \lambda(\theta_{-1}))\mu(\theta_{-1}) \tag{234}$$

where $\lambda(\theta_{-1}) = \frac{\tau^2(\theta_{-1})}{\tau^2(\theta_{-1}) + \frac{1}{\sigma^2(\theta_{-1})}}$ is the appropriate signal-to-noise ratio. Thus, the cross-sectional

distribution of actions is given by:

$$x|\theta,\theta_{-1} \sim N\left(\lambda(\theta_{-1})\frac{\beta(\theta,\theta_{-1})}{\int_\Theta \beta(\theta,\theta_{-1})\mathrm{d}\pi(\theta|\theta_{-1})}\gamma(\theta,\theta_{-1})\right.$$
$$\left. + (1-\lambda(\theta_{-1}))\mathbb{E}_{\pi_{\theta_{-1}}}\left[\frac{\beta(\theta,\theta_{-1})}{\int_\Theta \beta(\theta,\theta_{-1})\mathrm{d}\pi(\theta|\theta_{-1})}\gamma(\theta,\theta_{-1})\right], \frac{\lambda^2(\theta_{-1})}{\tau^2(\theta_{-1})}\right) \tag{235}$$

Say we empirically estimate an equation of the form:

$$x_{it} = \gamma_i + \chi_{j(i),t} + f_t(\theta_{it},\theta_{i,t-1}) + \varepsilon_{it} \tag{236}$$

which differs from our baseline specification in controlling flexibly for observed and lagged productivity.[33] The fitted values span $\mathbb{E}[x \mid \theta,\theta_{-1}]$ and capture state-dependent anchoring toward the prior mean. The residual ε_{it} captures the noise in the firm's action coming from the noise in the signal. The fact that the average action is no longer the unconditionally optimal action is an important departure from our baseline models in Section 2 and Appendix B. In the signal extraction model, the behavior of the stochastic residual captures some, but not all, of the effects of the "cognitive friction," since it does not directly speak to anchoring.

D.2.2   Interpreting Monotone Misoptimization

We now discuss the interpretation of our empirical exercise of studying stochastic volatility in ε_{it}. The variance of the residual is given in this model by:

$$\mathbb{V}_t[\varepsilon_{it}] = \frac{\lambda^2(\theta_{t-1})}{\tau^2(\theta_{t-1})} = \frac{\tau^2(\theta_{t-1})}{\left(\tau^2(\theta_{t-1}) + \frac{1}{\sigma^2(\theta_{t-1})}\right)^2} \tag{237}$$

Our empirical findings are consistent with $\theta_{t-1} \mapsto \mathbb{V}_t[\varepsilon_{it}]$ being an increasing function.[34] This holds in this model exactly when:

$$\frac{\partial\tau^2(\theta_{-1})}{\partial\theta_{-1}}\left(\frac{1}{2\lambda(\theta_{-1})} - 1\right) > \frac{\partial\frac{1}{\sigma^2(\theta_{-1})}}{\partial\theta_{-1}} \tag{238}$$

for all $\theta_{-1} \in \Theta$. Our own regression analysis in Section 5.7, as well as comprehensive studies of manufacturing establishments by Bloom et al. (2018) and Kehrig (2015), suggests that there is less fundamental dispersion in higher aggregate productivity states of the world. As a result, the RHS of this expression condition must be positive. Thus, there are two potential conditions under which this variant of our model is consistent with pro-cyclical misoptimization and counter-cyclical fundamentals

---

[33]In our main analysis, we consider some specifications with time-varying responsiveness to the fundamental shock. We have also considered specifications which depend more flexibly on lagged TFP and found broadly similar results to our baseline, but do not print these in the paper for brevity.

[34]Our specific empirical specification measured the correlation between contemporaneous output and contemporaneous dispersion of ϵ_{it}. If output is monotone in the state of nature and, along with the state of nature, very persistent, the translation to $\partial\mathbb{V}_t[\varepsilon_{it}]/\partial\theta_{t-1} \geq 0$ is immediate.

dispersion:

1. Firms acquire sufficiently less precise signals in higher states $\frac{\partial \tau^2(\theta_{-1})}{\partial \theta_{-1}} < 0$ and the signal to noise ratio is always such that $\lambda(\theta_{-1}) > \frac{1}{2}$

2. Firms acquire sufficiently more precise signals in lower states $\frac{\partial \tau^2(\theta_{-1})}{\partial \theta_{-1}} > 0$ and the signal to noise ratio is always $\lambda(\theta_{-1}) < \frac{1}{2}$

In the former case, "high attention" measured by high signal precision correlates with low residual variance. In the latter case, "low attention" measured by low precision correlates with low residual variance. An important difference between the signal-extraction model from our baseline, then, is that additional information is required to separately identify patterns in attention and residual variance. In both models, "misoptimization" in payoff terms and attention are perfectly correlated by construction. But residual variance is monotone in misoptimization in our baseline model, but not in the signal extraction model due to the role of anchoring.

We interpret our finding that firms discuss macroeconomic developments more during recessions as qualitatively inconsistent with model case 2, and therefore an identifying piece of evidence for case 1. Elsewhere in the literature, Coibion et al. (2018) find that firms have higher demand for information when presented with bad macroeconomic news. Chiang (2023) also finds evidence of higher attention (interpreted as precision of signals) in downturns. Thus, our preferred interpretation of the model is one in which residual variance inherits the monotonicity of signal precision and attention.

D.2.3 Monotone Endogenous Precision

We now extend the model to include endogenous choice of signal precision and derive conditions under which firms obtain less precise signals in high productivity states in the model. To this end, suppose that after θ_{-1} is realized, but before θ is realized, that the agent can pay a cost $\tilde{\phi}(\tau^2)$ to achieve signal precision of τ^2, and where $\tilde{\phi}', \tilde{\phi}'' > 0$. Concretely, the optimal $\tau^2(\theta_{-1})$ solves:

$$\max_{\tau^2(\theta_{-1}) \in \mathbb{R}_+} \mathbb{E}_{\pi_{\theta_{-1}}} \left[-\beta(\theta, \theta_{-1}) \left(x^*(s, \tau^2(\theta_{-1})) - \gamma(\theta, \theta_{-1}) \right)^2 \right] - \tilde{\phi}(\tau^2(\theta_{-1})) \tag{239}$$

We moreover parameterize the scaling of quadratic losses by writing $\beta(\theta, \theta_{-1}) = \kappa \hat{\beta}(\theta, \theta_{-1})$ for all (θ, θ_{-1}) and some $\kappa \geq 1$. Our first goal will be to derive conditions under which the optimally chosen τ^2 in Program 239 is monotone increasing in κ. This demonstrates the natural incentives for firms to choose more precise information when the utility cost of a fixed posterior variance about the stakes-adjusted optimal action is higher.

Toward this end, we first simplify the agent's objective function. Using the distribution of optimal

actions condition on τ^2 from in Equation 235, we write

$$
\mathbb{E}_{\pi_{\theta_{-1}}}\left[-\beta(\theta,\theta_{-1})\left(x^*(s,\tau^2(\theta_{-1}))-\gamma(\theta,\theta_{-1})\right)^2\right]
$$

$$
=\mathbb{E}_{\pi_{\theta_{-1}}}\left[\mathbb{E}\left[-\beta(\theta,\theta_{-1})\left(x^*(s,\tau^2(\theta_{-1}))-\gamma(\theta,\theta_{-1})\right)^2|\theta\right]\right]
$$

$$
=\mathbb{E}_{\pi_{\theta_{-1}}}\left[\mathbb{E}\left[-\beta(\theta,\theta_{-1})\left(x^*(s,\tau^2(\theta_{-1}))-\bar{x}(\theta,\theta_{-1})+\bar{x}(\theta,\theta_{-1})-\gamma(\theta,\theta_{-1})\right)^2|\theta\right]\right]
$$

$$
=\mathbb{E}_{\pi_{\theta_{-1}}}\left[-\beta(\theta,\theta_{-1})\mathbb{E}\left[\left(x^*(s,\tau^2(\theta_{-1}))-\bar{x}(\theta,\theta_{-1})\right)^2+(\bar{x}(\theta,\theta_{-1})-\gamma(\theta,\theta_{-1}))^2|\theta\right]\right]
$$

$$
=\mathbb{E}_{\pi_{\theta_{-1}}}\left[-\beta(\theta,\theta_{-1})\left(\frac{\lambda^2(\theta_{-1})}{\tau^2(\theta_{-1})}+\left[\lambda(\theta_{-1})\frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})}\gamma(\theta,\theta_{-1})+(1-\lambda(\theta_{-1}))\mu(\theta_{-1})-\gamma(\theta,\theta_{-1})\right]^2\right)\right]
$$

$$\tag{240}$$

where $\bar{x}(\theta,\theta_{-1})$ is the mean of the distribution in Equation 235 and $\bar{\beta}(\theta_{-1})=\int_\Theta\beta(\theta,\theta_{-1})\mathrm{d}\pi(\theta|\theta_{-1})$. Observe moreover that we simplify the second term as the following:

$$
\mathbb{E}_{\pi_{\theta_{-1}}}\left[-\beta(\theta,\theta_{-1})\left(\lambda(\theta_{-1})\frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})}\gamma(\theta,\theta_{-1})+(1-\lambda(\theta_{-1}))\mu(\theta_{-1})-\gamma(\theta,\theta_{-1})\right)^2\right]
$$

$$
=\mathbb{E}_{\pi_{\theta_{-1}}}\left[-\beta(\theta,\theta_{-1})\left((1-\lambda(\theta_{-1}))\left(\mu(\theta_{-1})-\frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})}\gamma(\theta,\theta_{-1})\right)+\left(\frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})}-1\right)\gamma(\theta,\theta_{-1})\right)^2\right]
$$

$$\tag{241}$$

A necessary condition for an interior and optimal $\tau^2(\theta_{-1})$ is then the following first-order condition:

$$
\tilde{\phi}'(\tau^2(\theta_{-1}))=-\bar{\beta}(\theta_{-1})\frac{\partial}{\partial\tau^2(\theta_{-1})}\left[\frac{\lambda^2(\theta_{-1})}{\tau^2(\theta_{-1})}\right]
$$
$$
+2(1-\lambda(\theta_{-1}))\frac{\partial\lambda(\theta_{-1})}{\partial\tau^2(\theta_{-1})}\mathbb{E}_{\pi_{\theta_{-1}}}\left[\beta(\theta,\theta_{-1})\left(\mu(\theta_{-1})-\frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})}\gamma(\theta,\theta_{-1})\right)^2\right]
$$
$$
+2\frac{\partial\lambda(\theta_{-1})}{\partial\tau^2(\theta_{-1})}\mathbb{E}_{\pi_{\theta_{-1}}}\left[\beta(\theta,\theta_{-1})\left(\mu(\theta_{-1})-\frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})}\gamma(\theta,\theta_{-1})\right)\left(\frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})}-1\right)\gamma(\theta,\theta_{-1})\right]
$$

$$\tag{242}$$

which reduces to:

$$
\tilde{\phi}'(\tau^2(\theta_{-1}))=-\bar{\beta}(\theta_{-1})\frac{\frac{1}{\sigma^2(\theta_{-1})}-\tau^2}{\left(\tau^2+\frac{1}{\sigma^2(\theta_{-1})}\right)^3}
$$
$$
+2\frac{\frac{1}{\sigma^2(\theta_{-1})}}{\tau^2+\frac{1}{\sigma^2(\theta_{-1})}}\frac{\frac{1}{\sigma^2(\theta_{-1})}}{\left(\tau^2+\frac{1}{\sigma^2(\theta_{-1})}\right)^2}\mathbb{E}_{\pi_{\theta_{-1}}}\left[\beta(\theta,\theta_{-1})\left(\mu(\theta_{-1})-\frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})}\gamma(\theta,\theta_{-1})\right)^2\right]
$$
$$
+2\frac{\frac{1}{\sigma^2(\theta_{-1})}}{\left(\tau^2+\frac{1}{\sigma^2(\theta_{-1})}\right)^2}\mathbb{E}_{\pi_{\theta_{-1}}}\left[\beta(\theta,\theta_{-1})\left(\mu(\theta_{-1})-\frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})}\gamma(\theta,\theta_{-1})\right)\left(\frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})}-1\right)\gamma(\theta,\theta_{-1})\right]
$$

$$\tag{243}$$

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We can now ask how $\tau^2(\theta_{-1})$ moves with κ. In particular, see that we can write:

$$\tilde{\phi}'(\tau^2(\theta_{-1})) = \xi(\tau^2(\theta_{-1}))\kappa \tag{244}$$

where:

$$
\begin{aligned}
\xi(\tau^2(\theta_{-1})) &= \frac{1}{\left(\tau^2 + \frac{1}{\sigma^2(\theta_{-1})}\right)^3}\left[-\bar{\beta}(\theta_{-1})\left(\frac{1}{\sigma^2(\theta_{-1})} - \tau^2\right)\right.\\
&\quad \left. + 2\left(\frac{1}{\sigma^2(\theta_{-1})}\right)^2\xi_1(\theta_{-1}) + 2\frac{1}{\sigma^2(\theta_{-1})}\left(\tau^2(\theta_{-1}) + \frac{1}{\sigma^2(\theta_{-1})}\right)\xi_2(\theta_{-1})\right]\\
\xi_1(\theta_{-1}) &= \mathbb{E}_{\pi_{\theta_{-1}}}\left[\beta(\theta,\theta_{-1})\left(\mu(\theta_{-1}) - \frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})}\gamma(\theta,\theta_{-1})\right)^2\right]\\
\xi_2(\theta_{-1}) &= \mathbb{E}_{\pi_{\theta_{-1}}}\left[\beta(\theta,\theta_{-1})\left(\mu(\theta_{-1}) - \frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})}\gamma(\theta,\theta_{-1})\right)\left(\frac{\beta(\theta,\theta_{-1})}{\bar{\beta}(\theta_{-1})} - 1\right)\gamma(\theta,\theta_{-1})\right]
\end{aligned}
\tag{245}
$$

Applying the implicit function theorem we then have that:

$$\frac{\partial\tau^2(\theta_{-1})}{\partial\kappa} = \frac{\xi(\tau^2(\theta_{-1}))}{\tilde{\phi}''(\tau^2(\theta_{-1})) - \xi'(\tau^2(\theta_{-1}))} = \frac{\tilde{\phi}'(\tau^2(\theta_{-1}))}{\tilde{\phi}''(\tau^2(\theta_{-1})) - \xi'(\tau^2(\theta_{-1}))} \tag{246}$$

where the denominator is positive as the marginal cost of precision is always positive. Thus, we have that:

$$\frac{\partial\tau^2(\theta_{-1})}{\partial\kappa} > 0 \iff \tilde{\phi}''(\tau^2(\theta_{-1})) > \xi'(\tau^2(\theta_{-1})) \tag{247}$$

A sufficient condition for $\frac{\partial\tau^2(\theta_{-1})}{\partial\kappa} > 0$, therefore, by convexity of the costs of precision is that $\xi'(\tau^2(\theta_{-1})) < 0$ for all $\tau^2(\theta_{-1})$. In words, if the benefit of precision is a concave function, then optimally set precision is increasing in κ.

Having shown the desired general comparative static, we now return to the context of our macroeconomic model. Recall that the curvature of firms profits is given by:

$$\beta(\theta,\theta_{-1}) = v_\Pi X(\theta,\theta_{-1})^{-1-\gamma+\chi(1+\epsilon)}\theta^{-1-\epsilon} \tag{248}$$

Thus:

$$\bar{\beta}(\theta_{-1}) = \mathbb{E}_{\pi_{\theta_{-1}}}[v_\Pi X(\theta,\theta_{-1})^{-1-\gamma+\chi(1+\epsilon)}\theta^{-1-\epsilon}] \tag{249}$$

Thus, whenever aggregate output is monotonically increasing in both θ and θ_{-1} and the prior $\pi_{\theta_{-1}}$ is monotone increasing in the FOSD order and $\gamma > \chi(1+\epsilon) - 1$, we have that $\bar{\beta}(\theta_{-1})$ is monotone decreasing in θ_{-1}. It then follows that $\tau^2(\theta_{-1})$ is monotone decreasing in θ_{-1} in equilibrium whenever $\xi' < 0$. Thus, the core logic of our baseline model translates exactly over to this setting with Gaussian signal extraction.

D.3 Rational Inattention

We now extend our results to the case of mutual information cost. As in the previous subsection, for notational simplicity, we describe this alternative model under the assumption that there is a uniform, scalar state variable θ, which represents each firm's productivity.

We first introduce the class of posterior-separable cost functionals. Denti (2020) provides this formulation as a representation theorem in stochastic choice space of the usual posterior-based definition of Caplin and Dean (2013):

Definition 3 (Posterior-Separable Cost Functionals). *A cost functional c has a posterior-separable representation if and only if there exists a convex and continuous ϕ such that:*

$$c(p) = \int_{\mathcal{X}} \hat{\phi}(\{p(x|\theta)\}_{\theta \in \Theta}) \, \mathrm{d}x \tag{250}$$

where:

$$\hat{\phi}(\{p(x|\theta)\}_{\theta \in \Theta}) = p(x)\phi\left(\left\{\frac{p(x|\theta)\pi(\theta)}{p(x)}\right\}_{\theta \in \Theta}\right) \tag{251}$$

whenever $p(x) > 0$ and $\hat{\phi} = 0$ otherwise.

Intuitively, such a cost functional considers the cost to the agent of arriving at any given posterior and adds that up over the distribution of posteriors that are realized. Important cost functionals such as the mutual information cost functional considered in the literature on rational inattention are members of this class. Indeed, mutual information is the special case of the above where ϕ returns the entropy of the distribution that is its argument.

The mathematical structure of posterior-separable cost functionals does not admit the same prior-independence property as likelihood-separable cost functionals. As a result, we will not be able to carry all of our results over to this setting. Nevertheless, as we will argue, the key qualitative forces apply.

In the setting with likelihood separable choice in the single-agent context, we showed that greater curvature of payoffs leads to more precise actions (Proposition 1). With posterior-separable choice, the above result does not hold in general. This is because the prior also influences the states in which the agent would like to learn precisely. In particular, even if a state features high curvature, if it is unlikely to arise, the agent may not care to acquire precise information in that state. A particular case where this complication can be bypassed is when costs are given by mutual information and all actions are exchangeable in the prior in the sense that all actions are *ex ante* equally attractive (Matějka and McKay, 2015). This is a natural case to consider and yields a particularly revealing structure to the optimal policy: the agent's actions in state θ are given by a normal distribution centered on the objective optimum and with variance inversely proportional to the curvature of their objective in that state – a normal mixture model.

Proposition 11. *Suppose that $u(x, \theta) = \alpha(\theta) - \beta(\theta)(x - \gamma(\theta))^2$ and costs are posterior separable with entropy kernel $\lambda\phi(\cdot)$ for some $\lambda > 0$. If all actions are exchangeable in the prior, then in the limit of the support of the action set to infinity, $\hat{x} \to \infty$ for $\overline{x} = -\underline{x} = \hat{x}$, the optimal stochastic choice rule is*

given by:[35]

$$p(x|\theta) = \frac{1}{\sqrt{\frac{\pi\lambda}{\beta(\theta)}}} \exp\left\{ -\frac{1}{2}\left(\frac{x - \gamma(\theta)}{\sqrt{\frac{\lambda}{2\beta(\theta)}}} \right)^2 \right\} \tag{253}$$

Which is to say that the agent's actions follow a normal mixture model with conditional action density given by:

$$x|\theta \sim N\left(\gamma(\theta), \frac{\lambda}{2\beta(\theta)} \right) \tag{254}$$

Proof. We first show that mutual information can be written in the claimed stochastic choice form. These arguments follow closely Matějka and McKay (2015) and Denti (2020). The agent can design an arbitrary signal space $\mathcal{S}$ and choose a joint distribution between signals and states $g \in \Delta(\mathcal{S} \times \Theta)$. As in Sims (2003), the mutual information is the reduction in entropy from having access to this signal relative to the prior:

$$I(g) = \int_{\mathcal{S}} \int_{\Theta} g(s, \theta) \log\left(\frac{g(s, \theta)}{\pi(\theta) \int_{\Theta} g(s, \tilde{\theta}) \, \mathrm{d}\tilde{\theta}} \right) \mathrm{d}\theta \, \mathrm{d}s \tag{255}$$

We now argue that it is without loss to consider a choice over stochastic choice rules $p : \Theta \to \Delta(\mathcal{X})$. Suppose x is an optimal action conditional on receiving any $s \in S_x$. Suppose that there exist $S_x^1, S_x^2 \in S_x$ of positive measure such that $g(\theta|s_1) \neq g(\theta|s_2)$ for all $s_1 \in S_x^1, s_2 \in S_x^2$. Now generate a new signal structure g' such $\tilde{s} \in S_x^1 \cup S_x^2$ is sent whenever any $s \in S_x^1 \cup S_x^2$ was sent under g. Clearly, x is optimal conditional on receiving $\tilde{s}$. Thus, expected payoffs under g' are the same as those under g. Moreover, g' is simply a garbling of g in the sense of Blackwell. Thus $C(g') < C(g)$ for any convex cost functional C. As I is convex, this is a contradiction. Thus, there must be at most one posterior (realized with positive density) associated with each action. As $g(s, \theta) = g(s|\theta)\pi(\theta)$, the choice of $g(s, \theta) \in \Delta(\mathcal{S} \times \Theta)$ is a choice over $g(\cdot|\cdot) : \Theta \to \Delta(S)$. Moreover, there is a unique posterior $\mu(\theta|s)$ associated with each (non-dominated) action which is determined exactly by $g(\cdot|\cdot)$. Hence, the agent directly chooses a mapping $p(\cdot|\cdot) : \Theta \to \Delta(\mathcal{X})$. The agent's problem can then be directly re-written in the claimed stochastic choice form for some c_I:

$$\max_{P \in \mathcal{P}} \int_{\Theta} \int_{\mathcal{X}} u(x, \theta) \, \mathrm{d}P(x|\theta) \, \mathrm{d}\pi(\theta) - c_I(P) \tag{256}$$

Moreover, separating terms, one achieves the following representation of c_I:

$$c_I(p) = \int_{\Theta} \int_{\mathcal{X}} p(x|\theta) \log p(x|\theta) \, \mathrm{d}x \, \mathrm{d}\pi(\theta) - \int_{\mathcal{X}} p(x) \log p(x) \, \mathrm{d}x \tag{257}$$

where:

$$p(x) = \int_{\Theta} p(x|\theta) \, \mathrm{d}\pi(\theta) \tag{258}$$

The stochastic choice problem can now be expressed by the Lagrangian: ($\kappa(x, \theta)$ are the non-negativity

---

[35] Formally, all actions are exchangeable in the prior if:

$$\int_{\Theta} \frac{\exp\{\beta(\theta)\lambda^{-1}(x - \gamma(\theta))^2\}}{\int_{\mathcal{X}} \exp\{\beta(\theta)\lambda^{-1}(\tilde{x} - \gamma(\theta))^2\}d\tilde{x}} \pi(\theta)d\theta = 1 \quad \forall x \in \mathcal{X} \tag{252}$$

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constraints and $\tilde{\gamma}(\theta)$ are the constraints that all action distributions integrate to unity)

$$
\begin{aligned}
\mathcal{L}(\{p(x|\theta), \kappa(x,\theta)\}_{x\in\mathcal{X}, \theta\in\Theta}, \{\gamma(\tilde{\theta})\}_{\theta\in\Theta}) = &\int_\Theta \int_\mathcal{X} u(x,\theta)p(x|\theta)\,\mathrm{d}x\,\mathrm{d}\pi(\theta)\\
&- \lambda\left(-\int_\mathcal{X} p(x)\log p(x)\,\mathrm{d}x + \int_\Theta \int_\mathcal{X} p(x|\theta)\log p(x|\theta)\,\mathrm{d}x\,\mathrm{d}\pi(\theta)\right)\\
&+ \kappa(x,\theta)p(x|\theta) + \tilde{\gamma}(\theta)\left(\int_\mathcal{X} p(x|\theta)\,\mathrm{d}x - 1\right)
\end{aligned}
\tag{259}
$$

Any time that $p(x|\theta) > 0$, taking the FOC pointwise with respect to $p(x|\theta)$ and rearranging we have that:

$$
p(x|\theta) = \frac{p(x)\exp\{u(x,\theta)\}}{\int_\mathcal{X} p(\tilde{x})\exp\{u(\tilde{x},\theta)\}\,\mathrm{d}\tilde{x}}
\tag{260}
$$

Moreover, we can plug the above back into the general problem and take the FOC. Re-arranging we have that for all x such that $p(x) > 0$:

$$
\int_\Theta \frac{\exp\{u(x,\theta)\}}{\int_\mathcal{X} p(\tilde{x})\exp\{u(\tilde{x},\theta)\}\,\mathrm{d}\tilde{x}}\,\mathrm{d}\pi(\theta) = 1
\tag{261}
$$

Up to now we have applied standard techniques from Matějka and McKay (2015). We now use our utility function and exchangeability assumption to derive our result. In particular, we take the utility function as:

$$
u(x,\theta) = \alpha(\theta) - \beta(\theta)(x - \gamma(\theta))^2
\tag{262}
$$

And assume exchangeability in the prior such that all actions are *ex-ante* equally attractive in the limit:

$$
\int_\Theta \frac{\exp\{-\beta(\theta)\lambda^{-1}(x - \gamma(\theta))^2\}}{\int_\mathcal{X} \exp\{-\beta(\theta)\lambda^{-1}(\tilde{x} - \gamma(\theta))^2\}\,\mathrm{d}\tilde{x}}\pi(\theta)\,\mathrm{d}\theta = 1 \quad \forall x \in \mathcal{X}
\tag{263}
$$

Under this condition, in the limit of the support to infinity, the unconditional action distribution converges to the improper uniform distribution $p(x) = p(x')$ for all $x \in \mathcal{X}$. The conditional action distribution then becomes:

$$
p(x|\theta) = \frac{\exp\{-\beta(\theta)\lambda^{-1}(x - \gamma(\theta))^2\}}{\int_\mathcal{X} \exp\{-\beta(\theta)\lambda^{-1}(\tilde{x} - \gamma(\theta))^2\}\,\mathrm{d}\tilde{x}}
\tag{264}
$$

The denominator of this expression can be computed:

$$
\begin{aligned}
\int_\mathcal{X} \exp\{-\beta(\theta)\lambda^{-1}(x - \gamma(\theta))^2\}\,\mathrm{d}x &= \int_\mathcal{X} \frac{\sqrt{2\pi\frac{\lambda}{2\beta(\theta)}}}{\sqrt{2\pi\frac{\lambda}{2\beta(\theta)}}}\exp\left\{-\frac{1}{2}\left(\frac{x - \gamma(\theta)}{\sqrt{\frac{\lambda}{2\beta(\theta)}}}\right)^2\right\}\,\mathrm{d}x\\
&= \sqrt{2\pi\frac{\lambda}{2\beta(\theta)}}\int_\mathcal{X} \frac{1}{\sqrt{2\pi\frac{\lambda}{2\beta(\theta)}}}\exp\left\{-\frac{1}{2}\left(\frac{x - \gamma(\theta)}{\sqrt{\frac{\lambda}{2\beta(\theta)}}}\right)^2\right\}\\
&= \sqrt{2\pi\frac{\lambda}{2\beta(\theta)}}
\end{aligned}
\tag{265}
$$

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It follows that:

$$p(x|\theta) = \frac{1}{\sqrt{\frac{\pi\lambda}{\beta(\theta)}}} \exp\left\{-\frac{1}{2}\left(\frac{x - \gamma(\theta)}{\sqrt{\frac{\lambda}{2\beta(\theta)}}}\right)^2\right\} \tag{266}$$

Which is to say that $X|\theta$ is a Gaussian random variable with mean $\gamma(\theta)$ and variance $\frac{\lambda}{2\beta(\theta)}$. $\qquad\square$

This result extends the known results on Gaussian optimality of stochastic choice with mutual information (Sims, 2003) to a domain with a stochastic weight on the deviation from optimality. For our purposes, the novel and interesting feature is that the variance of the action distribution in any given state is inversely-proportional to curvature. It follows that if all actions are exchangeable in the prior when:

$$\gamma(\theta) = x^*(X(\theta), \theta)$$
$$\beta(\theta) = \frac{1}{2}|\Pi_{xx}(X(\theta), \theta)| \tag{267}$$

where $X(\theta)$ is the unique equilibrium level of aggregate production, then the model with mutual information is exactly equivalent to the model with entropic likelihood-separable cost that we studied. All results from Section 3 then carry directly.

E   Additional Numerical Results

In this Appendix, we discuss robustness of our numerical findings as well as how the macroeconomic implications of attention cycles change under counterfactual scenarios.

E.1   Sensitivity of Main Results

E.1.1   Parameter Choice

To probe robustness to our choice of elasticity of substitution ϵ and the wage rule slope χ, we re-calibrate the model for alternative choices. For brevity, we summarize these experiments by considering "high" and "low" deviations for each parameter, holding fixed the others at baseline values, and present the proportional difference from the baseline in three summary statistics introduced in Section 6.3:

1. The relative output effect of negative and positive shocks, normalized in $\log\theta$ units such that the latter increases output by 3%;
2. The relative output effect of a "double dip" versus positive shock, holding fixed the size of the shock to productivity as above;
3. The ratio of output-growth volatility from the 10th to the 90th percentile of the output distribution.

We present our results in Figure A11. Lowering the elasticity of substitution or increasing the implied average markups can have ambiguous effects because it simultaneously increases the bite of a fixed level of misoptimization on misallocation, productivity, and output, while decreasing the bite of

the profit-curvature channel toward cyclical attention. We find numerically that increasing markups or decreasing ϵ, toward the level implied by De Loecker et al. (2020), significantly increases the extent of our predicted asymmetries (≈ 1.75x), while decreasing markups or increasing ϵ, toward the level implied by Edmond et al. (2018), modestly increases the extent of our predicted asymmetries.

Increasing the slope of the wage rule dampens our predictions, due to its dampening the economy's Keynesian-cross feedback. Decreasing the slope increases the bite of our predictions substantially by amplifying the same general-equilibrium effects.

E.1.2 Classical Labor Markets

For tractability in the theoretical section we equipped our model with a reduced-form *wage rule* rather than a micro-founded labor supply curve. As an alternative, we use the preferences of Greenwood et al. (1988) which replace Equation 1 with the following:

$$\mathcal{U}(\{C_{t+j}, L_{t+j}\}_{j \in \mathbb{N}}) = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{\left(C_{t+j} - \frac{L_{t+j}^{1+\phi}}{1+\phi}\right)^{1-\gamma}}{1-\gamma} \tag{268}$$

and remove the wage rule, Equation 3. These preferences generate a labor supply curve $w_t = L_t^{\phi}$ which closely resembles our reduced-form wage rule, but also takes seriously the implications for risk-pricing by making marginal utility a function of hours worked. We choose a parameterization of $\phi = \frac{\chi}{1-\chi\epsilon} = 0.153$, where χ and ϵ take our benchmark values indicated in Table 4. As indicated in the richer model of Appendix B, this calibration replicates an elasticity of $\chi = 0.095$ between real wages and real output.

Figure A12 is this model's analog to Figure 5, showing output, the attention wedge, and labor productivity. We find comparable behavior of the attention wedge and losses from misoptimization and inattention. Figure A13 is this model's analog to Figure 6, showing state-dependent shock response and stochastic volatility. Our results are quantitatively similar to our baseline calibration. These results demonstrate that classical labor markets do not undermine our main results in calibrations that are consistent with our calibrated wage rigidity.

E.2 Attention Cycles Under Counterfactual Scenarios

Because the main amplification mechanism in our model, the reallocation of attention, is endogenous to economic conditions, we can use our framework to study how attention cycles and all associated macro phenomena would behave under counterfactual conditions. In this subsection, we explore the interaction of our findings with recent trends in product markups; the decreasing pro-cyclicality of real wages; and external uncertainty shocks.

E.2.1 The Rise of Markups, via Lower Substitutability

An extensive recent literature documents a secular increase in markups charged by US public firms over the last half century (see, e.g., De Loecker et al., 2020; Edmond et al., 2018; Demirer, 2020). In

our empirical calibration, we targeted "modern" average markups as informed by this literature. Our framework would interpret any trends in aggregate markups as arising from changes in the elasticity of substitution ϵ between products, which would need to have been higher in the previous, low-markup era than it is today. In our model, a lower elasticity of substitution or higher markup increases the output cost of a fixed amount of misoptimization dispersion, as it intuitively makes each individual product more "essential" to the consumed good; and it has *a priori* ambiguous effects on the extent of equilibrium attention cycles.

In our model, we run the following simple experiment. First, we adjust ϵ upward to simulate a 15 percentage point decrease in the aggregate markup, to match the estimate of Demirer (2020) for markups since the 1960s; and second, we adjust ϵ downward to match a 15 percentage point increase. We find that lower markups correspond to more severe effects of attention cycles on business cycles, as summarized by the asymmetry and state-dependence of dynamics (second and third panel of Figure A14). These results provide only speculative clues about the future, owing to both uncertainty about the true "cause" of rising markups and the plausibility of the trend continuing. But, regardless of their precision, they highlight a potentially important pathway linking market structure, firm decisionmaking, and the aggregate cost of misallocation.

E.2.2 More Rigid Real Wages

The inverse relationship between wage inflation and real conditions has proved elusive in modern data, particularly since the financial crisis (see, e.g., Galí and Gambetti, 2019). Our baseline estimate of slope 0.095 between (detrended) real output and real wages since 1987 reflects this reality. In our model, more rigid real wages corresponded to a steeper Keynesian cross, and a steeper incentive toward high attention in low states of the world. For this reason, we may expect that the growing disconnect between factor prices and real conditions contributes toward the severity of our estimated macro effects.

In parallel to the previous experiment, we simulate both a "calibrated past" and "extrapolated future." For the former, we plug in the estimate of Galí and Gambetti (2019) that the wage Phillips curve has flattened by a factor of 1.9 over the last half century; for the latter, we extrapolate the same multiplicative trend as additional flattening.[36] We find, as shown in panels four and five of Figure A14, stronger effects of attention cycles in the regime with more rigid wages. This underscores the complementarity between attention cycles and the steepness of the Keynesian cross, and suggests a novel pathway by which factor price rigidity can influence patterns of macroeconomic volatility.

E.2.3 Elevated Uncertainty

Spikes in uncertainty around exceptional economic and political events have large documented effects on financial markets and firm decisionmaking (Bloom, 2009). Moreover, large, disorienting shocks are often either a natural consequence of poor economic performance (e.g., policy surprises during the 2007-2009 financial crisis) or their root cause (e.g., the Covid-19 pandemic). For this reason it is

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[36]In particular, we use the ratio of the 1964-2007 estimate and 2007-2017 estimates in Table 3A of Galí and Gambetti (2019).

natural to study how changes in the "level of uncertainty," formalized in our model as variation in the attention cost λ, might interact with our main business cycle predictions. Proposition 4 showed that increases in uncertainty depress output in our model. These shocks also, according to the results of Proposition 1, increase the *sensitivity* of dispersion to macroeconomic conditions and hence, based on extrapolation of this partial-equilibrium logic, may amplify the extent of misoptimization cycles. For this reason, we might predict that elevated uncertainty is also complementary to the asymmetry and state-dependence generated at the macro level.

We explore this relationship by solving for the model equilibrium under scenarios with depressed and elevated attention costs, and numerically verify the predicted complementarity (panels six and seven of Figure A14). Thus our theory predicts that business cycles caused and/or amplified by background uncertainty-inducing events may induce sharper fluctuations in aggregate volatility due to endogenous reallocation of attention.

F   Additional Empirical Results

F.1   Macro Attention With Conference Calls

In this Appendix, we describe an alternative construction of our textual Macro Attention measure using sales and earnings conference calls as an alternative source of information about firms' relative attention to different risks and events. This analysis mirrors and supplements our main analysis of forms 10-K and 10-Q.

Data and Measurement.   We obtain data from the Fair Disclosure (FD) Wire service, which records transcripts of sales and earnings conference calls for public companies around the world. We obtain an initial sample of 294,900 calls which cover 2003 to 2014. We next subset to documents that have reported firm names and stock tickers, which are automatically associated with documents by Lexis Nexis. When matches are probabilistic, we use the first (highest probability) match.[37] We finally restrict to firms that are listed on one of three US stock exchanges: the NYSE, the NASDAQ, or the NYSE-MKT (Small Cap). We finally connect tickers to firm identifiers (GVKEY) using the master cross-walk available on Wharton Research Data Services (WRDS). These operations together reduce the sample size to 164,805 calls.

We finally restrict to conference calls that are sales or earnings reports. This further reduces the sample to 158,810 total observations, by removing conference calls related to other activities (e.g., mergers). All in all, this sample is about 3,600 firm observations per quarter, or about 60% of the per-quarter observations we obtained via the SEC filings.

To tabulate histograms of words within documents, we use the CountVectorizer function in the FeatureExtraction module of the standard Python package Scikit Learn. We then replicate the exact methodology of Section 4.2, generating a new list of 73 macroeconomic words. Like the main list of words, they are a combination of very interpretable choices ("government," "unemployment," "monetary") and false positives related to structure and pedagogy ("theory," "chapter").

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[37]In the essentially zero-measure cases in which there is a tie, we take the alphabetically first ticker.

Results. Figure A9 plots the conference-call-derived measure, in log units and with quarterly fixed effects taken out, alongside the US unemployment rate. Conference-call-derived macro attention, like our main measure derived from forms 10-Q/K, is cyclical and persistent. To benchmark these facts in the same way we did in the main text, we first run linear regressions of the form

$$\log \text{MacroAttentionCC}_t = \alpha + \beta_Z \cdot Z_t + \epsilon_t \tag{269}$$

for $Z_t \in \{\text{Unemployment}_t/100, \log \text{SPDetrend}_t\}$. The first two columns of Table A10 show the coefficients, which are slightly larger in absolute value than their equivalents with our 10K/Q measure (1.529 and -0.104, respectively). The third column gives our estimate of an AR(1) process, which is close to a unit root.

Casual comparison of Figure A9 and Figure 4 in the main text suggests that, while our two measures of attention have similar cyclical patterns, they do not closely track each other at the aggregate level. Conference-call-derived attention is more sharply peaked around the onset of the Great Recession while 10-K/Q-derived attention remains elevated for several subsequent years. The correlation between the two measures on a common sample is a (statistically insignificant) 0.091. The relationship is closer, however, at the firm level. Columns 4-6 of Table A10 show the results of regressing the conference-call-derived measure 10K/Q-derived measure at the firm level, with increasingly more stringent fixed effects. The correlation is consistently positive, though strongest in terms of cross-firm differences as opposed to within-firm differences. Moreover, as indicated in column 1 of Table A7, our finding linking firm-level misoptimization with firm-level attention is stable based on the conference call measure.

F.2 Macro Attention With Word Stemming

Measurement. Our main method for constructing Macro Attention treats individual words as the unit of measurement. For this reason, words like "unemployment" and "unemployed" are counted separately despite likely communicating the same meaning in all contexts. This method, while appealingly simple, may systematically under-count words that have a number of different forms or tenses, while allowing the multiple forms of certain ubiquitous words to crowd out other distinct concepts.

As an alternative method, which allays some of these concerns, we re-do our calculation of macroeconomic language using *word stems*. For each word w in the macroeconomics references and/or regulatory filings, we use the Porter Stemmer implemented in Python's `nltk` software to determine a stem $s(w)$. Stemming is an algorithmic and imperfect process. In examples relevant to our context, the Porter Stemmer associates "unemployment" and "unemployed" with the common stem "unemploy." But it also, employing the same logic, associates "nominal" with "nomin," a stem which may match to words less often used to describe aggregate prices (e.g., "nominate").

We adapt our tf-idf calculation to the stem level by calculating, for each stem s that appears in the regulatory filings,

$$\text{tf-idf}(s)_{it} := \text{tf}(s)_{it} \cdot \log \left(\frac{1}{\text{df}(s)} \right) \tag{270}$$

where $\text{tf}(s)$ is the total term frequency of all words mapped to stem s, and $\text{df}(s)$ is the minimum

document frequency among words associated with the stem.[38] We calculate the top macro stems using the approach described in the main text (Section 4.2); construct the set of macro words $\mathcal{W}_M$ as the set of all words associated with a macro stem; and proceed in the standard way to calculate firm-level and aggregate macro attention.

Results. Table A11, in analogy to Table A10, presents a summary of the cyclical patterns of the stemmed Macro Attention measure as well as its relationship to our main measure. The two measures behave very similarly in the time series and are tightly connected at the firm level. Moreover, when we replicate our main model linking firm-level Macro Attention to firm-level misoptimization as in Table A7, we estimate a coefficient of -0.020 (SE: 0.004), which is comparable within error bars to our baseline estimate of -0.009 (SE 0.003).

F.3   Dispersion in TFPR and Value Marginal Products

In this Appendix, we review theoretically and empirically the behavior of revenue-TFP (TFPR) dispersion in our analysis. This analysis builds a bridge between our findings and those of Kehrig (2015), who shows that cross-firm variance in revenue total-factor-productivity, or the product of prices and physical productivity, is counter-cyclical for the US durable manufacturing sector between 1972 and 2007.

Definitions and Theoretical Context. In our model, with a three-input production function, log physical TFP is defined as

$$\log \theta_{it} = \log x_{it} - (\alpha_{j(i),M} \log M_{it} + \alpha_{j(i),K} \log K_{it} + \alpha_{j(i),L} \log L_{it}) \tag{271}$$

where x_{it} is physical sales, in quantity units; (M_{it}, K_{it}, L_{it}) are materials, capital, and labor; and $(\alpha_{j(i),M}, \alpha_{j(i),K}, \alpha_{j(i),L})$ are (industry-specific) weights on these inputs, which sum to one. Prices are defined by the demand curves $\log p_{it} = \frac{1}{\epsilon}(\log X_t - \log x_{it})$ and revenue-based TFP is therefore

$$\log \theta_{it}^R = \log \theta_{it} + p_{it} = \log \text{Sales}_{it} - (\alpha_{j(i),M} \log M_{it} + \alpha_{j(i),K} \log K_{it} + \alpha_{j(i),L} \log L_{it}) \tag{272}$$

where $\text{Sales}_{it} = p_{it} x_{it}$. We can calculate exactly what TFPR is in our empirical model with inattentive firms, introduced in Appendix C.3, by combining this definition with the input-choice policy functions derived in Lemma 2:

Lemma 4. *TFPR in our model is given by*

$$\log \theta_{it}^R = \tilde{\gamma}_i + \Xi_{j(i),t} - \frac{1}{\epsilon} m_{it} \tag{273}$$

where $\tilde{\gamma}_i$ is a constant at the firm level and $\Xi_{j(i),t}$ is a constant that varies at the industry-by-time level.

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[38] We use the minimum instead of the overall frequency due to a data limitation of having document frequencies at the word, not stem, level. We expect either method to produce broadly similar results.

Thus, the only sources of within-industry variation in revenue-based TFP in our model are the firm fixed effects and misoptimizations. This is another way of stating our theoretical result that misoptimizations matter for aggregate output and productivity via their effects on "misallocation," as TFPR measures the value marginal product of the (minimal cost) input bundle. A simple corollary, under our assumption of cost minimization, is that the value marginal product $\log \theta_{it}^Z = \log \frac{\text{Sales}_{it}}{Z_{it}}$, for any of the three inputs, can also be written as $\log \theta_{it}^Z = \tilde{\gamma}_{Z,i} + \Xi_{j(i),Z,t} - \frac{1}{\epsilon} m_{it}$ with now input-specific fixed effects (firm-level and industry-by-time level).

In our model, therefore, the presence of TFPR dispersion or value-marginal-product dispersion indicates that there is non-zero misoptimization. The monotonicity of TFPR or value-marginal-product dispersion over the business cycle, once we project out firm and industry-by-time fixed effects, therefore provides an alternative test of the model's prediction of procyclical misoptimization dispersion.

TFPR Dispersion and VMPL Dispersion. We calculate log TFPR in our data by first calculating

$$\log \tilde{\theta}_{it}^R = \log \text{Sales}_{it} - \left(\alpha_{M,j(i)} \cdot \log \text{MatExp}_{it} - \alpha_{L,j(i)} \cdot \log \text{Empl}_{it} - \alpha_{K,j(i)} \cdot \log \text{CapStock}_{it}\right) \quad (274)$$

where variable definitions follow the convention of Appendix C. We then remove industry-by-time fixed effects and firm fixed effects to remove factor prices, as suggested by Lemma 4, to generate the variable $\log \hat{\theta}_{it}^R$.[39] We also calculate the log value marginal product of labor (VMPL) by first calculating

$$\log \tilde{\theta}_{it}^L = \log \text{Sales}_{it} - \log L_{it} \quad (275)$$

and similarly removing industry-by-time and firm fixed effects to generate the final measure $\log \hat{\theta}_{it}^L$.

Appendix Figure A10 shows the cyclical behavior of TFPR dispersion, the variance of $\log \hat{\theta}_{it}^R$, and VMPL dispersion, the variance of $\log \hat{\theta}_{it}^L$. In line with our earlier results, VMPL dispersion is markedly pro-cyclical with respect to both the unemployment rate and the return on the S&P500. TFPR dispersion has no significant relationship with either unemployment or the S&P500. Taken together, these exercises suggest that value-marginal-product-based measures of misallocation give results consistent with our main finding of pro-cyclical misoptimization.

G State-Dependent Attention in Survey Data

In this Appendix, we test our interpretation of attention and misoptimization cycles using the dataset of Coibion et al. (2018) (henceforth, CGK), one of the most comprehensive datasets of firm-level operations and macro backcasts in an advanced economy. These data were assembled from a detailed survey of the general managers of a representative panel of firms in New Zealand from 2013 to 2016. The final subsection of this appendix, G.3, contains more details about the survey questions and data.

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[39]Compare with Equation 199, which resembles Equation 274 but for deflating the input shares by the markup. This is exactly the within-model adjustment for prices p_{it}.

G.1 Reported Attention and the Business Cycle

Although the CGK survey took place during relatively tranquil times for the New Zealand economy, it did ask two *hypothetical* questions directly revealing of the premise for this paper. Each concerned firm's desire to collect information on the macroeconomy conditional on either good (or poor) conditions:

> Suppose that you hear on TV that the economy is doing well [or poorly]. Would it make you more likely to look for more information?

Table A14 reports the percentage of answers in each of five bins, given the conditions of the economy doing "well" or "poorly." This self-reported demand for information increases in the context of *bad* news about the macroeconomy and, if anything, contracts with *good* news of the macroeconomy. This is consistent with our hypothesis that bad conditions increase the stakes for firms' decisions and hence make keen attention to macroeconomic conditions more important, while good news does not have a symmetric effect.

G.2 Reported Profit Function Curvature and Attention

A second test possible in the CGK data relates to our more specific prediction that higher curvature of the firm's objective, as a function of decision variables, should increase attentiveness to decision-relevant variables including macroeconomic aggregates. The CGK survey indirectly elicits information on this shifter via questions about purely hypothetical price changes and revenue increases to an "optimal point." In Section G.3 at the end of this appendix, we show exactly how one can use a pair of linked questions about firms' hypothetical optimal reset price, and the hypothetical percentage increase in profits that would be associated with that change, to develop an elicited measure of *firm profit curvature* in non-risk-adjusted units.[40]

As outcomes for macro attention, we can turn to two sources. The first is the absolute-value error in firms' one-year back-casts for three macro variables: inflation, output growth, and unemployment. The second is firm managers' reported (binary) interest in *tracking* one of the aforementioned variables.

For each of the aforementioned firm-level outcomes Y_{it}, we run the following regression on the firm-level profit curvature variable ProfitCurv_{it} and a vector of controls Z_{it}:

$$Y_{it} = \alpha + \beta \cdot \text{ProfitCurv}_{it} + \gamma' Z_{it} + \epsilon_{it} \tag{276}$$

Our prediction that profit curvature drives stakes for attention corresponds to $\beta < 0$ for back-cast errors and $\beta > 0$ for reported attention. We control for five bins in the firms' total reported output and the firms' 3-digit ANZ-SIC code industries. Finally, we cluster all standard errors by 3-digit industry.

Table A15 shows the results. For inflation we find strong evidence that higher-curvature firms make smaller errors, with some of the effect being absorbed by control variables when added. For GDP growth we find estimates that are much less precise but have the same signs; and for unemployment,

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[40]Table A16 shows that this curvature measure is higher for smaller firms with more within-industry competitors, though these patterns are immaterial for our reduced-form verification of our prediction.

results that are further imprecise and have the wrong signs. We take this as further suggestive evidence for the mechanism that our theory proposes: that the differential stakes of making mistakes is a contributing factor to macro attention.

G.3 Details of Data Construction

In this final subsection, we describe more precisely how we use the raw survey results of Coibion et al. (2018) in our empirical analysis. We make use of the full dataset contained in the replication files posted on the article's page hosted by the *American Economic Review* and OpenICPSR (Coibion et al., 2018b). All direct references to survey questions by wave or number match the "Appendix 5: Selected Survey Questions" in the online appendix available at the same link.

G.3.1 Profit Function Curvature

We draw our measure of profit function curvature from the answers to two survey questions about hypothetical price changes. These are jointly asked as Question 17 of Wave 5, Part B:

> If this firm was free to change its price (i.e. suppose there was no cost to renegotiating contracts with clients, no costs of reprinting catalogues, etc.) right now, by how much would it change its price? Please provide a percentage answer. By how much do you think profits would change as a share of revenues? Please provide a numerical answer in percent.

Denote the answer for prices as Δp_i and the answer for profits as $\Delta \Pi_i$. Under the assumption that the following second-order approximation holds for the deviation of profits from their frictionless optimum (e.g., a version of (10), in percentage units for the outcome and the choice variable), the following relationship holds between the measurable quantities and the profit function curvature $\mathrm{ProfitCurv}_i$:

$$\Delta \Pi_i = \mathrm{ProfitCurv}_i \cdot \Delta p_i^2 \tag{277}$$

We use this expression to calculate an empirical analog of profit curvature. The top panel of Table A16 provides summary statistics of measured profit curvatures among the 3,153 firms for which we can measure it. The median reported curvature is 0.12, which means that a one-percentage-point deviation from the optimal price for such a firm corresponds to a 0.12-percentage-point deviation from optimal profits as a fraction of revenue.

The bottom panel of Figure A16 shows firm and manager-level correlates for our measure in the CGK data. The table reports coefficients of the following regression:

$$\widehat{\mathrm{ProfitCurv}}_i = \beta \cdot \hat{X}_i + e_{it} \tag{278}$$

where the hat denotes that both variables have been normalized to z-score units (i.e., with means subtracted and standard deviation divided out), so the coefficient β is a "normalized" metric of the standard-deviation-to-standard-deviation effect. We find strong evidence that the firms with higher profit function curvature are smaller and have more competitors. There is only weaker evidence that

the associated managers are more skilled and/or better rewarded. We interpret this cautiously as evidence that likely confounds via manager skill and firm sophistication (i.e., better managers grow firms larger, and make better forecasts) are going the "wrong direction" to explain our reduced-form correlations between profit curvature and forecasting accuracy.

G.3.2 Outcomes: Back-cast Errors

For back-cast errors, we use the following questions that are split among waves of the survey. In survey wave 1, firms are asked the following question:

> During the last twelve months, by how much do you think prices changed overall in the economy?

Although the wording of the question is not entirely clear about what indicator is being referred to, we follow CGK and interpret this as the annual percent change in CPI, with realized value 1.6%. Firms are asked a similar question in wave 4, but we prefer the wave 1 version because the sample size is slightly larger. Table A17 recreates Table A15 from the main text, first for the wave 1 back-cast of inflation (reported for the main text) and next for the wave 4 back-cast of inflation (not reported in the main text, but quantitatively very similar).

For GDP growth, we use the following question from wave 4:

> What do you think the real GDP growth rate has been in New Zealand during the last 12 months? Please provide a precise quantitative answer in percentage terms.

and compare with a realized value of 2.5%. Finally, for unemployment, we use the following question also from wave 4:

> What do you think the unemployment rate currently is in New Zealand? Please provide a precise quantitative answer in percentage terms.

and compare with a realized value of 5.7%. All realized values are taken from the replication files of CGK, to deal with any ambiguity about statistical releases, and ensure comparability with that study.

G.3.3 Outcomes: Tracking Indicators

We finally use, for the lower panel of Table A15, the following questions from wave 4 about tracking different variables:

> Which macroeconomic variables do you keep track of? Check each variable that you keep track of.
>
> 1. Unemployment rate
> 2. GDP
> 3. Inflation
> 4. None of these is important to my decisions

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We code for each variable a binary indicator of whether the firm lists the variable of interest. We combine GDP in this question (by implication, in levels) with quantitative forecasts of GDP Growth in Table A15.

H   Supplemental Tables and Figures

List of Tables

List of Figures

Table A1: Misoptimization and Firm Performance

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		Outcome: R_{it}				Outcome: π_{it}		
$\hat{u}_{it}^2$	-0.236	-0.230	-0.060	-0.051	-0.316	-0.316	-0.106	-0.105
	(0.026)	(0.026)	(0.032)	(0.032)	(0.024)	(0.024)	(0.018)	(0.017)
Sector x Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Firm FE			✓	✓			✓	✓
TFP Control		✓		✓		✓		✓
N	41,578	41,578	41,206	41,206	51,015	51,015	50,966	50,996
R^2	0.238	0.261	0.384	0.403	0.117	0.131	0.663	0.681

Notes: R_{it} is the firm-level log stock return and π_{it} is firm-level profitability. $\hat{u}_{it}^2$ is the squared firm-level misoptimization residual, constructed using the methods described in Section 4.1. Standard errors are double-clustered at the year and firm level.

Table A2: Dynamic Effects of Misoptimization

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
					Outcome:				
		$\Delta \log \hat{\theta}_{i,t+k}$			$R_{i,t+k}$			$\pi_{i,t+k}$	
Horizon k	0	1	2	0	1	2	0	1	2
$\hat{u}_{it}^2$	-0.009	0.014	-0.007	-0.236	-0.252	-0.251	-0.316	-0.286	-0.265
	(0.007)	(0.008)	(0.010)	(0.026)	(0.027)	(0.038)	(0.024)	(0.018)	(0.019)
Sector x Time FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
N	50,455	40,671	32,362	41,578	34,643	28,103	51,015	42,014	33,934
R^2	0.231	0.245	0.263	0.238	0.241	0.248	0.117	0.123	0.126

Notes: Each column is the estimate of a separate projection regression. The outcomes are TFP growth $\Delta \log \hat{\theta}_{it}$ (first three columns), log stock returns R_{it} (second three columns), and profitability π_{it} (last three columns). $\hat{u}_{it}^2$ is the squared firm-level misoptimization residual, constructed using the methods described in Section 4.1. Standard errors are double-clustered at the year and firm level.

Table A3: Cyclicality of Misoptimization, with Alternative Measurement Schemes

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
					Outcome:	MisoptimizationDispersion$_t$					
Unemployment$_t$/100	-0.841	-0.439	-0.822	-0.749	-0.501	-0.695	-0.802	-0.813	-0.605	-0.812	-1.034
	(0.341)	(0.196)	(0.336)	(0.292)	(0.159)	(0.280)	(0.337)	(0.330)	(0.293)	(0.335)	(0.549)
Correlation	-0.493	-0.468	-0.485	-0.466	-0.546	-0.505	-0.479	-0.489	-0.494	-0.462	-0.379
Baseline	✓										
Adj. Control		✓									
Leverage Control			✓								
t, t^2 Control				✓							
Manufacturing					✓						
Sector Policy Fn.						✓					
t-varying Policy Fn.							✓				
Quadratic Policy Fn.								✓			
Pre-Period TFP									✓		
t-varying Prod. Fn.										✓	
OP (96) TFP											✓
N	31	31	31	31	31	31	31	31	20	31	31
R^2	0.243	0.219	0.235	0.326	0.298	0.255	0.230	0.239	0.244	0.213	0.144

Notes: The first row reports the coefficient from the regression of MisoptimizationDispersion$_t$ on Unemployment$_t$/100, with standard errors that are HAC-robust with a 3-year Bartlett Kernel. The following row reports the correlation of these variables (in column 4, conditional on projecting out controls). The "Adjustment Cost" and "Leverage" controls are described in the main text. The "Sector Policy Fn." estimates the policy function separately for each sector. The "t-varying Policy Fn." model interacts all coefficients in the policy function with time fixed effects. The "Quadratic Policy Fn." allows for quadratic dependence on TFP. The "Pre-Period TFP" model uses cost shares from before 1997 to construct the production function, and data after 1998 to estimate the policy function and misoptimizations. The "t-varying Prod. Fn." model estimates the Solow residual using industry-by-year-specific cost shares. The "OP (96)" model estimates productivity using the proxy-variable strategy of Olley and Pakes (1996), as detailed in Appendix C.2.

Table A4: The Effects of Misoptimization in Levels

	(1)	(2)	(3)	(4)	(5)	(6)
			Outcome:			
	R_{it}		π_{it}		$\hat{m}_{it}^2$	
$\hat{m}_{it}^2$	-0.042	-0.076	-0.021	-0.025		
	(0.028)	(0.033)	(0.010)	(0.011)		
$\hat{m}_{it}^2 \times R_t$		0.177		0.038		
		(0.078)		(0.053)		
$\log \mathrm{MacroAttention}_{it}$					-0.010	-0.020
					(0.006)	(0.007)
Firm FE	✓	✓	✓	✓		✓
Sector x Time FE	✓	✓	✓	✓	✓	✓
N	41,247	41,247	57,646	57,646	34,421	33,841
R^2	0.385	0.385	0.656	0.656	0.053	0.488

Notes: $\hat{m}_{it}$ is the firm-level misoptimization, constructed using the methods described in Section 4.1 These specifications replicate results in Tables 1, 2, 3, and A1 with with $\hat{m}_{it}$ in place of $\hat{u}_{it}$. Standard errors are double-clustered at the year and firm level.

Table A5: Pricing of Misoptimization, with Alternative Measurement Schemes

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					Outcome: R_{it}					
$\hat{u}_{it}^2$	-0.097	-0.239	-0.101	-0.168	-0.090	-0.099	-0.109	-0.062	-0.057	-0.096
	(0.034)	(0.941)	(0.035)	(0.045)	(0.036)	(0.035)	(0.034)	(0.028)	(0.021)	(0.034)
$\hat{u}_{it}^2 \times R_t$	0.443	0.941	0.415	0.680	0.420	0.330	0.447	0.227	0.231	0.417
	(0.171)	(0.370)	(0.169)	(0.182)	(0.156)	(0.163)	(0.163)	(0.130)	(0.098)	(0.168)
Sector x Time FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Baseline	✓									
Adj. Control		✓								
Leverage Control			✓							
Manufacturing				✓						
Sector Policy Fn.					✓					
t-varying Policy Fn.						✓				
Quadratic Policy Fn.							✓			
Pre-Period TFP								✓		
t-varying Prod. Fn.									✓	
OP (96) TFP										✓
N	41,206	35,388	41,016	22,902	41,197	41,203	41,203	26,206	40.078	41,166
R^2	0.385	0.387	0.385	0.367	0.385	0.384	0.385	0.429	0.382	0.385

Notes: R_{it} is the firm-level log stock return. $\hat{u}_{it}$ is the firm-level misoptimization residual, constructed using the methods described in Section 4.1 and the indicated variants described in the main text and the notes of Table A3. R_t is the log return of the S&P 500. Standard errors are double-clustered at the year and firm level. The scenarios are described in the main text and the notes of Table A3.

Table A6: Markets Punish Misoptimizations Harder in Low States, Additional Controls

	(1)	(2)	(3)	(4)	(5)	(6)
			Outcome: R_{it}			
$\hat{u}_{it}^2 \times R_t$	0.376	0.378	0.345	0.321	0.330	0.489
	(0.123)	(0.109)	(0.118)	(0.173)	(0.094)	(0.296)
Sector x Time FE	✓	✓	✓	✓	✓	✓
$\hat{u}_{it}^2$	✓	✓	✓	✓	✓	✓
TFP and Interaction		✓				
Leverage and Interaction			✓			
Lag Return and Interaction				✓		
Industry FE and Interaction					✓	
Firm FE and Interaction						✓
N	41,578	41,578	41,429	34,805	41,206	41,206
R^2	0.239	0.261	0.246	0.239	0.379	0.498

Notes: R_{it} is the firm-level log stock return. $\hat{u}_{it}$ is the firm-level misoptimization residual, constructed using the methods described in Section 4.1. R_t is the log return of the S&P 500. Column 1 reports the baseline estimate, Column 1 of Table 1. Columns 2-6 add additional variables and their interaction with $\hat{u}_{it}^2$. These variables are: the level of log firm TFP, $\log \hat{\theta}_{it}$; leverage, Lev$_{it}$; the previous year's stock return, $R_{i,t-1}$; an industry fixed effect $\chi_{j(i)}$; and a firm fixed effect γ_i. Standard errors are double-clustered at the year and firm level.

Table A7: Attention and Misoptimization, with Alternative Measurement Schemes

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
						Outcome: $\hat{u}_{it}^2$					
$\log \text{MacroAttention}_{it}$	-0.0081	-0.0163	-0.0035	-0.0076	-0.0127	-0.0107	-0.0084	-0.0062	-0.0140	-0.0080	-0.0140
	(0.0028)	(0.0066)	(0.0015)	(0.0028)	(0.0037)	(0.0028)	(0.0028)	(0.0026)	(0.0042)	(0.0028)	(0.0042)
Sector x Time FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Baseline	✓										
Conf. Call		✓									
Adj. Control			✓								
Leverage Control				✓							
Manufacturing					✓						
Sector Policy Fn.						✓					
t-varying Policy Fn.							✓				
Quadratic Policy Fn.								✓			
Pre-Period TFP									✓		
t-varying Prod. Fn.										✓	
OP (96) TFP											✓
N	28,279	5,997	24,024	28,133	14,891	28,283	28,275	28,275	24,785	28,266	24,785
R^2	0.053	0.072	0.060	0.053	0.041	0.054	0.051	0.056	0.046	0.053	0.046

Notes: $\hat{u}_{it}^2$ is the squared firm-level misoptimization residual, constructed using the methods described in Section 4.1 and the indicated variants described in the main text and the notes of Table A3. $\log \text{MacroAttention}_{it}$ is the measure of firm-level macroeconomic attention, constructed using the methods described in Section 4.2 and Appendix F.1. Standard errors are double-clustered at the year and firm level. The scenarios are described in the main text and the notes of Table A3.

Table A8: Policy Function Estimation

	Baseline	Adj. Cost	Leverage	Quadratic
	Panel A: Persistence of Misoptimization			
	Outcome: $\hat{m}_{it}^0$			
$\hat{m}_{i,t-1}^0$	0.696	0.016	0.696	0.683
	(0.021)	(0.005)	(0.003)	(0.003)
	Panel B: Quasi-Differenced Policy Function			
	Outcome: $\log L_{it} - \hat{\rho} \log L_{i,t-1}$			
$\log \hat{\theta}_{it}$	0.418	0.381	0.419	0.463
	(0.024)	(0.026)	(0.025)	(0.029)
$\log \hat{\theta}_{i,t-1}$	-0.031	-0.090	-0.026	0.006
	(0.018)	(0.015)	(0.019)	(0.020)
$\log \hat{\theta}_{it} \times \text{Lev}_{it}$			-0.008	
			(0.003)	
$\log \hat{\theta}_{i,t-1} \times \text{Lev}_{i,t-1}$			-0.025	
			0.006	
Lev_{it}			-0.020	
			(0.005)	
$\text{Lev}_{i,t-1}$			-0.050	
			(0.011)	
$(\log \hat{\theta}_{it})^2$				0.045
				(0.008)
$(\log \hat{\theta}_{i,t-1})^2$				0.031
				(0.006)
$\log L_{i,t-1}$		0.811		
		(0.012)		
$\log L_{i,t-2}$		-0.041		
		(0.010)		
N	51,891	44,051	51,664	51,891
R^2	0.896	0.990	0.896	0.904

Notes: The four columns correspond to four of our policy-function estimation methods, as described in the main text. The coefficient on $\hat{m}_{i,t-1}^0$ in panel A corresponds in the model to parameter ρ. Standard errors are double clustered by firm and year.

Table A9: Data Definitions in Compustat

	Quantity	Expenditure
Production, x_{it}	—	`sale`
Employment, L_{it}	`emp`	`emp` $\times$ industry wage
Materials, M_{it}	—	`cogs + xsga − dp` − wage bill
Capital, K_{it}	`ppegt` plus net investment	—

Table A10: Time-Series and Cross-Sectional Properties of Conference-Call Attention

	(1)	(2)	(3)	(4)	(5)	(6)
				Outcome:		
	$\log \mathrm{MacroAttnCC}_t$			$\log \mathrm{MacroAttnCC}_{it}$		
$\frac{\mathrm{Unemployment}_t}{100}$	2.481					
	(0.596)					
$\log \mathrm{SPDetrend}_t$		-0.270				
		(0.056)				
$\log \mathrm{MacroAttnCC}_{t-1}$			0.949			
			(0.068)			
$\log \mathrm{MacroAttn10K}_{it}$				0.463	0.372	0.121
				(0.034)	(0.036)	(0.028)
Firm FE						✓
Sector x Time FE					✓	✓
N	46	46	45	8,023	7,994	7,670
R^2	0.376	0.593	0.873	0.123	0.308	0.804

Notes: The "CC" MacroAttention is constructed using the methods described in Appendix F.1. The "10K" MacroAttention is our baseline measure from Section 5.5. In the first three columns, standard errors are HAC robust with a bandwidth (Bartlett kernel) of four quarters. In the second three columns, standard errors are double-clustered by year and firm ID.

Table A11: Time-Series and Cross-Sectional Properties of Word-Stemmed Attention

	(1)	(2)	(3)	(4)	(5)	(6)
				Outcome:		
	$\log \mathrm{MacroAttnStem}_t$			$\log \mathrm{MacroAttnStem}_{it}$		
$\frac{\mathrm{Unemployment}_t}{100}$	0.994					
	(0.330)					
$\log \mathrm{SPDetrend}_t$		-0.062				
		(0.031)				
$\log \mathrm{MacroAttnStem}_{t-1}$			0.811			
			(0.057)			
$\log \mathrm{MacroAttn10K}_{it}$				0.553	0.542	0.518
				(0.010)	(0.010)	(0.008)
Firm FE						✓
Sector x Time FE					✓	✓
N	92	92	92	46,612	46,590	45,458
R^2	0.118	0.140	0.675	0.561	0.639	0.867

Notes: The "Stem" MacroAttention is constructed using the methods described in Appendix F.2. The "10K" MacroAttention is our baseline measure from Section 5.5. In the first three columns, standard errors are HAC robust with a bandwidth (Bartlett kernel) of four quarters. In the second three columns, standard errors are double-clustered by year and firm ID.

Table A12: Selected Summary Statistics of Firm Micro-Data

Code	Name	1990 Sales millions	share	1990 Employees thousands	share	2010 Sales millions	share	2010 Employees thousands	share
11	Agriculture, Forestry, Fishing and Hunting	7935.69	0.22	117.03	0.54	16028.05	0.13	157.12	0.49
21	Mining, Quarrying, and Oil and Gas Extraction	143716.64	3.95	557.43	2.59	728283.11	5.88	1020.34	3.18
23	Construction	20221.05	0.56	79.43	0.37	76357.58	0.62	249.05	0.78
42	Wholesale Trade	92141.14	2.53	316.32	1.47	220566.67	1.78	286.52	0.89
44	Retail Trade (I)	90746.30	2.49	697.08	3.24	755083.01	6.10	1802.16	5.61
45	Retail Trade (II)	71844.36	1.98	606.25	2.82	103823.53	0.84	393.61	1.23
48	Transportation and Warehousing (I)	202207.84	5.56	1278.65	5.94	490525.14	3.96	1484.93	4.62
49	Transportation and Warehousing (II)	22033.22	0.61	337.61	1.57	60473.08	0.49	505.72	1.57
53	Real Estate and Rental and Leasing	29186.20	0.80	335.31	1.56	97644.47	0.79	414.75	1.29
54	Professional, Scientific, and Technical Services	39096.89	1.07	367.54	1.71	142888.17	1.15	878.65	2.74
56	Administrative and Support and Waste Management and Remediation Services	30757.47	0.85	1055.62	4.91	97139.76	0.78	1432.94	4.46
61	Educational Services	2830.78	0.08	21.45	0.10	12350.02	0.10	93.87	0.29
62	Health Care and Social Assistance	14676.48	0.40	278.49	1.29	113475.92	0.92	874.35	2.72
71	Arts, Entertainment, and Recreation	4299.00	0.12	77.61	0.36	14592.82	0.12	140.96	0.44
72	Accommodation and Food Services	28474.41	0.78	741.09	3.45	126452.99	1.02	1949.83	6.07
81	Other Services (except Public Administration)	818.90	0.02	13.61	0.06	6028.48	0.05	71.80	0.22
99	Nonclassifiable Establishments	72587.03	2.00	402.04	1.87	337554.38	2.73	957.49	2.98
311	Food Manufacturing	89888.93	2.47	393.90	1.83	280503.94	2.26	773.89	2.41
312	Beverage and Tobacco Product Manufacturing	81514.55	2.24	527.06	2.45	323117.33	2.61	1128.77	3.51
313	Textile Mills	8281.43	0.23	109.85	0.51	1698.84	0.01	14.17	0.04
314	Textile Product Mills	1642.40	0.05	15.52	0.07	6512.22	0.05	31.62	0.10
315	Apparel Manufacturing	18195.52	0.50	179.87	0.84	50193.90	0.41	204.02	0.64
316	Leather and Allied Product Manufacturing	3220.64	0.09	18.56	0.09	8007.68	0.06	26.47	0.08
321	Wood Product Manufacturing	17079.99	0.47	94.34	0.44	32329.60	0.26	79.89	0.25

Code	Name	1990 Sales millions	share	1990 Employees thousands	share	2010 Sales millions	share	2010 Employees thousands	share
322	Paper Manufacturing	80736.02	2.22	485.96	2.26	137924.31	1.11	366.40	1.14
323	Printing and Related Support Activities	6959.89	0.19	64.35	0.30	17960.41	0.14	98.55	0.31
324	Petroleum and Coal Products Manufacturing	707106.33	19.44	1054.61	4.90	2666606.41	21.53	1718.55	5.35
325	Chemical Manufacturing	376182.22	10.34	2146.13	9.98	1234732.55	9.97	2362.52	7.36
326	Plastics and Rubber Products Manufacturing	23886.12	0.66	206.51	0.96	35081.57	0.28	139.19	0.43
327	Nonmetallic Mineral Product Manufacturing	22485.57	0.62	190.56	0.89	72492.53	0.59	242.54	0.76
331	Primary Metal Manufacturing	81200.98	2.23	438.98	2.04	308524.32	2.49	883.98	2.75
332	Fabricated Metal Product Manufacturing	34872.77	0.96	296.50	1.38	51755.70	0.42	172.89	0.54
333	Machinery Manufacturing	126838.80	3.49	823.90	3.83	303757.26	2.45	1060.24	3.30
334	Computer and Electronic Product Manufacturing	169692.71	4.66	1377.51	6.40	575557.43	4.65	1976.28	6.15
335	Electrical Equipment, Appliance, and Component Manufacturing	48369.82	1.33	447.71	2.08	96056.38	0.78	394.72	1.23
336	Transportation Equipment Manufacturing	553821.60	15.22	3359.30	15.62	1210603.25	9.77	3201.16	9.97
337	Furniture and Related Product Manufacturing	2516.33	0.07	23.81	0.11	15259.16	0.12	68.65	0.21
339	Miscellaneous Manufacturing	16722.13	0.46	143.46	0.67	63225.24	0.51	259.74	0.81
511	Publishing Industries (except Internet)	18829.43	0.52	149.15	0.69	42552.96	0.34	170.81	0.53
512	Motion Picture and Sound Recording Industries	13821.56	0.38	53.44	0.25	36968.97	0.30	112.21	0.35
515	Broadcasting (except Internet)	23400.47	0.64	187.74	0.87	189150.10	1.53	435.67	1.36
517	Telecommunications	158686.32	4.36	1045.88	4.86	1107220.16	8.94	3041.71	9.47
518	Data Processing, Hosting, and Related Services	1032.31	0.03	9.77	0.05	74483.99	0.60	273.40	0.85
519	Other Information Services	77053.51	2.12	384.84	1.79	45026.66	0.36	168.24	0.52
Total		3637612	100	21512	100	12386570	100	32120	100
USA		5963000		125840		14990000		153890	
Compustat share of USA		61.00		17.09		82.63		20.87	

Table A13: Comparison of TFP Measures

	(1)	(2)	(3)
	Outcome: Cost-Share TFP		
OP TFP	1.117	1.141	1.025
	(0.009)	(0.008)	(0.006)
Firm FE		✓	✓
Sector x Time FE			✓
N	68,825	68,821	67,395
R^2	0.649	0.721	0.977

Notes: "Cost-Share TFP" is our baseline measure of $\log \hat{\theta}_{it}$ used in the main analysis. "OP TFP" is the alternative measure based on the method of Olley and Pakes (1996). Standard errors are double-clustered by year and firm ID.

Table A14: Changing Macro Attention in Response to News

Response	Poorly	Well
Much more likely	44.96	9.77
Somewhat more likely	30.91	19.42
No change	12.56	8.67
Somewhat less likely	7.16	53.35
Much less likely	4.40	8.79
Total	100.00	100.00

Notes: Data are from the Coibion et al. (2018) survey, as described in Appendix G.

Table A15: Profit-Function Curvature and Attention to Macro Variables

Panel 1: Back-cast Error (Absolute Value)

Variable	Inflation		GDP Growth		Unemployment	
ProfitCurv_{it}	-1.172	-0.328	-0.072	-0.042	0.075	0.121
	(0.195)	(0.091)	(0.041)	(0.041)	(0.072)	(0.077)
Controls		✓		✓		✓
R^2	0.024	0.457	0.001	0.006	0.001	0.032
N	3,153	3,145	1,257	1,237	1,257	1,256

Panel 2: Keeping Track

Variable	Inflation		GDP Growth		Unemployment	
ProfitCurv_{it}	0.170	0.050	0.015	0.019	-0.005	-0.022
	(0.039)	(0.029)	(0.022)	(0.028)	(0.035)	(0.081)
Controls		✓		✓		✓
R^2	0.032	0.332	0.000	0.074	0.000	0.065
N	1,255	1,235	1,255	1,235	1,255	1,235

Notes: Standard errors are clustered by three-digit industry. Data are from the Coibion et al. (2018) survey, as described in Appendix G.

Table A16: Profit Curvature in the Data

Summary Statistics						

Mean	Quantiles					
	5	25	50	75	95	
0.280	0.020	0.05	0.12	0.28	1.00	

Correlates				

	Variable	Norm. coef.	t-stat	R^2
	Frequency of price review	-0.106	-7.80	0.011
	log output	-0.066	-9.43	0.015
Firm	Firm age	-0.117	-10.17	0.014
	Employment	-0.122	-7.19	0.015
	Labor share	-0.138	-7.98	0.020
	Number of competitors	0.130	6.81	0.017
	log income	0.015	0.55	0.000
	Some or more college	0.043	1.87	0.002
Manager	Tenure at firm	-0.117	-5.73	0.014
	Tenure in industry	-0.058	-2.33	0.003
	Manager age	-0.091	-3.25	0.008

Notes: The top panel gives summary statistics. The bottom panel gives normalized regression coefficients for a number of possible correlates. Standard errors, used to calculate the t-statistics, are clustered by three-digit industry.

Table A17: Curvature and Inflation BCE in Waves 1 versus 4

	Outcome: absolute Inflation BCE			
Wave	1		4	
ProfitCurv$_{it}$	-1.172	-0.328	-0.884	-0.330
	(0.195)	(0.091)	(0.181)	(0.126)
Controls		✓		✓
R^2	0.024	0.457	0.033	0.268
N	3,153	3,145	1,257	1,256

Notes: Standard errors are clustered by three-digit industry.

Figure A1: Relationship of Misoptimization Dispersion with Other Statistics
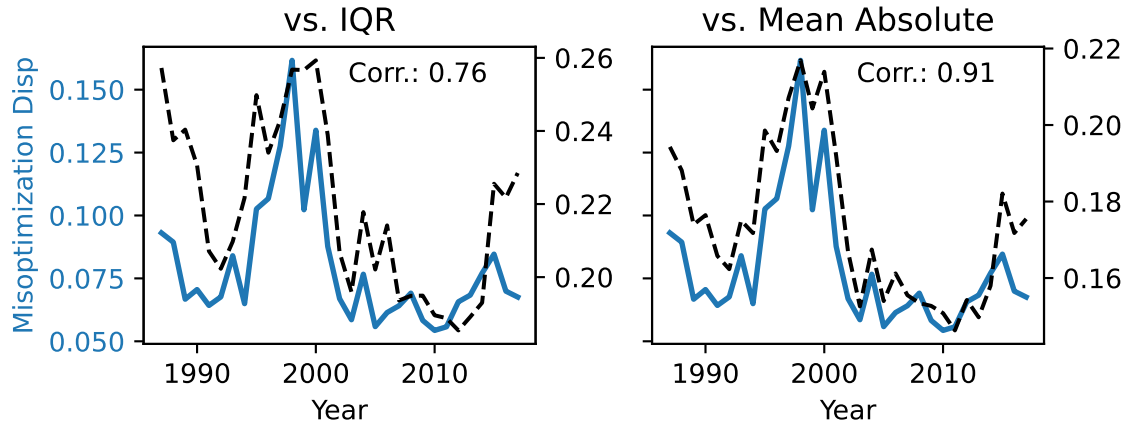
Notes: The blue line, measured by the left axis of each plot, is Misoptimization Dispersion as defined in Section 5. The black dashed line on the left is the (optimal-sale-weighted) interquartile range of the distribution of $\hat{u}_{it}$. The black dashed line on the right is the (optimal-sale-weighted) average of $|\hat{u}_{it}|$.

Figure A2: Misoptimization Dispersion is Pro-Cyclical, Long Sample
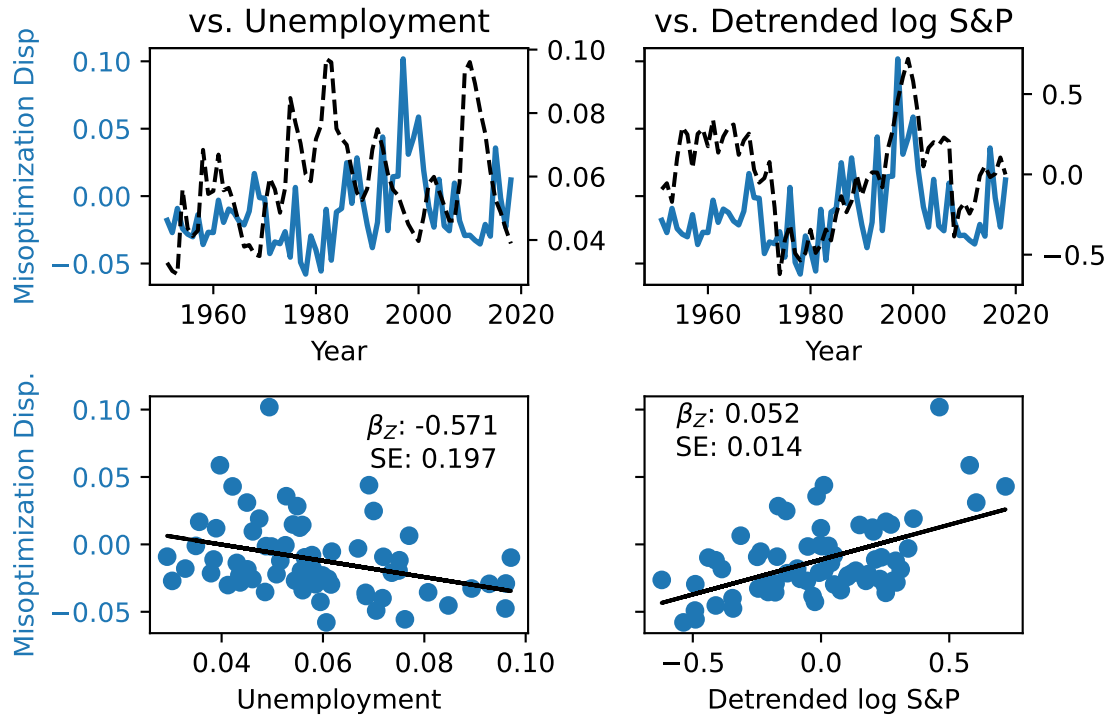
Notes: This Figure replicates Figure 2 using our long-sample (1950-2018) calculation of Misoptimization Dispersion, described in Section 5.2 ("Robustness to Studied Time Period."). The top two panels plot Misoptimization Dispersion (blue line, left axis) along with, respectively, unemployment and the linearly detrended S&P 500 price (black dashed lines, right axis). Because of the composition adjustment, the metric can be negative. The bottom two panels are scatterplots of Misoptimization Dispersion versus each macroeconomic aggregate. The black solid line is the linear regression fit. The standard errors are HAC robust based on a Bartlett kernel with a three-year bandwidth.

Figure A3: Relationship of Misoptimization Dispersion Across Inputs
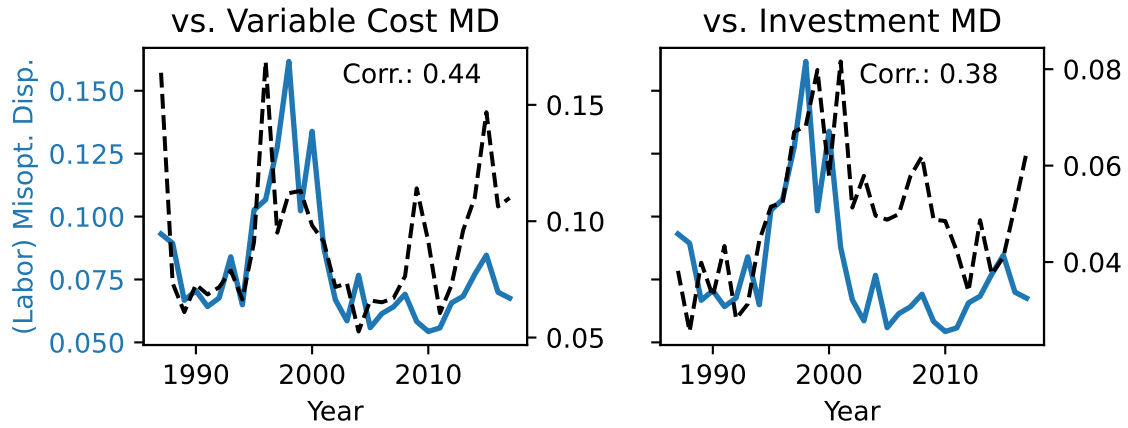
Notes: The blue line, measured by the left axis of each plot, is Misoptimization Dispersion as defined in Section 5. The black dashed line on the left is Misoptimization Dispersion for total variable cost expenditures. The black dashed line on the right is the same for investment rates.

Figure A4: Relationship of TFP Innovation Variance with Macro Variables
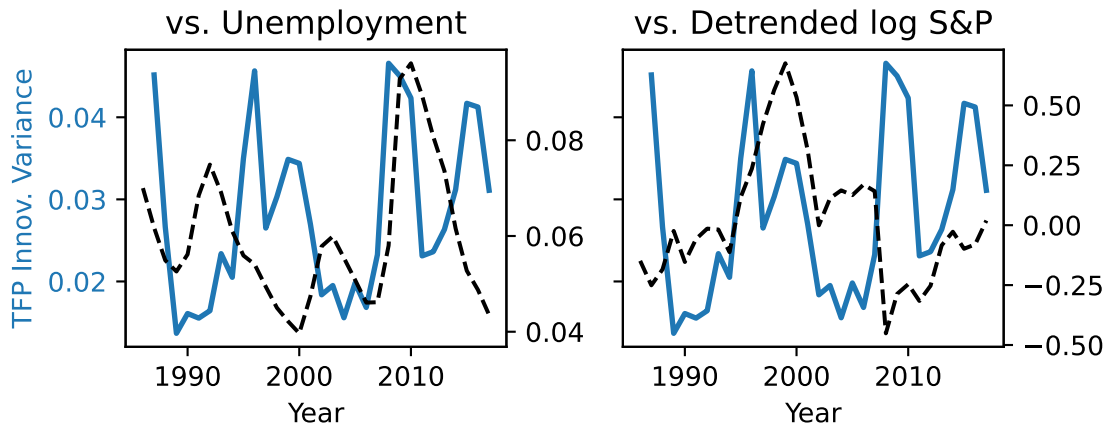
Notes: "TFP Innovations" are the residuals from an AR(1) model for $\log \theta_{it}$ with firm and sector-by-time fixed effects, as described in Section 5.7 of the main text. We calculate their average, consistent with our main calculations of Misoptimization Dispersion, using optimal-sales-weights.

Figure A5: Frequency over Time of Each Word in MacroAttention (Part I)

Figure A5: Frequency over Time of Each Word in MacroAttention (Part II)

Figure A6: Correlations with Unemployment by Word
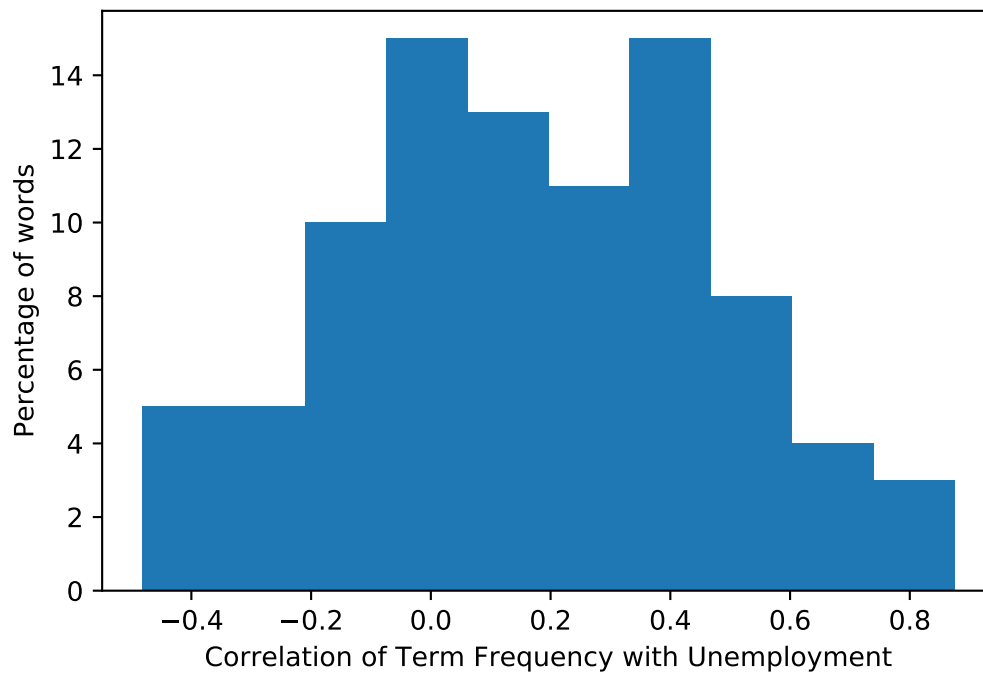
Notes: Correlations are calculated at the quarterly frequency.

Figure A7: Industry-Specific Cyclicality of Macro Attention
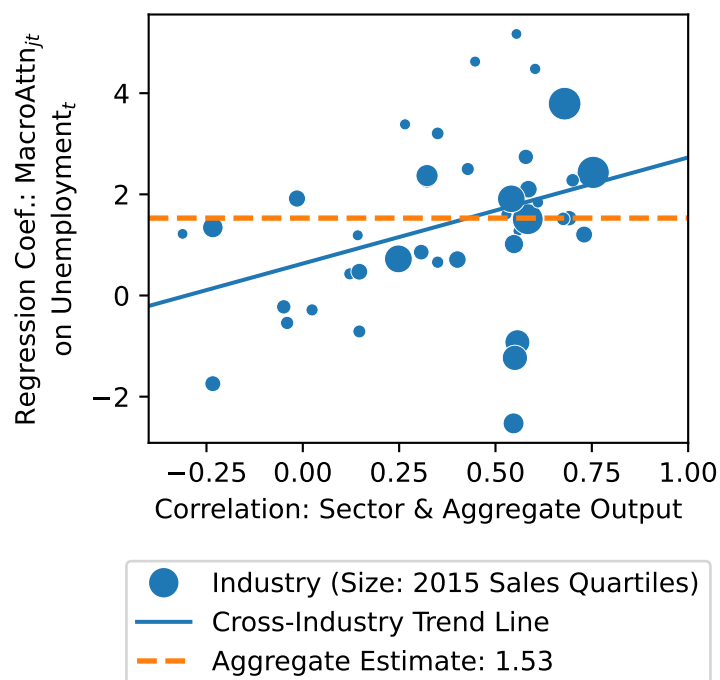
Notes: The horizontal axis is the correlation of sectoral and aggregate nominal GDP. The vertical axis is the regression coefficient of log sectoral macro attention, net of quarterly fixed effects, on the US unemployment rate. The dashed orange line is the estimate of the same using aggregate Macro Attention. The dots are sized based on quartiles of total sales in Compustat in 2015. The blue solid line is a cross-industry linear regression line.

Figure A8: The Attention Wedge and Its Derivatives
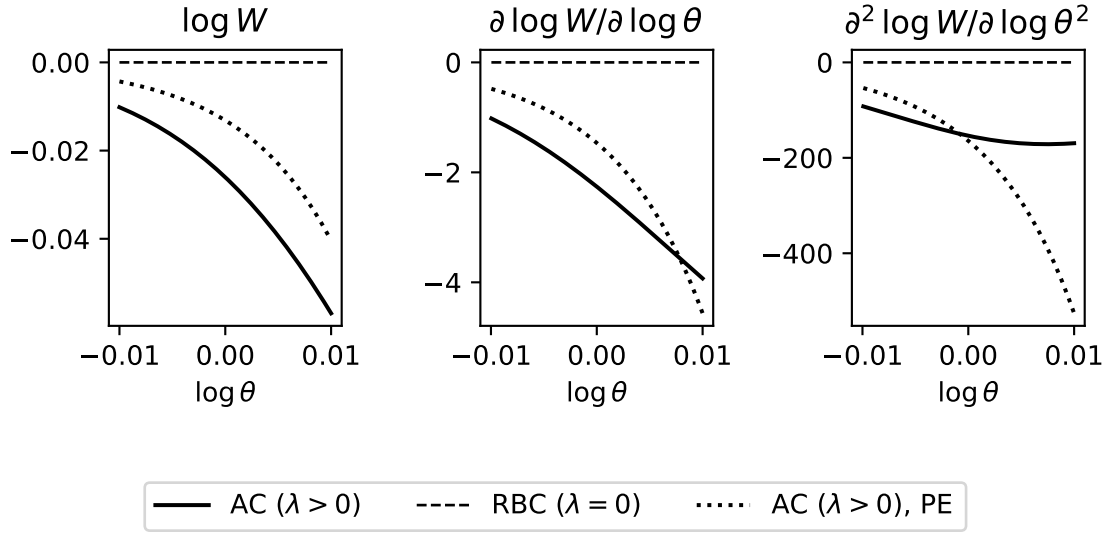
Notes: The panels respectively show the level, first derivative, and second derivative of the log attention wedge in the log state. The "Partial Equilibrium" thought experiment, plotted in each figure as a dotted line, is for firms to best-reply to the output and wages of the counterfactual RBC equilibrium.

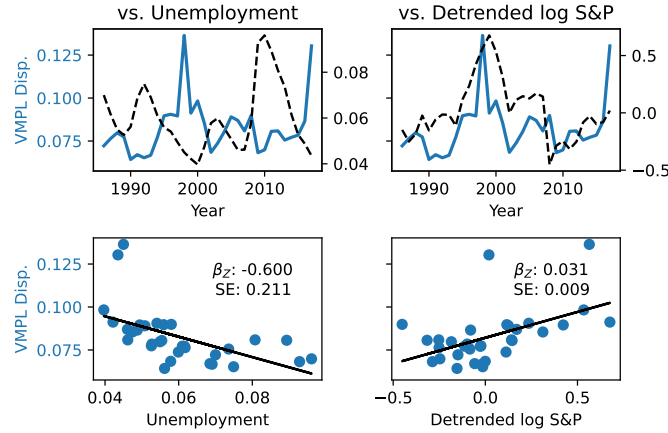**Figure A9:** Conference-Call Macro Attention and Unemployment
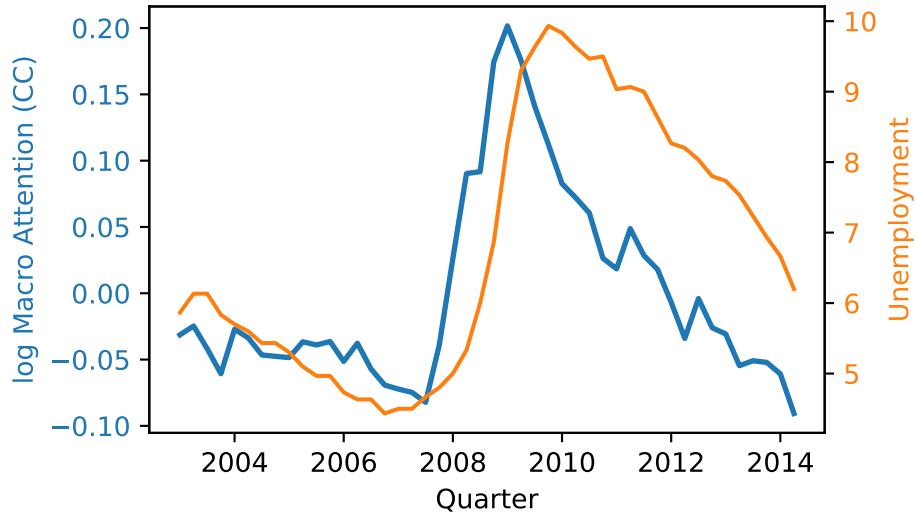
Notes: The left axis and blue line show our estimate of Macro Attention based on conference-call data, in log units net of seasonal trends. The right axis and orange line show the US unemployment rate.

**Figure A10:** The Cyclical Behavior of TFPR and VMPL Dispersion
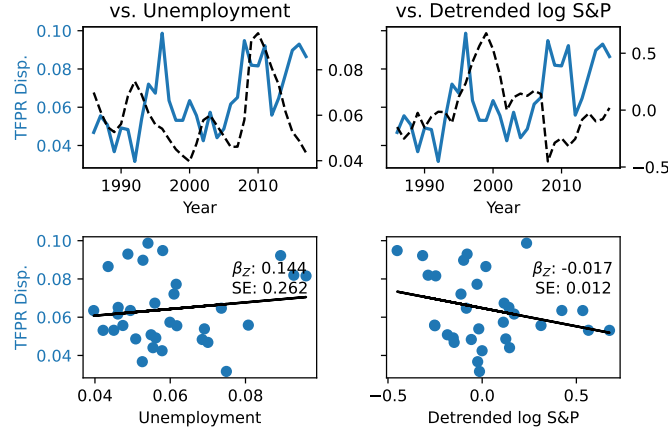
(a) TFPR Dispersion

(b) VMPL Dispersion

Notes: Each half of this Figure replicates Figure 2 using our measures of TFPR Dispersion (top) and VMPL Dispresion (bottom). Variable construction is defined in Appendix F.3. Because of the composition adjustment, the variance metrics can be negative. The standard errors for the linear fits are HAC robust baesd on a Bartlett kernel with a three-year bandwidth.

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**Figure A11:** Robustness of Numerical Results to Parameter Choices
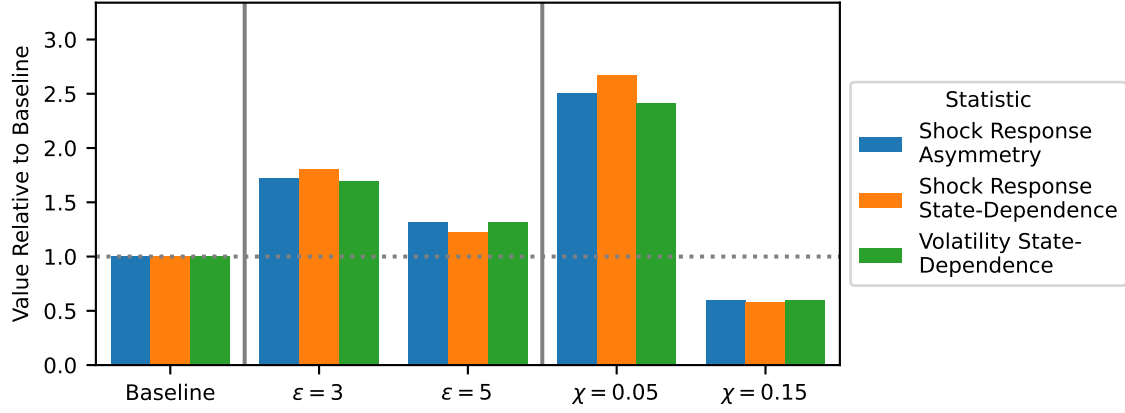
Notes: We re-calibrate the model under each indicated parameter choice. The outcomes and their interpretation are described in Appendix E.1.

**Figure A12:** Output, Wedge, and Labor Productivity with GHH Preferences
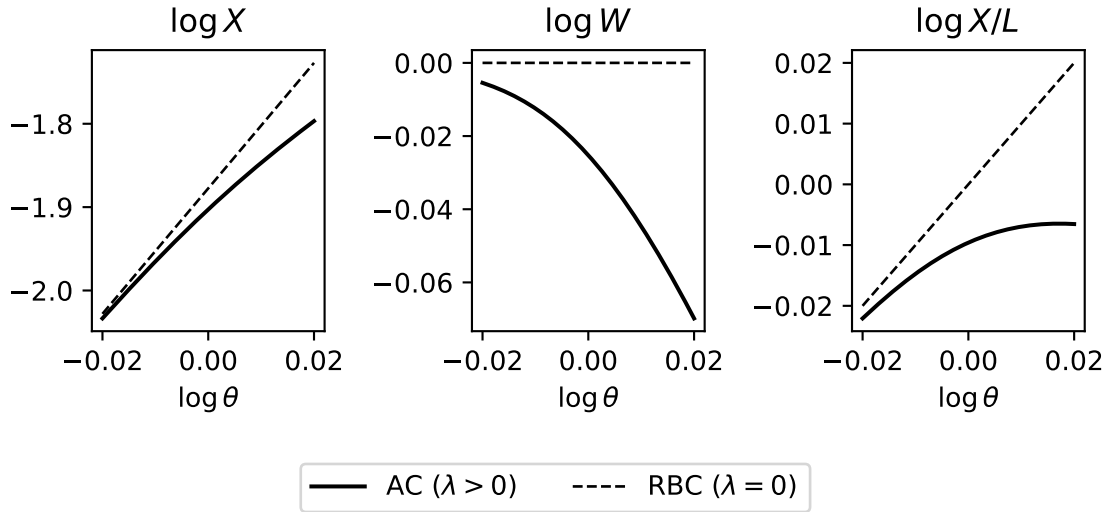
Notes: This recreates Figure 5 from the main analysis in the variant model with Greenwood et al. (1988) preferences, described in Appendix E.

Figure A13: Asymmetric Shock Response and Stochastic Volatility with GHH Preferences
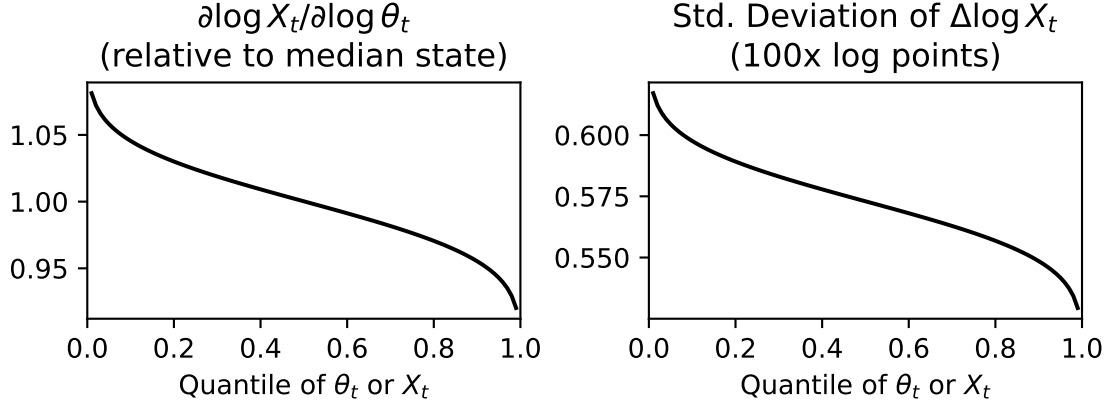
Notes: This recreates Figure 6 from the main analysis in the variant model with Greenwood et al. (1988) preferences, described in Appendix E.

Figure A14: Predictions in Counterfactual Scenarios
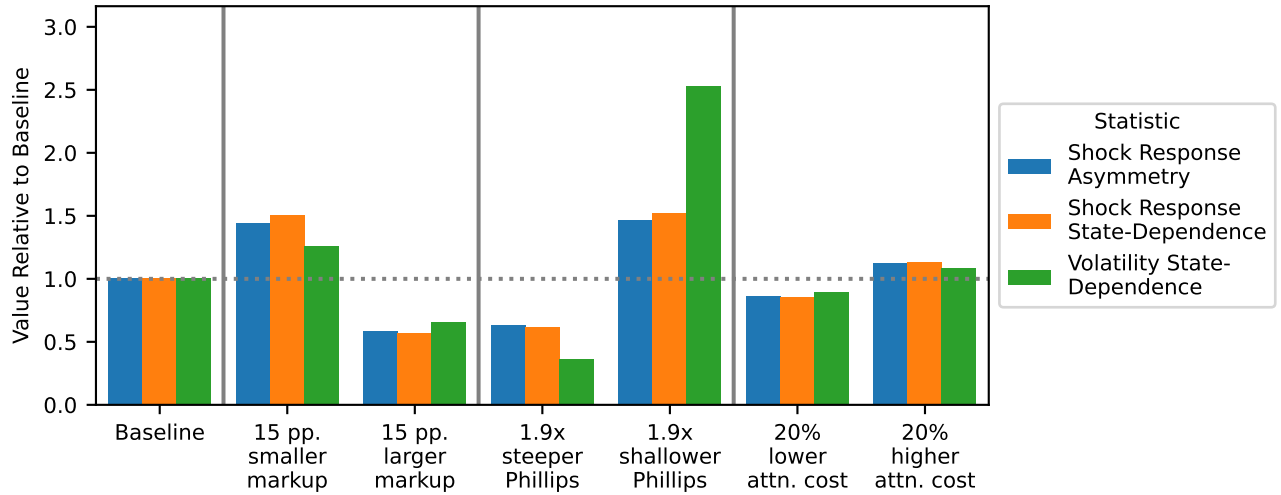
Notes: The scenarios are described in the main text. The outcomes are the same as in Figure A11, and are described in Appendix E.1.

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