What Can Measured Beliefs Tell Us About Monetary Non-Neutrality?*

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Abstract

This paper studies how measured beliefs can be used to identify the real effects of monetary policy. In a general equilibrium model with both nominal rigidities and endogenous information acquisition, we analytically characterize firms' optimal dynamic information policies and derive a closed-form representation of how their beliefs affect the response of output to monetary shocks. Next, we show that data on the cross-sectional distributions of uncertainty and pricing durations are both necessary and sufficient to identify monetary non-neutrality. Finally, implementing our approach in New Zealand survey data, we find support for the predictions of models with endogenous information acquisition and that information frictions approximately double monetary non-neutrality.

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1 Introduction

The real effects of monetary policy depend on the responsiveness of firms' beliefs (see *e.g.*, Woodford, 2003; Nimark, 2008), as they determine the extent to which firms set prices based on accurate information. However, beliefs are not something outside of a firm's control—firms can, and do, acquire information to make better decisions when it is valuable to do so (Sims, 2003; Maćkowiak and Wiederholt, 2009). These considerations are potentially important as a large body of recent survey evidence (see *e.g.,* Coibion, Gorodnichenko, and Kumar, 2018) demonstrates that the average firm holds highly inaccurate and diffuse beliefs about its economic environment and that there is substantial heterogeneity in these beliefs across firms. In this paper, we therefore ask three questions. How do firms choose their beliefs? How do their beliefs matter for the real effects of monetary policy? And whose beliefs matter? In studying these questions, we aim to arrive at a better understanding of what empirical measures of firms' beliefs tell us about monetary non-neutrality.

Model. To answer these questions, we begin by studying a general equilibrium monetary economy, in which firms face time-dependent pricing frictions: the probability that a firm can adjust its price depends on the duration of a firm's pricing spell. This model nests both the Calvo (1983) model, in which the rate at which firms can adjust their prices is constant, and the Taylor (1979) model, in which firms adjust their prices at fixed time intervals.

In this otherwise standard model, we add a novel component; we allow firms to acquire any possible dynamic information structure about their marginal costs of production by paying a cost that is proportional to the flow of the information that they acquire. We derive two main theoretical results that provide analytical characterizations of firms' optimal dynamic information policies and the general equilibrium output response to a monetary shock.

Theoretical Results: Optimal Uncertainty and Monetary Non-Neutrality. First, we show that firms' optimal information acquisition policy takes a simple form: acquire information only when changing prices and acquire exactly enough information to reset posterior uncertainty about the optimal price to some state-independent level, U^*. Intuitively, while being better informed reduces the costs of achieving any given level of uncertainty in the future as you need to acquire less information, it does not affect the *marginal cost* of reduced uncertainty. Moreover, we show that: the optimal level of uncertainty is decreasing in the firm's demand elasticity (as this increases the losses from setting the wrong price); increasing in the volatility of marginal costs (as this reduces the value of information acquired today for future decisions); and ambiguously affected by price stickiness (as this both increases the value of information for this pricing spell and decreases the value of information for all future pricing spells).

A key implication of this result is that a firm's uncertainty is increasing in the duration of its pricing spell. This implies that price-setting firms are the least uncertain firms in the economy. We call this phenomenon *selection in information acquisition* as price-setting firms are the most informed in the cross-section at any given point in time. This represents a testable implication of our model relative to alternatives with exogenous information frictions and nominal rigidities (as in Nimark, 2008; Angeletos and La'O, 2020) or models of endogenous information acquisition without nominal rigidities (as proposed by Sims, 2003; Maćkowiak and Wiederholt, 2009; Afrouzi and Yang, 2021): in both such cases, firms' uncertainty has no relationship with the duration of their pricing spell.

Second, we study the real effects of monetary policy by characterizing the cumulative impulse response (CIR) of aggregate output to a monetary policy shock. Normalizing this CIR by the size of the monetary policy shock, we denote the CIR by $\mathscr{M}^b$. Letting firms' marginal costs have variance σ^2 and letting $\bar{D}$ be the average duration of firms' pricing spells, we find a simple formula for the CIR:

$$\mathscr{M}^b = \bar{D} + \frac{U^*}{\sigma^2} \tag{1}$$

The first term is the usual effect of price stickiness (as in Carvalho, 2006; Carvalho and Schwartzman, 2015), which (all else equal) increases monetary non-neutrality as firms' prices are stuck for longer. The second term is new to our analysis and captures the lifetime lack of responsiveness of all firms in the economy in resetting their prices in light of their uncertainty. Intuitively, when price resetting firms are more uncertain, they respond to their current information to a lesser degree and so adjust their prices by less in response to a monetary shock. Moreover, when microeconomic volatility is higher (all else equal), firms know that their old information is less likely to be useful as things will have since changed by a larger amount; this makes firms more responsive to their information and lowers the extent of monetary non-neutrality. However, all else is not equal as optimal uncertainty is increasing in the volatility of marginal costs and moves ambiguously with respect to price stickiness. As a result, price stickiness and marginal cost volatility have theoretically ambiguous effects on the efficacy of monetary shocks.

This result establishes that uncertainty amplifies the real effects of monetary policy relative to a full-information benchmark. However, because of selection in information acquisition, we would systemically overstate the real effects of monetary policy if we looked at the data through the lens of a model with exogenous information, which would imply that the uncertainty of the average firm is what matters.

Our final theoretical results establish that the sufficient statistics that determine the CIR (as per Equation 1) can be estimated given cross-sectional data on firms' uncertainty and the time since they last reset their price. Thus, survey data on theses quantities is sufficient to identify the model. Moreover, such data are necessary in the sense that access to data that provide the distribution of price changes are insufficient to identify the model in the presence of endogenous information acquisition. This is because the firm's choice of information structure causes the distribution of price changes to be invariant to the uncertainty of price setters.

Micro-To-Macro: Using Survey Data to Quantify the Model. Finally, we adopt a "micro-to-macro" approach of combining measured beliefs with the structure of the model to quantify the extent to which imperfect information and endogenous information acquisition matter for monetary non-neutrality.

First, by integrating a new question into a survey of New Zealand firms between Q4 2017 and Q2 2018 (as run by Coibion et al., 2018; Coibion, Gorodnichenko, Kumar, and Ryngaert, 2021), we obtain information on firms' uncertainty about their optimal reset prices and the length of their current pricing spell.

Second, we use these survey data to test the key prediction of our model with endogenous information acquisition with nominal rigidities: firms that recently changed their prices should be less uncertain about their optimal reset prices. We find an upward-sloping empirical relationship between pricing duration and uncertainty that is robust to controlling for sector fixed effects and a suite of firm-level and manager-level controls. Thus, we argue that selection in information acquisition is not only quantitatively important but also present in the data. Moreover, this result provides evidence in favor of models that feature information costs and against models that feature only fixed information capacities or exogenous information.

Third, applying the estimators for the CIR from the theory to the survey data, we find that accounting for uncertainty approximately doubles the CIR that one would obtain under full information. Moreover, the endogeneity of information acquisition would lead us to overstate the size of the CIR by approximately 50%. Thus, we argue that both uncertainty and selection in information acquisition are quantitatively important.

Finally, by using the firm's first-order condition for its optimal uncertainty, we can derive and implement estimators of the effect of counterfactually increasing microeconomic volatility and price stickiness on the CIR. We find that greater microeconomic volatility significantly dampens the real effects of monetary policy. This is because the direct effect of reducing firms' reliance on past information quantitatively dominates the indirect effect that firms optimally choose to be less informed in the face of this increase. We also find that greater price stickiness increases monetary non-neutrality, but by approximately 20% less than with full information. This is because we find that firms would become better informed in the face of increased stickiness. This is itself because the effect of increasing the duration over which information gathered today is used quantitatively dominates the reduction in the value of information for future pricing spells.

Related Literature. This paper builds on and contributes to several strands of literature. First, we contribute to the literature studying the real effects of monetary policy shocks under price stickiness or informational frictions. The seminal work by Golosov and Lucas (2007) shows that a reasonably calibrated standard menu cost model cannot generate sizable monetary non-neutrality because of the strong selection effects of price changes. Following the seminal work by Sims (2003), the rational inattention literature provides another mechanism through which monetary policy shocks can have real effects.[1] Maćkowiak and Wiederholt (2009) develop a stylized rational inattention model and find that firms pay less attention to aggregate shocks, which are less volatile than idiosyncratic shocks, leading to large monetary non-neutrality. In our model, we study both sticky prices and rational inattention in a unified framework to study the real effects of monetary policy shocks.

The theoretical model we study in Section 2 is different from previous models with both nominal rigidities and informational frictions. For example, Gorodnichenko (2008) studies a menu cost model with a partial information acquisition with a fixed observational cost. Alvarez, Lippi, and Paciello (2011), Alvarez, Lippi, and Paciello (2017), and Bonomo, Carvalho, Garcia, Malta, and Rigato (2023) study models with both menu costs and observational costs, where firms decide when they observe either idiosyncratic shocks or aggregate shocks by paying a fixed cost. In these models, firms can perfectly observe the underlying shocks that whenever they pay the fixed cost. Woodford (2009) and Stevens (2019) develop models with consideration costs, where firms' price reviews incur a fixed cost and the review decision is made on the basis of incomplete information. However, in these models, firms have perfect information once they pay the consideration costs. Yang (2022) develops a model with both menu costs and rational inattention for multi-product firms and shows that price adjusters choose to be better informed about underlying shocks. This selection effect in information processing leads to a leptokurtic distribution of firms' desired price changes. Our new contribution to this literature is to develop a continuous-time model with both rational inattention and nominal rigidities and study its implications for monetary non-neutrality in an analytical framework.

This relates our analysis to recent literature studying how firms form their expectations and how their expectation affects their decisions. Using the survey of New Zealand firms' macroeconomic beliefs that we also use in this paper, Kumar, Afrouzi, Coibion, and Gorodnichenko (2015), Coibion et al. (2018), and Coibion et al. (2021) study determinants of firms' inattentiveness to aggregate economic conditions, how firms update their beliefs in response to new information, and how changes in their belief affect their decisions. While informative, as these analyses do not bridge theory and data, they do not speak to the quantitative relevance of uncertainty. Afrouzi (2023) shows that firms facing more competitors are better informed about

[1] See, for instance, Sims (2010) and Maćkowiak, Matějka, and Wiederholt (2023) for comprehensive reviews.

aggregate inflation while Yang (2022) shows firms with a greater product scope have better information about aggregate economic conditions. We use the same New Zealand survey data to estimate and test our model. Furthermore, our analysis relates to the literature that studies the implications of different specifications for the cost of information acquisition in different settings (e.g., Caplin and Dean, 2013; Hébert and Woodford, 2018; Caplin, Dean, and Leahy, 2022). Our finding of selection in the data provides evidence in favor of cost-based information acquisition as opposed to models with exogenous capacities and may be of interest beyond macroeconomics.

2 Model: Sticky Prices with Information Acquisition

We study a general equilibrium monetary economy with endogenous information acquisition by firms that are subject to general, time-dependent pricing frictions. To make the role of information acquisition as clear as possible, the macroeconomic side of the model follows Golosov and Lucas (2007), Alvarez and Lippi (2014), and Alvarez, Le Bihan, and Lippi (2016). Conditional on this canonical structure, our goal is to answer two questions: how do firms optimally acquire information? And how does the optimal choice of information affect our understanding of the propagation of monetary shocks?

2.1 Households

Primitives. Time is continuous and indexed by $t \in [0,\infty]$. A representative household has preferences over consumption C_t, real money balances M_t/P_t (where M_t is money and P_t is the price of consumption), and labor L_t given by:

$$\int_0^\infty e^{-rt}\left[\frac{C_t^{1-\gamma}}{1-\gamma}+\log\left(\frac{M_t}{P_t}\right)-\alpha L_t\right]\mathrm{d}t \tag{2}$$

where $r>0$ is the discount rate, $\gamma^{-1}>0$ is the elasticity of intertemporal substitution, and $\alpha>0$ indexes the extent of labor disutility. Consumption is a constant elasticity of substitution aggregate of a continuum of varieties, indexed by $i\in[0,1]$:

$$C_t=\left(\int_0^1 A_{i,t}^{\frac{1}{\eta}}C_{i,t}^{\frac{\eta-1}{\eta}}\,\mathrm{d}i\right)^{\frac{\eta}{\eta-1}} \tag{3}$$

where $\eta>1$ is the elasticity of substitution between varieties and $A_{i,t}$ is a variety-specific taste shock. The household can also trade a risk-free nominal bond in zero net supply that pays a nominal interest rate of R_t. Thus, the household's lifetime budget constraint is:

$$M_0 + \int_0^\infty \exp\left(-\int_0^t R_s \, \mathrm{d}s\right) \left[w_t L_t + \int_0^1 \Pi_{i,t} \, \mathrm{d}i - \int_0^1 P_{i,t} C_{i,t} \, \mathrm{d}i - R_t M_t \right] \mathrm{d}t = 0 \tag{4}$$

where w_t is the wage, $P_{i,t}$ is the price of variety i at time t, and $\Pi_{i,t}$ is the net nominal profit of firm i at time t.

The money supply is constant and equal to $\bar{M}$. Later, when we do monetary experiments, we will shock $\bar{M}$ to $\bar{M} + \delta$ for some small value of $\delta \in \mathbb{R}$.

Optimality Conditions. As is well-known, this setup implies the following optimality conditions, which reduce understanding aggregate dynamics to understanding the price-setting decisions of each firm in the economy. First, the household's demand for consumption variety i at time t is given by:

$$C_{i,t} = A_{i,t} C_t \left(\frac{P_{i,t}}{P_t} \right)^{-\eta} \tag{5}$$

where aggregate price index is given by:

$$P_t = \left(\int_0^1 A_{i,t} P_{i,t}^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{6}$$

and aggregate consumption demand is given by:

$$C_t^{-\gamma} = \alpha \frac{P_t}{w_t} \tag{7}$$

Moreover, nominal wages and the interest rate are given by:

$$w_t = \alpha r M_t \quad \text{and} \quad R_t = r \tag{8}$$

This final equation is key to the environment's tractability: the endogenous component of marginal costs (wages) is pinned down solely by the level of the money supply.

2.2 Firms' Production, Pricing, and Profits

Production Technology. Each variety $i \in [0,1]$ is produced by a firm with the same index. Firms produce output $Y_{i,t}$ according to the linear production technology:

$$Y_{i,t} = \frac{1}{Z_{i,t}} L_{i,t} \tag{9}$$

where $L_{i,t}$ is the labor input and $Z_{i,t}$ is a marginal cost shock to the firm. As in Alvarez and Lippi (2014), we make the simplifying assumption that $Z_{it}^{1-\eta} A_{i,t} = 1$, which implies that (log) marginal cost is perfectly correlated with (log) demand. Moreover, we assume that:

$$Z_{i,t} = \exp\{\sigma W_{i,t}\} \tag{10}$$

where $\{W_{i,t}\}_{t\geq 0}$ is a standard Brownian motion that is independent across $i \in [0,1]$.

Time-Dependent Pricing. Firms are price setters and subject to a time-dependent pricing friction. Formally, price change opportunities for firm i are governed by the Poisson process $N_{i,t}$ which is independent across $i \in [0,1]$. We assume that the distribution of price reset opportunities is exogenously given by G. We moreover assume that G admits a density g and define its hazard rate as $\theta(h) = g(h)/(1-G(h))$.

This general model of time-dependent pricing nests several important benchmarks, including Calvo (1983) pricing in which price reset opportunities arise at a constant rate:

Example 1 (Calvo Pricing)**.** Price reset opportunities arise at a constant rate $\theta(h) = \theta$. △

A more general formulation, in which G does not admit a density, also allows for Taylor (1979) pricing, under which firms reset their prices periodically. All of our results hold under this specification:

Example 2 (Taylor Pricing)**.** Price reset opportunities arise every $k \in \mathbb{R}_+$ periods and so $G = \delta_k$, where δ_k is a Dirac delta function on k. △

We can also capture richer patterns that combine random and periodic resets, such as the following example:

Example 3 (Hybrid Pricing)**.** Price reset opportunities are uniform on $[0,k]$. That is, $g(h) = k^{-1}\mathbb{I}[h \in [0,k]]$. △

Approximating Firms' Profits. Given their price at a given time, firms commit to hiring enough labor to meet demand at their given price. Define the (log) optimal price of the firm as $q_{i,t} = \log\left(\frac{\eta}{\eta-1} w_t Z_{it}\right)$ and the (log) price of the firm as $p_{i,t} = \log P_{i,t}$. Approximating the firm's profit function to second-order around $p_{i,t} = q_{i,t}$, we obtain that the firm's loss from mispricing relative to the optimum is given by:

$$\mathscr{L}(p_{i,t}) = -\frac{B}{2}(p_{i,t} - q_{i,t})^2 \tag{11}$$

where $B = \eta(\eta-1)$. Intuitively, when the firm faces more elastic demand, the losses from mispricing are larger.

2.3 Firms' Costly Information Acquisition

So far we have followed the textbook model of firm pricing in general equilibrium. We now introduce the novel feature of our analysis: endogenous information acquisition. We assume firms are aware of their price change opportunities, *i.e.,* they observe the process $N_{i,t}$, but cannot directly observe the shock to their marginal costs and acquire information about this process subject to a cost.

Formally, given the joint measure for the process $\{(W_{i,t}, N_{i,t}) : t \geq 0\}$, firm i chooses a joint measure for $\{(W_{i,t}, N_{i,t}, s_{i,t}) : t \geq 0\}$, observes realizations of the process $s_{i,t}$ along with $N_{i,t}$ and makes decisions at time t given the information set $S_i^t \equiv \{(s_{i,h}, N_{i,h}) : h \leq t\} \in \mathscr{S}^t$.

We assume that the cost of acquiring information is given by mutual information à la Sims (2003). Formally, given an information structure $\{S_i^t : t \geq 0\}$, we measure the amount of information acquired by firm i up to time t as the mutual information between the history of the marginal cost shock, $\mathscr{W}_i^t \equiv \{W_{i,h} : h \leq t\}$, and the information set S_i^t. Thus, letting $\mu_{i,t}^{\mathscr{W}S}$ be the measure for the process $\{(W_{i,h}, s_{i,h}, N_{i,h}) : h \leq t\}$, and $\mu_{i,t}^{\mathscr{W}} \otimes \mu_{i,t}^{S}$ be the product measure induced by $\mu_{i,t}^{\mathscr{W}S}$, mutual information is defined by:

$$\mathbb{I}(\mu_{i,t}^{\mathscr{W}S}) \equiv \int \log\left(\frac{\mathrm{d}\mu_{i,t}^{\mathscr{W}S}}{\mathrm{d}\left(\mu_{i,t}^{\mathscr{W}} \otimes \mu_{i,t}^{S}\right)}\right) \mathrm{d}\mu_{i,t}^{\mathscr{W}S} \tag{12}$$

where the term inside the logarithm is the Radon-Nikodym derivative between the joint measure $\mu_{i,t}^{\mathscr{W}S}$ and the product measure $\mu_{i,t}^{\mathscr{W}} \otimes \mu_{i,t}^{S}$. We also define the amount of information processed in the time interval $(h, t]$ as $\mathbb{I}(\mu_{i,t}^{\mathscr{W}S}) - \mathbb{I}(\mu_{i,h}^{\mathscr{W}S})$ and let $\mathrm{d}\mathbb{I}(\mu_{i,t}^{\mathscr{W}S})$ denote the differential form of this object—i.e., the amount of information processed at the "instant" t.

As is standard in the rational inattention literature (see *e.g.,* Maćkowiak et al., 2023), in our baseline model we assume that the cost of the flow of information to the firm is linear in the information that the firm acquires, with scaling parameter $\omega > 0$. That is, the cost of the information flow is $\omega\,\mathrm{d}\mathbb{I}$.

2.4 The Firm's Problem

Putting together the firm's profit function and its information costs, we obtain that the firm's problem is to choose a pricing and information policy to maximize the expected discounted value of its profits net of its information costs. Formally, a pricing policy for the firm is a map that returns the price that the firm charges after each history at each time $\hat{p}_{i,t} : \mathscr{S}^t \to \mathbb{R}$. A pricing policy is feasible if it is constant whenever the firm does not receive a price change opportunity. The firm chooses its information policy $\mu_{\mathscr{W}S}^{i,t}$ along with a feasible pricing policy to maximize its expected discounted profits net of information costs:

$$\sup_{\{\mu_{i,t}^{\mathscr{W}S}, \hat{p}_{i,t}\}_{t\geq 0}} \mathbb{E}_0 \left[\int_0^\infty e^{-rt} \left[-\frac{B}{2}(p_{i,t} - q_{i,t})^2 \mathrm{d}t - \omega \, \mathrm{d}\mathbb{I}_t \right] \right] \tag{13}$$

2.5 Equilibrium

An *equilibrium* is a path for all endogenous variables such that the household maximizes its expected utility, the firm maximizes its profits, and all markets clear.

Definition 1 (Equilibrium). *An equilibrium is a sequence of random variables:*

$$\left\{ C_t, P_t, L_t, R_t, w_t, \{\Pi_{i,t}, P_{i,t}, C_{i,t}, L_{i,t}, Y_{i,t}\}_{i\in[0,1]} \right\}_{t\in\mathbb{R}_+} \tag{14}$$

and a collection of policy functions $(\mu_{i,t}^{\mathscr{W}\mathscr{S}}, \hat{p}_{i,t})_{i\in[0,1], t\in\mathbb{R}_+}$ *such that:*

1. *The policy functions solve Equation* (13)
2. *Production occurs according to Equation* (9)
3. *The household optimizes its expected discounted utility (Equation* (2)*) subject to its intertemporal budget constraint (Equation* (4)*) and so Equations* (5)*,* (6)*,* (7)*,* (8) *hold.*
4. *The markets for labor, goods, bonds, and money clear.*

In the following sections, we will study equilibrium firm policies and characterize the resulting implications for monetary non-neutrality.

3 Firms' Information Acquisition

We solve for firms' optimal pricing and information strategies and explore their testable implications. Optimal information policies take a striking form: only acquire information when resetting prices and *always* acquire exactly enough information to reset uncertainty about optimal prices to some fixed level, regardless of the current state of your uncertainty.

3.1 Optimal Information Acquisition

We begin by fully characterizing firms' optimal information and pricing policies. Once the information policy is pinned down, optimal pricing is simple, as firms simply set prices equal to the conditional expectation of the optimal price:

$$p_{i,t} = \mathbb{E}[q_{i,t} | S_i^t] \tag{15}$$

Toward characterizing the optimal information policy, define firm i's posterior uncertainty about its optimal reset price at time t as $U_{i,t} = \mathbb{V}[q_{i,t}|S_i^t]$. We let $U_{i,t-}$ denote the corresponding prior uncertainty about $q_{i,t}$ at time t. The following result characterizes optimal information acquisition.

Theorem 1 (Optimal Dynamic Information Policy)**.** *The firm only acquires information when it changes its price. When the firm changes its price, there exists a threshold level of uncertainty U^* such that:*

1. *If $U_{i,t-} \leq U^*$, then the firm acquires no information and $U_{i,t} = U_{i,t-}$.*
2. *If $U_{i,t-} > U^*$, then the firm acquires a Gaussian signal of its optimal price such that its posterior uncertainty is $U_{i,t} = U^*$.*

Moreover, U^* is the unique solution to:

$$\frac{1}{U^*} = \frac{B}{\omega r}\left(1 - \mathbb{E}^h[e^{-rh}]\right) + \mathbb{E}^h\left[e^{-rh}\frac{1}{U^* + \sigma^2 h}\right] \tag{16}$$

Proof. See Appendix A.1. □

We prove this result in three steps. First, we show that firms should only wish to acquire information when they change their prices. The intuition for why this is optimal comes from two observations: (i) because of discounting, acquiring information further in the future is preferable, (ii) as the firm's marginal cost moves over time, information becomes stale over time. By acquiring information only when it is used, the firm pushes information acquisition further into the future and never acquires information that becomes stale.

Second, we show that the firm should always acquire Gaussian signals when they reset their prices. Intuitively, as the firm sets $p_{i,t} = \mathbb{E}_{i,t}[q_{i,t}|S_i^t]$, the firm's expected per period loss until it resets its price is proportional to $\mathbb{V}[q_{i,t}|S_i^t]$. Thus, the firm's payoffs depend only on a sequence of conditional variances of a Gaussian random variable. Under mutual information, the cheapest way to achieve such a sequence is with a sequence of signals that maximizes entropy. The highest entropy distribution for any expected variance-covariance matrix is the Gaussian one. Thus, the firm should always acquire a Gaussian signal of its optimal reset price:

$$s_{i,t} = q_{i,t} + \hat{\sigma}_{i,t}\varepsilon_{i,t} \tag{17}$$

where $\varepsilon_{i,t}$ is an independent and identically distributed standard normal random variable and $\hat{\sigma}_{i,t}$ is an adapted sequence of signal standard deviations.

Third, we characterize the optimal noise in signals. To do this, we observe that the firm's posterior variance about optimal reset prices is a sufficient statistic for the firm's dynamic problem.

Thus, letting $U_{i,t-}$ be the firm i's prior uncertainty in period t, we have that firms solve:

$$V(U_{i,t-}) = \max_{U_{i,t} \le U_{i,t-}} -U_{i,t}\frac{B}{2}\mathbb{E}^h\left[\int_0^h e^{-r\tau}\,\mathrm{d}\tau\right] + \mathbb{E}^h\left[e^{-rh}V(U_{i,t}+\sigma^2 h)\right] + \frac{\omega}{2}\ln\left(\frac{U_{i,t}}{U_{i,t-}}\right) \tag{18}$$

The first term is the expected loss from mispricing, which is $U_{i,t} \times \frac{B}{2}$ per period, for the expected discounted duration of the pricing spell. The second term is the continuation value. If you reset your price in h periods, uncertainty at that point is your posterior uncertainty today plus the volatility of the ideal price multiplied by h. These two terms give rise to a trade-off: information today is more valuable the more likely it is that you reset your price soon because you will have better information the next time you set your price, but losses from mispricing are lower if you reset your prices sooner. The final term is simply the cost of achieving a given level of posterior uncertainty given the mutual information form of costs. These trade-offs yield the claimed first-order condition.

An important feature of the optimal solution is that the optimal level of posterior uncertainty does not depend on uncertainty when firms come to reset prices. Intuitively, having better prior information reduces the cost of obtaining better posterior information. However, under mutual information, it does not change the *marginal cost* of better information and so the optimal policy is invariant to $U_{i,t-}$.

3.2 The Economic Forces That Shape Optimal Uncertainty

We now study how changes in price stickiness, the volatility of marginal costs, and the costs and benefits of more precise information affect the optimal level of uncertainty.

Comparative Statics. The following result characterizes how the optimal reset level of uncertainty depends on various features of the underlying economic environment:

Corollary 1 (Comparative Statics for Optimal Uncertainty)**.** *The optimal level of uncertainty upon resetting the price, U^*, is:*

1. *Decreasing in the price elasticity of demand, η*
2. *Increasing in the cost of information, ω*
3. *Increasing in the volatility of marginal costs, σ^2*
4. *Increasing in the discount rate, r*

Changes in the distribution of price reset opportunities, G, in the sense of first-order stochastic dominance have an ambiguous effect on U^*.

Proof. See Appendix A.2 □

Intuitively, a greater price elasticity of demand increases the profit losses form mispricing and leads firms to acquire more precise information. Moreover, when marginal costs become more volatile, it becomes more expensive to target a given level of uncertainty and the benefits do not change. Thus, when marginal cost volatility increases, so too does optimal uncertainty. When the discount rate increases, future losses from mispricing become smaller and the value of information for future decisions is smaller. Thus, higher discount rates lead to greater uncertainty. Changes in the flexibility of prices have ambiguous impacts because of two countervailing effects. First, as price reset opportunities become more frequent, the value of information until you next reset prices is lower because you keep your price fixed based on this information for a shorter period of time. Second, when price adjustment is more frequent, information acquired today is more valuable for future price resetting opportunities because marginal costs are likely to have changed by less when you next come to reset your prices. Which of these effects dominates depends on the other parameters of the problem and the total effect of price flexibility on optimal uncertainty is ambiguous.

Special Cases and bounds on uncertainty. To illustrate these results, it is informative to consider the special limit cases in which marginal costs are infinitely volatile and marginal costs are constant over time. In these cases, we can solve for the optimal level of uncertainty in closed-form. As per our earlier comparative statics, these cases respectively also provide upper and lower bounds on firms' optimal uncertainty, respectively.

Corollary 2 (Special Cases and Bounds for Optimal Uncertainty)**.** *In the limit of infinite volatility, optimal reset uncertainty is:*

$$\lim_{\sigma^2\to\infty} U^* = \frac{\omega r}{B} \frac{1}{1-\mathbb{E}^h[e^{-rh}]} \equiv \bar{U} \tag{19}$$

In the limit of zero volatility, optimal reset uncertainty is:

$$\lim_{\sigma^2\to 0} U^* = \frac{\omega r}{B} \equiv \underline{U} \tag{20}$$

Moreover, any optimal reset uncertainty is such that $\underline{U} \le U^* \le \bar{U}$.

Proof. Immediate from Theorem 1 and Corollary 1. □

Intuitively, when marginal costs are infinitely volatile, information acquired today has no value in making future price-setting decisions because the current state of marginal costs is completely uninformative about the future state of marginal costs. In this case, as price adjustment becomes more frequent, firms' optimal uncertainty increases. Intuitively, because information today has no continuation value, the only effect of more frequent price adjustment is that losses from mispricing based on information today occur for fewer periods. This makes information today less valuable and increases the optimal level of uncertainty. As this case minimizes the continuation value of information, this case also places an upper bound on the optimal uncertainty that a firm will choose.

Conversely, when marginal costs are close to constant, information today is equally useful today as it will be when the firm resets prices. Thus, the frequency of price adjustment is irrelevant for optimal uncertainty. As this case maximizes the continuation value of information, this case places a lower bound on firms' optimal uncertainty.

3.3 Selection and Uncertainty

Our model of endogenous information acquisition implies an important property: firms that are setting prices are the least uncertain.

Corollary 3 (Uncertainty and Time Since Changing Price)**.** *Consider a firm i at time t that changed its price h periods ago. The firm's uncertainty about its optimal price follows:*

$$U_{i,t} = U^{*} + \sigma^2 h \tag{21}$$

Proof. See Appendix A.3. □

An important implication of this result is that it is not average uncertainty that is relevant for the price-setting decisions of firms, but rather the optimal reset level of uncertainty. We call this phenomenon *selection in information acquisition*: it is the price-setting firm whose uncertainty matters and, as these are the firms that most recently acquired information, they are the least uncertain firms.

Absence of Selection Under Exogenous Information and Capacity Constraints. This predicted relationship between a firm's uncertainty and the duration of its pricing spell is not present in models with exogenous information processing capacity or Gaussian signals with constant precision. In both models, the firm's beliefs follow a Kalman-Bucy filter. Thus, under either model, the firm's level of uncertainty is constant and does not depend on the time since the firm reset its price. This motivates a simple test of our model of information acquisition that we will later perform: do firms that reset their prices longer ago have greater uncertainty? An affirmative answer to this question would reject the constant capacity and exogenous information models in favor of our proposed model. Moreover, this is not a dry, theoretical point: we will shortly see how the presence of selection effects has large qualitative and quantitative implications for monetary non-neutrality.

4 Implications for Monetary Non-Neutrality

Having characterized firms' optimal dynamic information policies, we now explore the implications of endogenous information acquisition for the propagation of monetary shocks. We find that uncertainty affects the cumulative impulse response of output to a monetary shock in a surprisingly simple way: it is equal to the benchmark with perfect information plus the ratio of the uncertainty of price-setting firms to the instantaneous variance of their marginal costs. This highlights the importance of the selection mechanism: it is not average uncertainty that matters, it is the uncertainty of price setters. Thus, the effects of a monetary shock with endogenous information acquisition always lie between those with perfect information and the benchmark under exogenously given imperfect information. Finally, we show formally how data on firms' uncertainty and pricing durations are sufficient to identify the model. Moreover, we show that data on firms' price changes are insufficient to identify the model as the distribution of firm price changes is invariant to firms' uncertainty.

4.1 From Firm-Level Price Gaps to The Aggregate Output Gap

We begin by decomposing the aggregate response to shocks into firm-level responses to shocks. From the household's optimality conditions (Equations (7) and (8)), we have that aggregate output follows:

$$y_t = \frac{1}{\gamma}(m_t - p_t) \tag{22}$$

where $y_t = \log Y_t - \log Y_0$, $m_t = \log M_t - \log M_0$, and $p_t = \log P_t - \log P_0$. Following the literature on the propagation of monetary shocks (see *e.g.,* Alvarez and Lippi, 2014), we will primarily be interested in studying the cumulative impulse response (CIR) of output to a monetary shock from the steady state at time $t = 0$:

$$\mathcal{M} = \int_0^\infty y_t \,\mathrm{d}t \tag{23}$$

To compute this CIR, we can re-express the aggregate output gap as an integral of firm-level output gaps and then integrate this over time. Formally, by log-linearizing the ideal price index (Equation (6)), we have that:

$$p_t = \int_0^1 p_{i,t} \,\mathrm{d}i \tag{24}$$

Thus, we decompose the aggregate output gap as the integral of firm-level output gaps, $y_t = \int_0^1 y_{i,t} \,\mathrm{d}i$, where firm-level output gaps follow:

$$y_{i,t} = -\frac{1}{\gamma}(p_{i,t} - q_{i,t}) \tag{25}$$

Hence, to characterize the response to monetary shocks, we need only consider how firms' prices respond to the shock. To do this, we decompose firms' output gaps into two components. The first is the belief gap, $y_{i,t}^b = \frac{1}{\gamma}\left(q_{i,t} - \mathbb{E}_{i,t}[q_{i,t}]\right)$, which measures the output effects of firms' errors in pricing from having incorrect information. The second is the perceived gap, $y_{i,t}^x = \frac{1}{\gamma}\left(p_{i,t} - \mathbb{E}_{i,t}[q_{i,t}]\right)$, which arises from a firm's price not having adjusted since it receives information. For a firm that last changed its price h periods ago and that has an initial belief gap y^b, perceived gap y^x, we define the firm-level cumulative output gap as

$$Y(y^b, y^x, h) = \mathbb{E}\left[\int_0^\infty y_{i,t}\,\mathrm{d}t \mid y_{i,0}^b = y^b, y_{i,0}^x = y^x, D_{i,0} = h\right] \tag{26}$$

Following the monetary shock, we define the initial joint distribution of belief gaps and perceived gaps, and lengths of pricing spells as $\mathscr{F} \in \Delta(\mathbb{R}^3)$. Moreover, we define the respective marginal distributions as $\mathscr{F}^b$, $\mathscr{F}^x$, and $\mathscr{F}^h$. As pricing is time-dependent, the distribution of pricing durations is exogenous to any monetary shock. Thus, $\mathscr{F}^h = F$, which is the distribution of pricing spell lengths in the cross-section of firms, and y^b and y^x are independent of h. We therefore have that the CIR is given by:

$$\mathscr{M}(\mathscr{F}) = \int_{\mathbb{R}^3} Y(y^b, y^x, h)\,\mathrm{d}\mathscr{F}(y^b, y^x, h) \tag{27}$$

This reduces the question of how monetary shocks to the understanding two questions. First, how do firms' lifetime output gaps depend on their initial belief gap, initial perceived gap, and the time since they last changed their price via Y. Second, how do we aggregate firms' lifetime output gaps to compute the CIR.

4.2 Characterization of Lifetime Output Gaps

We first characterize a firm's expected lifetime output gap. To do this, we make use of the following definitions. We define the average conditional duration as $\bar{D}_h = \mathbb{E}_g^{h'}[h'|h]$, which is simply how long a firm that reset its price h periods ago expects to wait before resetting its price. By Theorem 1, we have that the Kalman gain for a firm that resets its price τ periods after last resetting its price is $\kappa_\tau = \frac{\sigma^2\tau}{U^* + \sigma^2\tau}$. We define the average conditional Kalman gain as $\bar{\kappa}_h = \mathbb{E}_g^{h'}[\kappa_{h'+h}|h]$, which is the expected Kalman gain at the next price reset opportunity for a firm that last reset its price h periods ago. With these objects in hand, the following Proposition characterizes the expected lifetime output gap of a firm

Proposition 1 (Lifetime Output Gap Characterization)**.** *The expected lifetime output gap of a* *firm with initial pricing duration h, initial belief gap y^b, and initial perceived gap y^x is given by:*

$$Y(y^b, y^x, h) = \bar{D}_h y^x + \left(\bar{D}_h + \bar{D}_0 \frac{1-\bar{\kappa}_h}{\bar{\kappa}_0}\right) y^b \tag{28}$$

Proof. See Appendix A.4 □

To understand this result, consider first the lifetime output effect of a perceived gap. Importantly, as the firm knows its perceived gap, it persists only until the firm can reset its price, at which point any perceived gap is reset to zero. Thus, as the firm on average will take $\bar{D}_h$ periods to reset its price, the lifetime effect of a perceived gap y^x is simply $\bar{D}_h y^x$.

Second, in contrast to perceived gaps, belief gaps persist forever. Initially, a belief gap operates in much the same way as a perceived gap. Until the firm next resets its price, in expectation its belief gap remains y^b and so until the first price reset a belief gap also contributes $\bar{D}_h y^b$ to the expected lifetime output gap of the firm. After this point, its behavior becomes more complicated. In particular, when a firm that reset its price h periods ago comes to reset its price in h' periods, Theorem 1 implies that it acquires a Gaussian signal of its marginal costs with a Kalman gain of $\kappa_{h+h'}$. Hence, if this firm had a belief gap of y^b at time t, it would have an expected belief gap of $\mathbb{E}_g^{h'}[1-\kappa_{h+h'}|h]y^b = (1-\bar{\kappa}_h)y^b$ at time $t+h'$. Moreover, on average, this belief gap persists for $\bar{D}_0$ periods before the firm's next price reset opportunity. Thus, between the first price reset and the second, the expected total output gap of a firm is $\bar{D}_0(1-\bar{\kappa}_h)y^b$. After this point, if a further h'' periods elapse before the firm next resets its price, its Kalman gain at that point would be $\kappa_{h''}$ and so the firm's expected output gap at the second price reset opportunity would be $\mathbb{E}_g^{h''}[1-\kappa_{h''}]\mathbb{E}_g^{h'}[1-\kappa_{h+h'}|h]y^b = (1-\bar{\kappa}_0)(1-\bar{\kappa}_h)y^b$. Thus, once again integrating over the expected duration of the third pricing spell, this period contributes $\bar{D}_0(1-\bar{\kappa}_0)(1-\bar{\kappa}_h)y^b$ to the expected lifetime output gap. The same process now happens *ad infinitum* for all future spells: the initial belief gap gets down-weighted by $1-\bar{\kappa}_0$ because of the acquisition of new information and each spell lasts $\bar{D}_0$ periods on average. Hence, the total effect of the belief gap on the lifetime output gap is given by the following geometric series:

$$\bar{D}y^b + \sum_{k=0}^{\infty} \bar{D}_0(1-\bar{\kappa}_0)^k(1-\bar{\kappa}_h)y^b = \bar{D}y^b + \bar{D}_0 y^b \frac{1-\bar{\kappa}_h}{\bar{\kappa}_0} \tag{29}$$

which collapses to the claimed expression in Proposition 1.

4.3 The Propagation of Monetary Shocks

We now characterize the propagation of monetary shocks conditional on the distribution of output gaps that they induce on impact. This is simply the integral of the expected lifetime output gaps of firms over the joint distribution of price gaps and pricing spells. As price gaps and spell duration are independent, Proposition 1 immediately implies that:

$$\mathcal{M}(\mathscr{F}) = \mathbb{E}_{\mathscr{F}}[y^x]\bar{D} + \mathbb{E}_{\mathscr{F}}[y^b]\left(\bar{D} + \bar{D}_0 \frac{1-\bar{\kappa}}{\bar{\kappa}_0}\right) \tag{30}$$

where $\bar{D} = \mathbb{E}_f^h[\bar{D}_h]$ is the average pricing duration in the population and $\bar{\kappa} = \mathbb{E}_f^h[\bar{\kappa}_h]$ is the average across all firms of the expected Kalman gain when they next reset their prices. These objects are in principle quite complicated: they are double integrals of Kalman gains and durations with respect to two different distributions—the conditional distribution of price reset opportunities G and the cross-sectional distribution of pricing spell durations F. However, Theorem 2 shows that they collapse to a simple formula in terms of only the uncertainty of price-setters U^* and the instantaneous variance of marginal costs σ^2:

Theorem 2 (CIR Characterization)**.** *Given an initial distribution $\mathscr{F} \in \Delta(\mathbb{R}^3)$, the CIR is given by:*

$$\mathcal{M}(\mathscr{F}) = \mathbb{E}_{\mathscr{F}}[y^x]\bar{D} + \mathbb{E}_{\mathscr{F}}[y^b]\left(\bar{D} + \frac{U^*}{\sigma^2}\right) \tag{31}$$

Proof. See Appendix A.5. □

This result follows from showing that the net present value of the average Kalman gain in the cross-section is given by the ratio of price-setters' uncertainty to the instantaneous variance of marginal costs. Moreover, it has two important implications: imperfect information about monetary shocks amplifies their real effects and selection effects in information acquisition dampen the importance of imperfect information.

Imperfect Information Amplifies Monetary Non-Neutrality. Theorem 2 highlights that the effects of a monetary policy shock hinge on whether monetary policy shocks are observed (thus affecting perceived gaps) or unobserved (thus affecting belief gaps). Concretely, if there is a permanent monetary expansion of amount $m = \log M_t - \log M_0$ and it is unobserved, then all firms' initial belief gaps are $y_m^b = \frac{m}{\gamma}$. We let the normalized CIR in this case be given by $\mathcal{M}^b = \mathcal{M}(\delta_0, \delta_{\frac{m}{\gamma}}, F)/\frac{m}{\gamma}$. By contrast, if the monetary shock m is observed, then $y^x = \frac{m}{\gamma}$. We let the normalized CIR in this case be given by $\mathcal{M}^x = \mathcal{M}(\delta_{\frac{m}{\gamma}}, \delta_0, F)/\frac{m}{\gamma}$. The following corollary characterizes the relative expansion of the economy under these two scenarios:

Corollary 4 (Imperfect Information Amplifies Monetary Non-Neutrality)**.** *The difference between the normalized CIRs to a permanent and unobserved monetary shock and a permanent and observed monetary shock of the same size is:*

$$\Delta^{Info} \equiv \mathcal{M}^b - \mathcal{M}^x = \frac{U^*}{\sigma^2} > 0 \tag{32}$$

Proof. Immediate from Theorem 2. □

The intuition for this result is simple: if firms are more sluggish in their adjustment of prices, then monetary policy has larger effects. Moreover, when firms have imperfect information, they are slower to adjust because they only learn about the shock over time.

Selection Dampens Monetary Non-Neutrality. Importantly, Theorem 2 shows that it is the uncertainty of price-setters alone that determines the non-neutrality of shocks and not the average uncertainty in the population. We let $\mathcal{M}^{exo}$ be the CIR of an unobserved monetary shock when firms' uncertainty is exogenously fixed at some level $\bar{U}$. The following result characterizes the importance of selection, or the fact that price-setters' uncertainty is what matters and not the average level of uncertainty in the population:

Corollary 5 (Selection Dampens Monetary Non-Neutrality)**.** *The difference between the normalized CIRs to permanent and unobserved monetary shocks under exogenous uncertainty and endogenous uncertainty is given by:*

$$\Delta^{Select} \equiv \mathcal{M}^{exo} - \mathcal{M}^{b} = \frac{\bar{U} - U^*}{\sigma^2} > 0 \tag{33}$$

Proof. Immediate from Theorem 2. □

Intuitively, as uncertainty is lowest for price-setters by Theorem 1, and greater uncertainty amplifies monetary non-neutrality it is immediate that selection effects in information acquisition dampen monetary non-neutrality relative to a benchmark model in which all firms have exogenous uncertainty equal to some level $\bar{U}$. Moreover, our characterization from Theorem 2 gives us a simple formula by which selection effects can be quantified in the data.

4.4 Identification of the Real Effects of Monetary Policy

We now show how data on firms' uncertainty about their optimal reset prices and the duration of their pricing spells are sufficient to identify the parameters of the model. Formally, let l be the density of firms' uncertainty. An implication of Theorem 1 is that the distribution of firms' uncertainty and the distribution of firms' spell lengths f are closely related:

Proposition 2 (Distribution of Uncertainty)**.** *The cross-sectional density of uncertainty about optimal reset prices $l \in \Delta(\mathbb{R}_+)$ is given by:*

$$l(z) = \begin{cases} 0, & z < U^*, \\ \frac{1}{\sigma^2} f\left(\frac{z-U^*}{\sigma^2}\right), & z \geq U^*. \end{cases} \tag{34}$$

where $f(\cdot) = \frac{1}{\bar{D}_0}(1 - G(\cdot))$ *is the density of ongoing spell lengths in the cross-section.*

Proof. See Appendix A.6 □

This result tells us that knowledge of the distribution of uncertainty l and the length of ongoing pricing spells f is sufficient to identify the uncertainty of price-setters U^*, the instantaneous variance of marginal costs σ^2, and the average expected duration of pricing spell $\bar{D}$, which in turn identify the CIR $\mathscr{M}$.

Moreover, it suggests a simple methodology by which U^* and σ^2 can be estimated from data. First, observe that the uncertainty of price-setters is given by the mode of the uncertainty distribution $U^* = \text{mode}_l[U]$. Thus, given an empirical estimate of the uncertainty distribution $\hat{l}$, we obtain the following estimator for U^*:

$$\hat{U}^* = \text{mode}_{\hat{l}}[U] \tag{35}$$

Second, for $z \geq U^*$, we have that $l(z) = \frac{1}{\sigma^2} f\left(\frac{z-U^*}{\sigma^2}\right)$. Thus, given an empirical estimate $\hat{f}$ of the distribution of ongoing spell lengths and our estimate of the uncertainty of price-setters $\hat{U}^*$, we can determine the model implied uncertainty distribution as:

$$l^M(z;\sigma^2) = \mathbb{I}[z \geq \hat{U}^*] \frac{1}{\sigma^2} \hat{f}\left(\frac{z - \hat{U}^*}{\sigma^2}\right) \tag{36}$$

which depends on a single parameter, the volatility of marginal costs σ^2. We can then therefore estimate σ^2 by minimizing the distance between $l^M(\sigma^2)$ and $\hat{l}$:

$$\hat{\sigma}^2 \in \arg\min \int_{\hat{U}^*}^{\infty} \left(\hat{l}(z) - l^M(z;\sigma^2)\right)^2 \mathrm{d}z \tag{37}$$

Data on Price Changes is Insufficient to Identify the Model. We now show that data on uncertainty is *necessary* in the sense that data on price changes and pricing durations are *insufficient* to identify the CIR in the absence of information about uncertainty. As is well known (see *e.g.*, Alvarez and Lippi, 2014), data on price changes are sufficient to identify the CIR in many models with both state-dependent pricing and time-dependent pricing frictions. Moreover, as data on the uncertainty of firms are not typically available in commonly used datasets of price changes it is natural to ask if data on price changes (potentially alongside data on pricing durations) are sufficient to identify the CIR in the presence of endogenous information acquisition. The following result presents a characterization of the distribution of price changes that answers this question in the negative:

Proposition 3 (Distribution of Price Changes)**.** *The distribution of price changes conditional on a firm changing its price $H \in \Delta(\mathbb{R})$ is invariant to U^* and follows:*

$$H(\Delta p) = \int_0^\infty \Phi\left(\frac{\Delta p}{\sigma\sqrt{h}}\right) \mathrm{d}G(h) \tag{38}$$

where Φ is the standard normal CDF.

Proof. See Appendix A.7. □

We prove this result by first deriving the conditional distribution of price changes conditional on a firm's last pricing spell lasting h periods and conditional on a firm's information set at the beginning of its last pricing spell S_i^{t-h}. We show that the conditional variance of such price changes is invariant to the information set of the firm. Intuitively, the nature of the firm's optimal information acquisition makes its price change independent of the prices that it previously charged. Moreover, from the form of the firm's optimal information policy derived in Theorem 1, the conditional variance of price changes depends only on the volatility of marginal costs σ and the length of the pricing spell h and is given by $\sigma^2 h$. By mixing this distribution over the distribution of pricing durations, we obtain the distribution of price changes.

The important upshot of this result is that data on price changes, even in conjunction with data on pricing durations, are insufficient to identify U^* and, therefore, the real effects of monetary policy when there is endogenous information acquisition. Thus, data on uncertainty are not only sufficient for identifying U^*, they are also necessary.

5 Using Survey Data to Quantify and Test the Model

We have shown how to identify the effects of uncertainty on the real effects of monetary shocks given information about firms' uncertainty and the volatility of their marginal costs. We now show how to use survey micro data on firms' uncertainty and the duration of their pricing spells to identify and estimate these parameters. Using a survey of New Zealand firms from Coibion et al. (2018), we perform this estimation. Moreover, we test the core prediction of our model with endogenous information acquisition—that firms that more recently reset their prices are less uncertain about optimal prices—and find strong empirical support in the data. We find that the effect of uncertainty is of comparable magnitude to the effects of price stickiness itself and that the effect of selection is of a comparable magnitude. From this, we conclude that uncertainty is critical for understanding the real effects of monetary policy and that understanding selection effects in information acquisition are equally important.

5.1 Survey Data on Firms' Uncertainty and Pricing Duration

Motivated by our identification results, we need data on firms' uncertainty about their optimal reset prices and how long ago they last reset their price. To obtain these data, we use the survey of firm managers in New Zealand described in Coibion et al. (2021), implemented between 2017Q4 and 2018Q2. The survey included 515 firms with six or more employees. These firms were a random sample of firms in New Zealand with broad sectoral coverage.[2]

These data contain two questions that allow us to measure the key objects of interest. First, firms are asked about their subjective uncertainty about their ideal prices:

> Q1: *If your firm was free to change its price (i.e. suppose there was no cost to renegotiating contracts with clients, no costs of reprinting catalogues, etc...) today, what probability would you assign to each of the following categories of possible price changes the firm would make? Please provide a percentage answer.*[3]

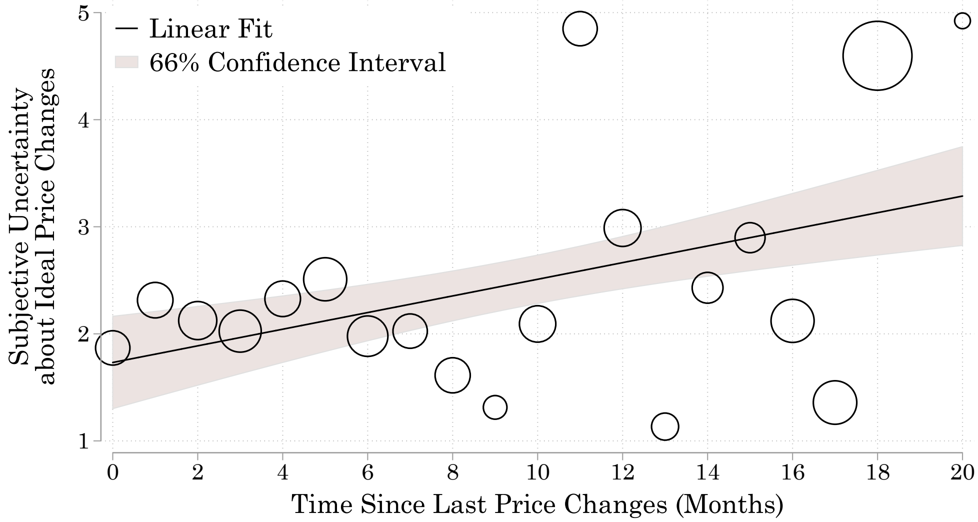
As the survey was conducted by phone, firms' answers are consistent in that they feature no probabilities below zero and all probabilities sum to one. To compute an estimate of the firm's uncertainty, we first compute an estimate of the firm's expectation of its optimal price by taking the midpoint of each bin and computing its expected value under the probabilities the firm manager provides. Then, we construct an estimate of the firm's uncertainty by computing the variance under the elicited probability distribution.[4] This gives us a measure U_i of firm i's uncertainty about its optimal reset price for each of the firms in our sample.

Second, firms are asked the time that has elapsed since they last changed their price:

> Q2: *When did your firm last change its price (in months) and by how much (in % change)?*

This straightforwardly gives us a measure D_i of the duration of firm i's pricing spell.

Figure 1: Firms That Recently Changed Their Prices Are Less Uncertain



Notes: This figure plots the time elapsed since firms' last price changes versus firms' subjective uncertainty about their ideal price changes. The black line is a linear fitted line and the shaded area is 66% confidence interval. We drop the outliers with the implied subjective uncertainty greater than 20. The size of bins represents the average size of employment of firms in each percentile.

5.2 Testing for Endogeneity of Information Acquisition in the Data

First, we can use the survey data to test the key mechanism from Theorem 1 that generates selection: firms that reset their price longer ago have greater uncertainty.

To this end, Figure 1 plots the binned scatterplot relationship between time since firms' last price changes and subjective uncertainty about firms' ideal price changes. We find an upward-sloping relationship, consistent with our hypothesis and theoretical prediction from Corollary 3 that price-setters are the least uncertain.

We can also test our hypothesis in more demanding specifications that control for industry fixed effects, firm-level controls (firms' log age, employment, foreign trade share, number of competitors, the slope of the profit functions, firms' expected size of price changes in 3 months,

[2] Previous works have used the survey data to characterize how firms form their expectations. For example, Afrouzi (2023) shows that strategic complementarity decreases with competition and reports that firms with more competitors have more certain posteriors about aggregate inflation. Also, Coibion et al. (2021) evaluate the relation between first-order and higher-order expectations of firms, including how they adjust their beliefs in response to a variety of information treatments. Yang (2022) shows that firms producing more goods have both better information about inflation and more frequent but smaller price changes. See Coibion et al. (2021) for a comprehensive description of the survey.

[3] Firms assigned probabilities to the following 16 bins: less than -25%, from -25% to -15%, from -15% to -10%, from -10% to -8%, from -8% to -6%, from -6% to -4%, from -4% to -2%, from -2% to 0%, from 0% to 2%, from 2% to 4%, from 4% to 6%, from 6% to 8%, from 8% to 10%, from 10% to 15%, from 15% to 25%, more than 25%.

[4] When we calculate the variance, we assume a uniform distribution within each bin. For example, if a firm assigns 100% on the bin "2-4 percent", then the implied variance is $\frac{1}{12}(4-2)^2 = 1/3$.

Table 1: The Relationship Between Uncertainty and Time Since Changing Price

	(1)	(2)	(3)	(4)
Dependent variable: Subjective uncertainty about firms' ideal price changes				
Dummy for price changes	-0.488***	-0.577***	-0.647***	-0.643***
in the last 12 months	(0.137)	(0.141)	(0.148)	(0.151)
Observations	469	469	465	467
R-squared	0.0341	0.103	0.151	0.153
Industry Controls		Yes	Yes	Yes
Firm-level Controls			Yes	Yes
Manager Controls				Yes

Notes: This table reports results for the Huber robust regression of Equation (39). The dependent variable is the subjective uncertainty about firms' ideal price changes in the 2018Q1 survey, which is measured by the variance implied by each firm's reported probability distribution over different outcomes of their ideal price changes if firms are free to change their prices. Industry fixed effects include dummies for 13 sub-industries. Firm-level controls include a log of firms' age, a log of firms' employment, foreign trade share, number of competitors, the slope of the profit function, firms' expected size of price changes in 3 months, and firms' subjective uncertainty about their ideal prices in next three months reported in the 2017Q4 survey. Manager controls include the age, education, and tenure at the firm of the respondent (each firm's manager). Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit Australian and New Zealand Standard Industrial Classification level) are reported in parentheses. *** denotes statistical significance at 1% level.

and firms' subjective uncertainty about their ideal prices in 3 months), and manager-level controls (age, education, and tenure at the firm) by estimating the following regression equation:

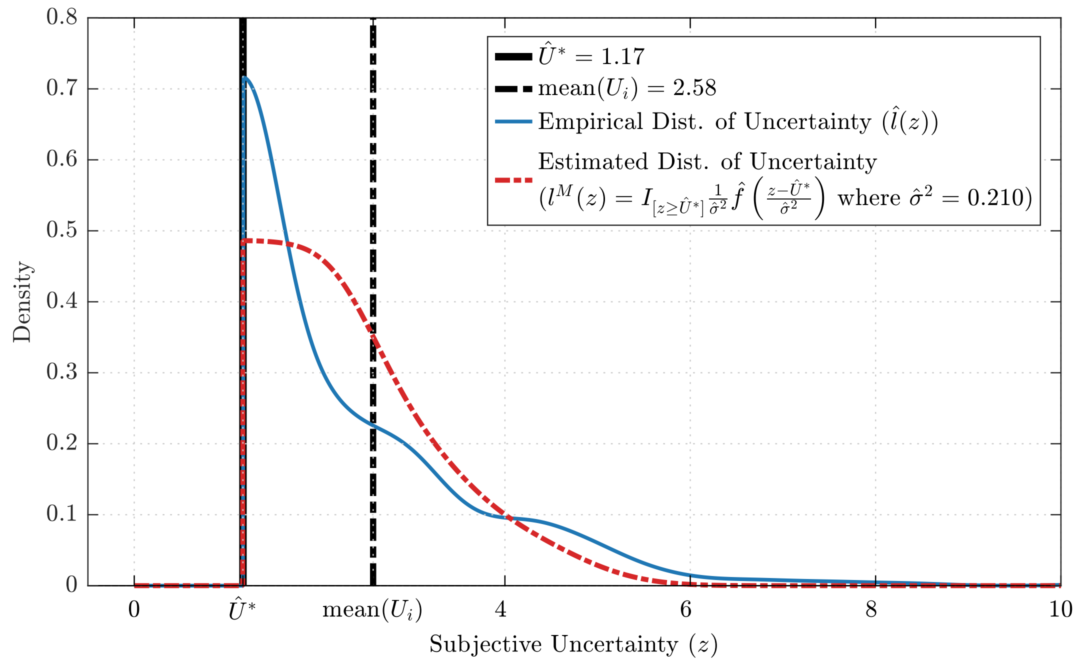
$$U_i = \beta \times \mathbb{1}_i^{(\Delta p)} + \gamma_{j(i)} + X_i^{f'}\delta + X_i^{m'}\lambda + \varepsilon_i \tag{39}$$

where U_i is firm i's subjective uncertainty about their optimal price, $\mathbb{1}_i^{(\Delta p)}$ is a dummy variable equal to one if the firm reset its price in the last 12 months and zero otherwise, $\gamma_{j(i)}$ is an industry fixed effect where $j(i)$ is the industry of firm i, X_i^f is the previously mentioned vector of firm-level controls, X_i^m is the previously mentioned vector of manager-level controls, and ε_i is an error term.

We present the results from estimating this relationship by ordinary least squares in Table 1. We find a strong and highly statistically significant negative relationship between having changed prices in the last year and subjective uncertainty about optimal prices. This relationship strengthens when industry fixed effects and firm and manager controls are added. The result is also quantitatively significant, firms that changed prices in the last year have uncertainty that is 0.2 standard deviations lower than the firms that did not.

This result not only provides evidence for the relevance of selection in information acquisition, but it also rejects models with exogenous information and endogenous information ac-

Figure 2: Distributions of Firms' Subjective Uncertainty in the Data and the Model



Notes: This figure shows the distribution of firms' subjective uncertainty about their ideal prices. The black vertical solid line shows the mode of the empirical distribution of subjective uncertainty ($\hat{U}^*$) and the black vertical dashed line shows the mean of the subjective uncertainty observed in the survey data. The blue solid line is the empirical distribution of uncertainty $\hat{l}(z)$. The red dashed line shows the estimated distribution of uncertainty ($l^M(z)$) from Equation (36) using the empirical distribution of time since the last price changes ($\hat{f}$) and the estimated uncertainty of shocks ($\hat{\sigma}^2$)

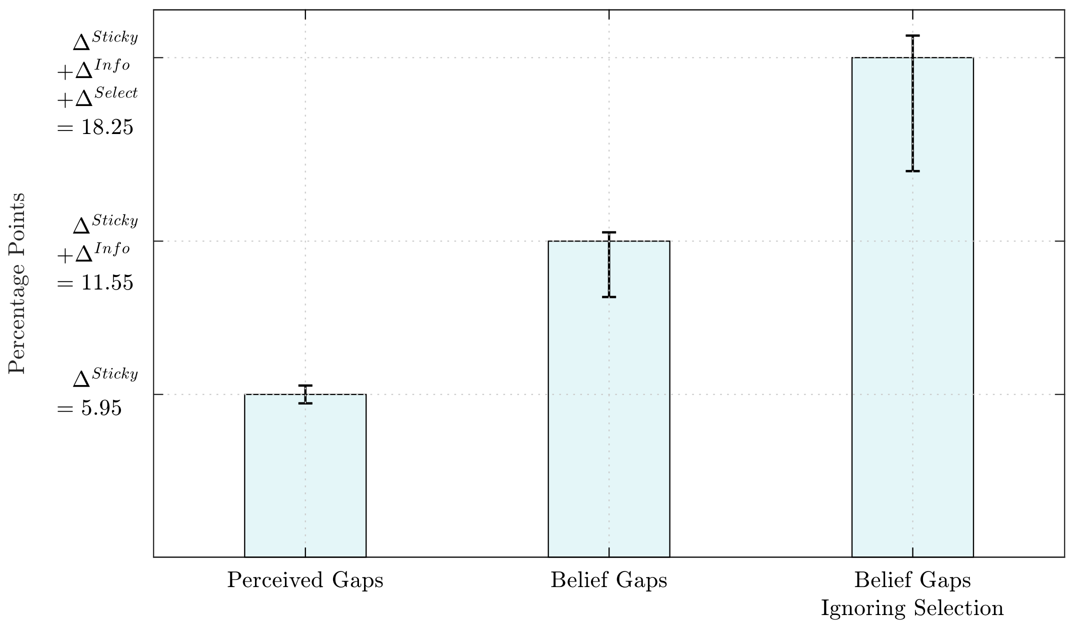
quisition subject to a capacity constraint. As many seminal papers regarding business cycles with information frictions (see *e.g.,* Maćkowiak and Wiederholt, 2009; Angeletos and La'o, 2010) use exogenous information or capacity-constraints to generate uncertainty, this result suggests that revisiting these analyses with information costs may better fit the data. At the same time, as we have shown here, the selection in information acquisition that costs generate may have important qualitative and quantitative macroeconomic implications beyond the specific model that we have studied.

5.3 The Quantitative Impact of Uncertainty and Selection

We now use these data to quantify the importance of both uncertainty and selection for monetary non-neutrality. We first estimate the density of pricing durations and uncertainty using standard kernel density methods to obtain $\hat{f}$ and $\hat{l}$.[5] We then obtain $\hat{U}^*$ and $\hat{\sigma}^2$ using our estimators from Equations (35) and (37). For all estimated objects, we construct standard errors

[5]We estimate $\hat{l}$ using a standard kernel density function with a bandwidth of 0.5 on $[0,50]$. We then obtain $\hat{U}^*$ as the mode of $\hat{l}$ and reestimate the kernel density on $[\hat{U}^*, 50]$. We estimate $\hat{f}$ with a bandwidth of 2.4 on $[0,40]$.

Figure 3: Estimated Monthly Cumulative Impulse Responses to an Initial 1 Percentage Point Output Gap under Different Scenarios



Notes: This figure shows the output effects of a 1 percentage point shock to perceived gaps (left bar), to belief gaps (middle bar), and to belief gaps ignoring the selection effect (right bar). The output effect of a 1pp perceived gap is the average duration of firms' pricing spells $\Delta^{Sticky} = \bar{D}$, the effect of a 1pp belief gap is the effect of a perceived gap plus $\Delta^{Info} = \frac{U^*}{\sigma^2}$, and the effect of 1pp belief gap without selection effect is $\Delta^{Stikcy} + \Delta^{Info}$ plus $\Delta^{Select} = \frac{\bar{U}-U^*}{\sigma^2}$. We present 95% confidence intervals as black vertical lines.

using the bootstrap.[6] From this exercise, we obtain that $\hat{U}^* = 1.17$ (S.E.: 0.02) and $\hat{\sigma}^2 = 0.21$ (S.E.: 0.03). In Figure 2, we plot the estimated uncertainty distribution (in red) alongside the empirical uncertainty distribution (in blue). The fit, while not perfect, is surprisingly good given we only have one degree of freedom (the volatility of marginal costs σ^2) to match the entire distribution. In Appendix Figure 1, we plot the estimated conditional durations of pricing spells $\bar{D}_h$ as well as the estimated conditional Kalman gains $\bar{\kappa})h$ that these estimates imply. In Appendix Figure 2, we plot the estimated distribution of price reset opportunities G and the corresponding hazard function θ, which is increasing in the duration of the pricing spell.

Using Theorem 2, we now estimate the extent to which uncertainty affects monetary non-neutrality as well as the extent to which selection effects in information acquisition matter. Figure 3 shows the monthly CIR of a 1 percentage point (pp) shock to output gaps under different scenarios (*i.e.,* to obtain the annual CIRs simply divide the following numbers by 12). First, we recall as a baseline that the output effect of a 1pp perceived gap is simply the average duration

[6]Formally, for $d = 1, \ldots, 10,000$, we uniformly resample $N = 515$ data points which are the same with the number of observations in the survey data. We re-estimate $\hat{f}_d$ and $\hat{l}_d$ using these data. We then re-estimate any model-implied quantity under these distributions and compute the distribution of the resulting estimates over the 10,000 bootstrap samples. We then compute the standard error as the standard deviation of the resulting distribution.

of firms' pricing spells $\Delta^{Sticky} = \bar{D}$, which we estimate to be 5.95pp (S.E.: 0.17). The effect of a 1pp belief gap is the effect of a perceived gap plus $\Delta^{Info} = \frac{U^*}{\sigma^2}$, which we estimate to be 5.60pp (S.E.: 0.59). Thus, accounting for uncertainty is approximately as important for monetary non-neutrality as accounting for the mechanical effects of price stickiness. We also estimate the importance of selection $\Delta^{Select} = \frac{\bar{U}-U^*}{\sigma^2}$, which is the error in what we would have estimated Δ^{Info} to be if we naively used firms' average uncertainty rather than the uncertainty of price-setters, which we find to be 6.71pp (S.E.: 0.80). Thus, explicitly accounting for uncertainty is about as important as accounting for price stickiness itself. Moreover, accounting for selection is slightly more important than accounting for price stickiness itself. Indeed, computing the effects of shocks ignoring selection would massively overstate the non-neutrality of monetary shocks.

***Ex Ante* Heterogeneity.** We have assumed in our analysis that all firms are *ex ante* identical and differ only because they experience different productivity shocks and pricing spells. Of course, firms may be heterogeneous in several respects and this could matter for the propagation of monetary shocks. However, Theorem 2 tells us how heterogeneity can matter in very precise ways. In particular, if we augment the model to allow for arbitrary cross-firm heterogeneity in pricing durations G_i, the costs of mispricing B_i, the costs of information acquisition ω_i, and the volatility of marginal costs σ_i, we have that the CIR to a belief shock is given by:

$$\mathcal{M}^b = \mathbb{E}[\bar{D}_i] + \mathbb{E}\left[\frac{U_i^*}{\sigma_i^2}\right] \tag{40}$$

where $\bar{D}_i$ is the average expected duration implied by G_i and U_i^* is the posterior uncertainty of price setter i. Moreover, as $\mathbb{E}[\bar{D}_i] = \bar{D}$, heterogeneity does not matter for the mechanical term coming from price stickiness. Heterogeneity therefore matters precisely insofar as there is heterogeneity in $\frac{U_i^*}{\sigma_i^2}$. Moreover, by allowing for unrestricted heterogeneity in pricing hazards across firms, this formula holds under many recently developed extensions of the simple Calvo model, such as the mixed proportional hazard model proposed by Alvarez, Borovičková, and Shimer (2021).

To gauge the potential importance of such heterogeneity, we re-estimate U_i^* and σ_i^2 across different sectors, which are potentially quite likely to differ along each of the possible margins highlighted above. We present the results of this analysis in Table 2. We find estimates of U^* that are very similar across sectors, ranging between 1.1 and 1.2, while finding more substantial heterogeneity in the instantaneous variance of marginal costs, ranging between 0.16 and 0.30. Weighting each sector by its GDP contribution, we find that $\Delta^{Info} = \hat{\mathbb{E}}\left[\frac{\hat{U}_i^*}{\hat{\sigma}_i{}^2}\right] = 4.98$ (S.E.: 0.31), which is close to our baseline estimate of 5.60 without sectoral heterogeneity. More advanced

Table 2: Estimates of Sectoral Heterogeneity in Uncertainty and Marginal Cost Volatility

	GDP Share	Obs.	$\hat{U}^*$	$\hat{\sigma}^2$
Manufacturing and Construction	0.284	195	1.209	0.161
Trade, Transportation, Accommodation, and Food Services	0.290	150	1.107	0.302
FIRE and Professional Services	0.426	170	1.090	0.241
GDP-Weighted Average of Three Sectors	1	515	1.129	0.236
All sector (Baseline)	1	515	1.173	0.210

Notes: This table shows the estimation results of $\hat{U}^*$ and $\hat{\sigma}^2$ for three groups of sectors. We also present the GDP-weighted average of these estimates as well as the baseline estimates with all sectors. GDP share is computed using the 2018 New Zealand GDP by sectors. FIRE stands for Financial Activities, Information, and Real Estate services sectors.

modeling of heterogeneous pricing hazards across firms, such as that performed by Alvarez et al. (2021), would require panel data to which we do not have access from this survey. Extending the analysis to account for heterogeneity of this sort is an interesting avenue for future work.

6 Counterfactuals: How Microeconomic Uncertainty and Price Stickiness Affect Monetary Non-Neutrality

In a final quantitative analysis, we study how changes in microeconomic uncertainty and price stickiness affect the degree of monetary non-neutrality. While our theoretical results show that such changes have potentially ambiguous effects, we can leverage our empirical estimates to both sign and quantify the extent to which greater microeconomic uncertainty and price stickiness would affect the efficacy of monetary policy. We find that elevated microeconomic uncertainty dampens the real effects of monetary policy while increased price stickiness increases the real effects of monetary policy, but by 20% less than would be the case in a model without endogenous information acquisition.

6.1 Microeconomic Volatility

We first use the model and data to ask how changes in microeconomic volatility matter for the propagation of monetary shocks. As evidence from Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018) shows that microeconomic volatility is significantly higher in recessions, this allows us to gauge the implications of this fact, through the lens of our model, for the relative efficacy of monetary policy in booms versus recessions. By Theorem 2, we have that the effect of microeconomic volatility on the CIR is given by:

$$\frac{\partial \mathcal{M}^b}{\partial \sigma^2} = \frac{\frac{\partial U^*}{\partial \sigma^2} - \frac{U^*}{\sigma^2}}{\sigma^2}$$

The theoretical sign of this effect is ambiguous. First, there is a direct effect of increasing the volatility of firms' marginal costs. This makes them pay attention less to their priors as they know that their past information is less accurate. This means that firms pay more attention to their information and dampens the real effects of monetary shocks. Second, there is an indirect effect on firms' optimal information choice. By Theorem 1, we have that the effect of a change in microeconomic volatility on firms optimal posterior uncertainty is given by:

$$\frac{\partial U^*}{\partial \sigma^2} = \frac{\mathbb{E}^h\left[e^{-rh}\frac{h}{\left(U^{*2}+\sigma^2 h\right)^2}\right]}{\frac{1}{U^{*2}} - \mathbb{E}^h\left[e^{-rh}\frac{1}{\left(U^{*2}+\sigma^2 h\right)^2}\right]} \tag{41}$$

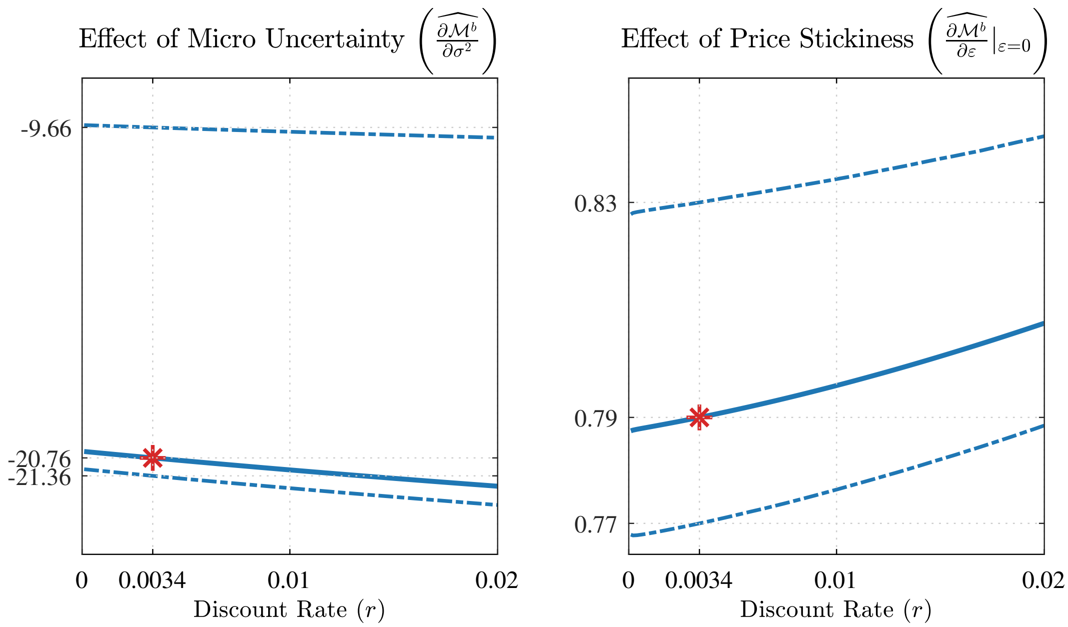
which is always positive. Intuitively, greater volatility of marginal costs makes information gathered today less valuable when the firm makes future decisions as marginal costs will have moved by more when those future decisions are made. As a result, when there is greater volatility, the firm acquires less information and becomes more uncertain. This makes firms pay more attention to their priors and less to their current information. As a result, the indirect effect goes in the opposite direction to the direct effect and amplifies the real effects of monetary policy.

Despite this theoretical ambiguity, we can estimate both the sign and magnitude of $\frac{\partial \mathcal{M}^b}{\partial \sigma^2}$ by using the structure of our model and our estimates of firms' pricing durations $\hat{g}$, optimal uncertainty $\hat{U}^*$, and microeconomic volatility $\hat{\sigma}^2$. Together, this information pins down the effects of microeconomic volatility on the CIR up to a single parameter, the discount rate of firms r:

$$\widehat{\frac{\partial \mathcal{M}^b}{\partial \sigma^2}}(r) = \frac{1}{\hat{\sigma}^2}\left(\frac{\mathbb{E}^h_{\hat{g}}\left[e^{-rh}\frac{h}{\left(\hat{U}^{*2}+\hat{\sigma}^2 h\right)^2}\right]}{\frac{1}{\hat{U}^{*2}} - \mathbb{E}^h_{\hat{g}}\left[e^{-rh}\frac{1}{\left(\hat{U}^{*2}+\hat{\sigma}^2 h\right)^2}\right]} - \frac{\hat{U}^*}{\hat{\sigma}^2}\right) \tag{42}$$

We can then assess the likely effects of local changes in microeconomic volatility for any possible discount rate by plotting $\widehat{\frac{\partial \mathcal{M}^b}{\partial \sigma^2}}(r)$ as a function of r. We plot the results of this exercise as we vary the monthly discount rate from $r = 0$ to $r = 0.02$ (equivalent to an annual discount factor of approximately 0.8) in Figure 4. For all such discount rates, we find strong statistical evidence that $\frac{\partial \mathcal{M}^b}{\partial \sigma^2}$ is negative, largely insensitive to the exact chosen value for the discount rate–with a point estimate of approximately −21 over the entire range of discount rates. As a result, the di-

Figure 4: Microeconomic Volatility, Price Stickiness, and Monetary Non-Neutrality



Notes: This figure shows two counterfactual analyses on how micro uncertainty and price stickiness affect monetary non-neutrality. The left panel shows the effect of microeconomic uncertainty on monetary non-neutrality induced by information friction, $\widehat{\partial \mathcal{M}^b / \partial \sigma^2}$ in Equation (42), as a function of the discount rate (r). The right panel shows the effect of price stickiness on monetary non-neutrality, $\widehat{\partial \mathcal{M}^b / \partial \epsilon}|_{\epsilon=0}$ in Equation (45), as a function of the discount rate (r). Red stars show the estimates with the baseline discount rate $r = 0.0034$, which implies $\beta = \frac{1}{1+r} = 0.96^{(1/12)}$. We present 95% confidence intervals as blue dashed lines.

rect effects, which are empirically equal to $-\frac{\hat{U}^*}{\hat{\sigma}^4} \approx -27$, dominate the indirect effects by a factor of approximately 4-5.

Thus, we find that microeconomic volatility significantly dampens the real effects of monetary policy. This point has been made in the context of models of lumpy adjustment (see *e.g.,* Vavra, 2014), because firms are more likely to adjust prices when marginal costs are more volatile. However, the mechanism that underlies this result in our model is entirely different and instead follows because firms pay less attention to prior information when marginal costs are more volatile and are therefore more responsive to current information and monetary shocks. Moreover, the current literature on monetary non-neutrality with information frictions (see *e.g.,* Lucas, 1972; Afrouzi and Yang, 2021) largely emphasizes the role of macroeconomic uncertainty for monetary non-neutrality, while this result emphasizes the importance of microeconomic uncertainty (as recently studied by Flynn, Nikolakoudis, and Sastry, 2023).

6.2 Price Stickiness

We now use the model to analyze how changes in price stickiness affect monetary non-neutrality. Suppose that the distribution of price reset opportunities changes and a firm that would have reset its price at time h now resets its price at time $h+\varepsilon$ for some $\varepsilon > 0$. More formally, the distribution of price reset times changes from G to $\tilde{G}$, where $\tilde{G}(x) = G(x-\varepsilon)$ for all $x \geq \varepsilon$. Theorem 2 then implies that the effects on monetary non-neutrality of a small such increase in price stickiness are given by:

$$\frac{\partial \mathcal{M}^b}{\partial \varepsilon}\Big|_{\varepsilon=0} = 1 + \frac{\frac{\partial U^*}{\partial \varepsilon}|_{\varepsilon=0}}{\sigma^2} \tag{43}$$

where the first term is the direct effect of an increase in stickiness, which increases average excepted durations one-for-one. The second term is the indirect effect, which comes from how price stickiness affects the optimal level of uncertainty. Theorem 1 implies that this indirect effect is given by:[7]

$$\frac{\partial U^*}{\partial \varepsilon}\Big|_{\varepsilon=0} = \frac{r\left(\mathbb{E}^h\left[\frac{1}{U^*+\sigma^2 h}\right] - \frac{B}{\omega r}\mathbb{E}^h[e^{-rh}]\right) + \sigma^2 \mathbb{E}^h\left[e^{-rh}\frac{1}{(U^*+\sigma^2 h)^2}\right]}{\frac{1}{U^{*2}} - \mathbb{E}^h\left[e^{-rh}\frac{1}{(U^*+\sigma^2 h)^2}\right]} \tag{44}$$

As discussed earlier, this has a theoretically ambiguous sign because of two countervailing effects. First, longer pricing durations make information more valuable for the current pricing spell because you are keeping your price fixed for longer. This encourages a lower level of optimal uncertainty. Second, longer pricing durations make information less valuable for all future pricing spells because today's information is less valuable further into the future. This encourages a higher level of optimal uncertainty.

Once again, despite this theoretical ambiguity, we can estimate both the sign and magnitude of $\frac{\partial \mathcal{M}^b}{\partial \varepsilon}|_{\varepsilon=0}$ using our data, up to calibrating the firms' discount rate:

$$\widehat{\frac{\partial \mathcal{M}^b}{\partial \varepsilon}}\Big|_{\varepsilon=0}(r) = 1 + \frac{\frac{r\mathbb{E}^h_{\hat{g}}\left[\frac{1}{\hat{U}^*+\hat{\sigma}^2 h}\right] - \widehat{\left(\frac{B}{\omega}\right)}(r)\mathbb{E}^h_{\hat{g}}[e^{-rh}] + \hat{\sigma}^2\mathbb{E}^h_{\hat{g}}\left[e^{-rh}\frac{1}{(\hat{U}^*+\hat{\sigma}^2 h)^2}\right]}{\frac{1}{\hat{U}^{*2}} - \mathbb{E}^h_{\hat{g}}\left[e^{-rh}\frac{1}{(\hat{U}^*+\hat{\sigma}^2 h)^2}\right]}}{\hat{\sigma}^2} \tag{45}$$

[7] To see this, observe that:

$$\frac{1}{U^*} = \frac{B}{\omega r}\left(1 - \int_\varepsilon^\infty e^{-rh} g(h-\varepsilon)\,\mathrm{d}h\right) + \int_\varepsilon^\infty e^{-rh}\frac{1}{U^*+\sigma^2 h} g(h-\varepsilon)\,\mathrm{d}h$$

Differentiating both sides, we obtain that:

$$-\frac{1}{U^{*2}}\frac{\partial U^*}{\partial \varepsilon}\Big|_{\varepsilon=0} = \frac{B}{\omega}\mathbb{E}^h[e^{-rh}] - r\mathbb{E}^h\left[\frac{1}{U^*+\sigma^2 h}\right] - \mathbb{E}^h\left[\left(\frac{\partial U^*}{\partial \varepsilon}\Big|_{\varepsilon=0} + \sigma^2\right)e^{-rh}\frac{1}{(U^*+\sigma^2 h)^2}\right]$$

Rearranging this expression yields Equation (44).

where we estimate the ratio of the losses from mispricing parameter B to the information cost parameter ω, $\widehat{\left(\frac{B}{\omega}\right)}$, by finding the value of $\frac{B}{\omega}$ that rationalizes the U^* we see in the data. That is, we find the exact value of $\frac{B}{\omega}$ that solves the firm's first-order condition for the optimal choice of U^* given the U^*, σ^2 and g that we see in the data and any fixed value for r:

$$\widehat{\left(\frac{B}{\omega}\right)}(r) = \frac{r}{1 - \mathbb{E}^h_{\hat{g}}[e^{-rh}]}\left(\frac{1}{\hat{U}^*} - \mathbb{E}^h_{\hat{g}}\left[e^{-rh}\frac{1}{\hat{U}^* + \hat{\sigma}^2 h}\right]\right) \tag{46}$$

We again plot the results of this exercise as we vary r from 0 to 0.02 (*i.e.,* the annual discount factor ranges from 1 to 0.8) in Figure 4. We find that the indirect effect is negative, *i.e.,* when stickiness increases, firms respond by reducing their uncertainty. Quantitatively, this offsets approximately 20% of the increase in monetary non-neutrality that stickiness would induce in a model without endogenous information acquisition. Thus, we find that information acquisition dampens the effects of changes in price stickiness on monetary non-neutrality.

7 Conclusion

In this paper, we study how to use firms' measured beliefs to quantify the degree of monetary non-neutrality in a general equilibrium model with nominal rigidities and endogenous information acquisition. We showed that the combination of these two ingredients leads to selection in information acquisition: the price-setting firms are the most informed in the cross-section at any given time and it is their beliefs that ultimately determine the degree of monetary non-neutrality. Implementing our approach in a survey of firms' beliefs in New Zealand, we estimate that endogenous information acquisition doubles the degree of monetary non-neutrality relative to the benchmark model with no information costs. Finally, we showed that data on beliefs is not only sufficient to identify the real effect of monetary policy but also necessary: commonly used data on the distribution of price changes are insufficient for identification in the presence of endogenous information acquisition.

More broadly, our framework has implications for how measured beliefs (*e.g.,* from surveys) can be used to uncover the macroeconomic impacts of imperfect and endogenous information. This is useful because it is *ex ante* unclear whose beliefs, and which aspects of those beliefs, matter for any given outcome. For instance, within a standard general model of price-setting with endogenous information acquisition, we showed that the relevant moment of beliefs for monetary non-neutrality is *price-setters' uncertainty* about their optimal prices. This highlights how, fixing an outcome of interest, one can use theory to narrow down whose beliefs to measure, what aspects of these beliefs to measure, and how to use these measured beliefs to understand macroeconomic phenomena at both quantitative and qualitative levels. Interestingly, in our case, our results imply that the ideal survey would use a *selected sample* of price-setters—as opposed to a representative sample of *all* firms, which is usually the targeted pool for firm surveys—and measure their uncertainty about their desired prices. We believe this implication should also hold in some form for settings where economic agents make infrequent decisions, such as households buying houses or other durable goods or firms making lumpy investment decisions. In all such settings, agents might prefer to acquire information when the decision is relevant and so averages of uncertainty from representative samples might exaggerate the degree of information rigidities that are relevant for macroeconomic outcomes.

Our analysis also highlights several questions for future research. Our model shows how a given and exogenous process for arrival of price adjustments affects the dynamic information acquisition policy of firms. Nonetheless, the process for adjustment of prices can itself be affected by the information acquisition policies of firms. While we abstracted away from this feedback in this paper to focus on how the arrival process affects incentives for acquiring information over time, studying this feedback effect is an open question for future research, which can be achieved by extending our formulation of nominal rigidities by including menu costs for changing prices. In this regard, previous work by Alvarez et al. (2011, 2017) shows how such interactions work in models where agents can pay a fixed cost and update their information set to that of a fully informed agent. However, our analysis shows that when updating to such information sets are not cost effective, these interactions could take more complicated forms as the histories of previous beliefs now matter by forming the agents' priors. While these models are analytically complex to solve, we think that our main model mechanism would still operate in a model with state-dependent pricing frictions. Indeed, previous work on menu cost models with flexible information costs demonstrate that firms do acquire additional information when they change their prices (Gorodnichenko, 2008; Yang, 2022). Thus, how state-dependent pricing frictions affect the implications of uncertainty for monetary non-neutrality is a quantitative question that we leave to future research.

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Appendices

A Proofs

A.1 Proof of Theorem 1

Proof. We first characterize optimal pricing conditional on an arbitrary information policy $\mu^{i,t}_{\mathscr{W}S}$. Let $\nu_{i,t}$ be the firm's belief regarding W_{it} at time t. Suppose that the firm has received a pricing opportunity at some date t. The firm's price policy problem is given by:

$$J(\nu_{i,t}) = \sup_p \mathbb{E}^h \left[\int_0^h e^{-r\tau} \left[-\frac{B}{2}(p - q_{i,\tau})^2 \right] \mathrm{d}\tau + e^{-rh} J(\nu_{i,t+h}) \mid \nu_{it} \right] \tag{47}$$

Thus, any optimal price solves:

$$p_{i,t}(\nu_{i,t}) \mathbb{E}^h \left[\int_0^h e^{-r\tau} \,\mathrm{d}\tau \right] = \mathbb{E}^h \left[\int_0^h e^{-r\tau} \mathbb{E}[q_{i,\tau} \mid \nu_{i,t}] \,\mathrm{d}\tau \right] \tag{48}$$

Using the fact that $\mathbb{E}[q_{i,\tau} \mid \nu_{i,t}] = \bar{q} + \sigma \mathbb{E}[W_{i,t} \mid \nu_{i,t}]$, we obtain:

$$p_{it}(\nu_{it}) = \bar{q} + \sigma \mathbb{E}[W_{i,t} \mid \nu_{i,t}] \tag{49}$$

We can therefore compute the value function $J(\nu_{it})$ as:

$$J(\nu_{i,t}) = \mathbb{E}^h \left[\int_0^h e^{-r\tau} \left[-\frac{B}{2} \sigma^2 \mathbb{V}[W_{i,\tau} \mid \nu_{i,t}] \right] \mathrm{d}\tau \right] + \mathbb{E}\left[e^{-rh} J(\nu_{i,t+h}) \mid \nu_{i,t} \right] \tag{50}$$

We now show that the firm only acquires information when it changes its price. Fix a time t at which the firm cannot change its price. The value of a given information policy is given by:

$$V(\nu_{i,t}) = \mathbb{E}^h \left[-\omega \int_0^h e^{-r\tau} \frac{\mathrm{d}\mathbb{I}_{i,\tau}}{\mathrm{d}\tau} \,\mathrm{d}\tau + e^{-rh} J(\nu_{i,t+h}) \mid \nu_{i,t} \right] \tag{51}$$

Fix the horizon at which the firm next adjusts its price h. For each such h, suppose that the information policy yields $\nu_{i,t+h}$ and let the information be $\mathbb{I}_{i,\tau}$ under this policy. Consider instead an information policy that acquires no information until time $t+h$ and achieves the same $\nu_{i,t+h}$ and let the information be $\tilde{\mathbb{I}}_{i,\tau}$ under the policy. As both policies attain the same posterior at the next price-setting opportunity, the difference in the values of these policies is just the difference in the information costs. Moreover, we have that this difference in information costs satisfies:

$$\begin{aligned}
\omega\left(\int_0^h e^{-r\tau}\frac{\mathrm{d}\mathbb{I}_{i,\tau}}{\mathrm{d}\tau}\,\mathrm{d}\tau - e^{-rh}(\tilde{\mathbb{I}}_{i,t+h}-\tilde{\mathbb{I}}_{i,t})\right) &\geq \omega e^{-rh}\left(\int_0^h \frac{\mathrm{d}\mathbb{I}_{i,\tau}}{\mathrm{d}\tau}\,\mathrm{d}\tau - (\tilde{\mathbb{I}}_{i,t+h}-\tilde{\mathbb{I}}_{i,t})\right)\\
&= \omega e^{-rh}\left((\mathbb{I}_{i,t+h}-\mathbb{I}_{i,t}) - (\tilde{\mathbb{I}}_{i,t+h}-\tilde{\mathbb{I}}_{i,t})\right)\\
&= \omega e^{-rh}\left(\mathbb{I}_{i,t+h}-\tilde{\mathbb{I}}_{i,t+h}\right)
\end{aligned} \tag{52}$$

where the inequality follows as $e^{-rh} \leq e^{-r\tau}$ for $\tau \leq$ h, the first equality follows by the fundamental theorem of calculus, and the final equality follows as the initial information under both policies is the same. Thus, acquiring information only when there is a price reset opportunity yields a higher value if this policy leads to acquiring less information in total. Consider the following garbling of the signals obtained under the baseline information policy: receive a perfect signal about $\nu_{i,t+h}$, *i.e.,* garble $\{s_{i,\tau}\}_{\tau\in[t,t+h]}$ into the induced posterior at time $t+h$. As this is a garbling, and mutual information is monotone in the Blackwell order, we have that $\mathbb{I}_{i,t} \geq \tilde{\mathbb{I}}_{i,t+h}$.

It remains to characterize optimal information acquisition when firms reset their price. First, we show that any optimal information structure is Gaussian. Fix a path of price reset times $\mathscr{R}$, let such a reset time be t, and let $\nu_{i,t-}$ be the belief at the start of time t. We have that $\nu_{i,0} = N(0,\sigma_0^2)$. Let $\{p_t\}_{t\in\mathscr{R}}$ be the sequence of random variables corresponding to the firm's reset prices at each reset date and let S^t be the information set implied by this price sequence. Now define a sequence of Gaussian random variables $\{\hat{p}_t\}_{t\in\mathscr{R}}$ such that for all $t\in\mathscr{R}$: $\mathbb{V}[W_{i,t}|\hat{p}_t] = \mathbb{E}\left[\mathbb{V}[W_{i,t}|S^t]\right]$. The expected nominal profits of the firm are the same under both policies. Thus, $\{\hat{p}_t\}_{t\in\mathscr{R}}$ yields a payoff improvement if and only if its total mutual information is lesser. This is immediate as, for any given expected variance-covariance matrix, the Gaussian random variable maximizes entropy (see Chapter 12 in Cover, 1999). Thus, as $\mathscr{R}$ was arbitrary, the firm should acquire a Gaussian signal at each price reset opportunity regardless of the sequence of price reset times.

Second, we write their dynamic optimization problem using this structure. We observe that, $W_{i,t+h} = W_{i,t+h} - W_{i,t} + W_{i,t} - W_{i,0}$, $W_{i,t+h} - W_{i,t} \perp W_{i,t} - W_{i,0}$, and $W_{i,t+h} - W_{i,t}|\nu_{i,t} \sim N(0,h)$. Thus, $\nu_{i,t+h-}$ is the convolution measure of $\nu_{i,t}$ with $N(0,h)$, which we will denote by $\nu_{i,t} * N(0,h)$. Moreover, we know that $\mathbb{V}[W_{i,\tau}|\nu_{i,t}] = \tau + \mathbb{V}[W_{i,t}|\nu_{i,t}]$. As the firm acquires a Gaussian signal, we have that their problem reduces to:

$$V(U_{i,t-}) = \max_{U_{i,t}\leq U_{i,t-}} -U_{i,t}\frac{B}{2}\mathbb{E}^h\left[\int_0^h e^{-r\tau}\,\mathrm{d}\tau\right] + \mathbb{E}^h\left[e^{-rh}V(U_{i,t}+\sigma^2 h)\right] + \frac{\omega}{2}\ln\left(\frac{U_{i,t}}{U_{i,t-}}\right) \tag{53}$$

Taking the first-order condition we have that (if the constraint that $U_{i,t} \leq U_{i,t-}$ is slack):

$$0 = -\frac{B}{2}\mathbb{E}^h\left[\int_0^h e^{-r\tau}\,\mathrm{d}\tau\right] + \mathbb{E}^h\left[e^{-rh}V'(U_{i,t}+\sigma^2 h)\right] + \frac{\omega}{2}\frac{1}{U_{i,t}} \tag{54}$$

By the envelope theorem, we also have that:

$$V'(U_{i,t}+h) = -\frac{\omega}{2}\frac{1}{U_{i,t}+\sigma^2 h} \tag{55}$$

Thus, we obtain the following condition for the optimality of $U_{i,t}$:

$$\frac{1}{U_{i,t}} = \frac{B\sigma^2}{\omega}\mathbb{E}^h\left[\int_0^h e^{-r\tau}\,\mathrm{d}\tau\right] + \mathbb{E}^h\left[e^{-rh}\frac{1}{U_{i,t}+\sigma^2 h}\right] \tag{56}$$

To see that this equation has a unique solution, we rewrite it as:

$$1 - U_{i,t}\frac{B}{\omega}\mathbb{E}^h\left[\int_0^h e^{-r\tau}\,\mathrm{d}\tau\right] = \mathbb{E}^h\left[e^{-rh}\frac{U_{i,t}}{U_{i,t}+\sigma^2 h}\right] \tag{57}$$

The right-hand side is a strictly positive and strictly increasing function of $U_{i,t}$ and the left-hand side is a strictly decreasing function that attains a value of 1 at $U_{i,t} = 0$ and attains a value of 0 at $\bar{z} = \frac{1}{\frac{B}{\omega}\mathbb{E}^h\left[\int_0^h e^{-r\tau}\mathrm{d}\tau\right]}$. Thus, this equation has a unique solution U^*, which moreover satisfies $U^* \leq \bar{z}$. Moreover, computing the second derivative of the objective function, we obtain:

$$-\frac{\omega}{2}\left(\frac{1}{U_{i,t}^2} - \mathbb{E}^h\left[e^{-rh}\frac{1}{(U_{i,t}+\sigma^2 h)^2}\right]\right) < -\frac{\omega}{2}\mathbb{E}^h\left[e^{-rh}\left(\frac{1}{U_{i,t}^2} - \frac{1}{(U_{i,t}+\sigma^2 h)^2}\right)\right] \leq 0 \tag{58}$$

Thus, as the problem is strictly concave, we have this solution is simply the minimum between $U_{i,t-}$ and U^*. As a result, if $U_{i,t-} \leq U^*$ the firm acquires no information, and if $U_{i,t-} > U^*$, the firm acquires a Gaussian signal of $W_{i,t}$ that resets its posterior uncertainty about Z_{it} to U^*. □

A.2 Proof of Corollary 1

Proof. By Theorem 1, the optimal level of uncertainty solves:

$$\begin{aligned}\mathrm{LHS}(U^*;B,\omega,r,G) \equiv 1 - U^*\frac{B}{\omega}\mathbb{E}^h\left[\int_0^h e^{-r\tau}\,\mathrm{d}\tau\right] \\ = \mathbb{E}^h\left[e^{-rh}\frac{U^*}{U^*+\sigma^2 h}\right] \equiv RHS(U^*;r,\sigma^2,G)\end{aligned} \tag{59}$$

Given the existence of a unique solution U^* (from Theorem 1), the results are immediate from the observations that: LHS is decreasing in U^*, B (which is increasing in η), and G (in the sense of first-order stochastic dominance) and increasing in ω and r, and RHS is decreasing in σ^2, r, and G and increasing in U^*. □

A.3 Proof of Corollary 3

Proof. By Theorem 1, the firm's uncertainty at a price-setting opportunity is reset to U^* and they acquire no information between price-setting opportunities. Thus, in h periods, their uncertainty is given by:

$$\begin{aligned} U_{i,t} &= \mathbb{V}[q_{i,t} \mid S_i^t] = \mathbb{V}[q_{i,t} \mid S_i^{t-h}] = \mathbb{V}\left[\sigma\left(W_{i,t} - W_{i,t-h}\right) + \sigma W_{i,t-h} \mid S_i^{t-h}\right] \\ &= \sigma^2 h + \mathbb{V}\left[\sigma W_{i,t-h} \mid S_i^{t-h}\right] = \sigma^2 h + U^* \end{aligned} \tag{60}$$

as claimed. □

A.4 Proof of Proposition 1

Proof. From Theorem 1, we know that firms do not acquire information between price resetting opportunities. Thus, $\mathbb{E}_{i,t}[q_{i,t}] = \mathbb{E}_{i,0}[q_{i,0}]$ until the firm next resets its price, which we will suppose happens in h' periods. As firms' marginal costs follow a martingale, this implies that the firm's expected belief gap until period h' is simply the firm's initial belief gap, y^b. From Theorem 1, we have that when firms reset their prices, they acquire a Gaussian signal of their marginal costs with a signal noise $\tilde{\sigma}_{h+h'}$ that resets their posterior uncertainty to U^*:

$$s_{i,t+h'} = W_{i,t+h'} + \tilde{\sigma}_{h+h'}\varepsilon_{i,t+h'} \tag{61}$$

where $\varepsilon_{i,t+h'} \sim N(0,1)$. Because of this, a resetting firm has a conditional expectation of the random component of their marginal costs that is given by:

$$\begin{aligned} \mathbb{E}_{i,t+h'}[W_{i,t+h'}] &= \kappa_{h+h'} s_{i,t+h'} + (1-\kappa_{h+h'})\mathbb{E}_{i,t}[W_{i,t}] \\ &= W_{i,t+h'} + (1-\kappa_{h+h'})(\mathbb{E}_{i,t}[W_{i,t}] - W_{i,t+h'}) + \kappa_{h+h'}\tilde{\sigma}_{h+h'}\varepsilon_{i,t+h'} \end{aligned} \tag{62}$$

This implies that the belief gap is given by:

$$\begin{aligned} y^b_{i,t+h'} &= (1-\kappa_{h+h'})y^b_{i,t} + (1-\kappa_{h+h'})(W_{i,t+h'} - W_{i,t}) - \frac{\sigma}{\gamma}\kappa_{h+h'}\tilde{\sigma}_{h+h'}\varepsilon_{i,t+h'} \\ &= (1-\kappa_{h+h'})y^b_{i,t} + Z_{i,t+h'} \end{aligned} \tag{63}$$

where $Z_{i,t+h'} \sim N(0, \hat{\sigma}^2_{h+h'})$.

We can then proceed recursively to characterize expected lifetime output gaps by observing that:

$$Y(y^b, y^x, h) = \mathbb{E}^{h',Z}\left[\int_0^{h'} y^b \, d\tau + \int_0^{h'} y^x \, d\tau + Y\left((1-\kappa_{h+h'})y^b + Z_{h'}, 0, 0\right)\right] \tag{64}$$

We now guess and verify that $Y(y^b, y^x, U, h) = \beta(h) y^x + m(h) y^b$. Plugging this guess into Equation 64 and matching coefficients, we obtain that $\beta(h)$ and $m(h)$ must satisfy:

$$\beta(h) = \mathbb{E}_g[h'|h] = \bar{D}_h \tag{65}$$

$$m(h) = \mathbb{E}_g[h'|h] + m(0)\mathbb{E}_g^{h'}[1-\kappa_{h+h'}|h] = \bar{D}_h + m(0)(1-\bar{\kappa}_h) \tag{66}$$

$$m(0) = \frac{\mathbb{E}_g[h']}{1-\mathbb{E}_g[1-\kappa_{h'}]} = \bar{D}_0 \frac{1}{\bar{\kappa}_0} \tag{67}$$

completing the proof. □

A.5 Proof of Theorem 2

Proof. First, by Proposition 1, we have that the CIR is given by Equation 30. We now show that $\bar{D}_0 \frac{1-\bar{\kappa}}{\bar{\kappa}_0} = \frac{U^*}{\sigma^2}$. By definition, we have that:

$$\begin{aligned} 1-\bar{\kappa} = \mathbb{E}_f[1-\bar{\kappa}_h] = \mathbb{E}_f\left[1 - \frac{\sigma^2 h}{U^* + \sigma^2 h}\right] = \mathbb{E}_f\left[\frac{U^*}{U^* + \sigma^2 h}\right] = \mathbb{E}_f\left[\frac{\frac{U^*}{\sigma^2}}{\frac{U^*}{\sigma^2} + h}\right] \\ = \int_0^\infty \left[\int_h^\infty \frac{\frac{U^*}{\sigma^2}}{\tau + \frac{U^*}{\sigma^2}} \frac{g(\tau)}{1-G(h)} d\tau\right] f(h) \, dh \end{aligned} \tag{68}$$

We now state and prove an ancillary result that characterizes the cross-sectional distribution of durations in terms of the expected duration of a price setting firm and the distribution of price-setting opportunities.[8]

[8] As this result uses the fact that G admits a density, it does not nest Taylor pricing. However, our result still goes through. Concretely, we observe that $h' = k - h$ and f is uniform over $[0, k]$. Thus, we have that:

$$\mathbb{E}_f^h\left[\mathbb{E}_g^{h'}[h'|h]\right] = \mathbb{E}_f^h[k-h] = \frac{k}{2} \tag{69}$$

Moreover, we have that:

$$\mathbb{E}_g[h'|h=0] \frac{\mathbb{E}_f^h\left[\mathbb{E}_g^{h'}\left[\frac{U^*}{U^*+\sigma^2(h+h')}|h\right]\right]}{1-\mathbb{E}_g^{h'}\left[\frac{U^*}{U^*+\sigma^2 h'}|h=0\right]} = k\frac{\frac{U^*}{U^*+\sigma^2 k}}{1-\frac{U^*}{U^*+\sigma^2 k}} = \frac{U^*}{\sigma^2} \tag{70}$$

And the conclusion of Theorem 2 still holds.

Lemma 1. *The distribution of pricing durations in the cross-section is given by:*

$$f(h) = \frac{1}{\bar{D}_0}(1 - G(h)) \tag{71}$$

Proof. To derive f, define $p_h = \mathbb{P}[\tilde{h} \in [h-\delta, h]]$ and observe that $p_h = p_{h-\delta} \times (1 - \mathbb{P}[\text{Reset between } h-\delta \text{ and } h | \text{Not reset by } h-\delta])$. Thus, we have that:

$$p_h - p_{h-\delta} = -p_{h-\delta} \frac{G(h) - G(h-\delta)}{1 - G(h-\delta)} \tag{72}$$

diving by δ and taking the limit $\delta \to 0$, we obtain:

$$f'(h) = -f(h)\theta(h) \tag{73}$$

Integrating this expression yields:

$$f(h) \propto \exp\left\{-\int_0^h \theta(s)\,\mathrm{d}s\right\} = \exp\left\{-\int_0^h \frac{g(s)}{1-G(s)}\,\mathrm{d}s\right\} = 1 - G(h) \tag{74}$$

Using the fact that $G(0) = 0$, we then have that $f(h) = f(0)(1 - G(h))$. Integrating both sides of this expression, we then have that:

$$1 = \int_0^\infty f(h)\,\mathrm{d}h = f(0)\int_0^\infty (1 - G(h))\,\mathrm{d}h = f(0)\mathbb{E}_g[h] = f(0)\bar{D}_0 \tag{75}$$

which implies that $f(h) = \frac{1}{\bar{D}_0}(1 - G(h))$, as claimed. □

Combining Equations 68 and 71, we obtain that:

$$\begin{aligned}
1 - \bar{\kappa} &= \int_0^\infty \left[\int_h^\infty \frac{\frac{U^*}{\sigma^2}}{\tau + \frac{U^*}{\sigma^2}} \frac{g(\tau)}{1 - G(h)}\,\mathrm{d}\tau\right] \frac{1}{\bar{D}_0}(1 - G(h))\,\mathrm{d}h \\
&= \frac{1}{\bar{D}_0}\int_0^\infty \int_h^\infty \frac{\frac{U^*}{\sigma^2}}{\tau + \frac{U^*}{\sigma^2}} g(\tau)\,\mathrm{d}\tau\,\mathrm{d}h = \frac{1}{\bar{D}_0}\int_0^\infty \left[\int_0^\tau \frac{\frac{U^*}{\sigma^2}}{\tau + \frac{U^*}{\sigma^2}} g(\tau)\,\mathrm{d}h\right]\mathrm{d}\tau \\
&= \frac{1}{\bar{D}_0}\int_0^\infty \frac{\frac{U^*}{\sigma^2}\tau}{\tau + \frac{U^*}{\sigma^2}} g(\tau)\,\mathrm{d}\tau = \frac{1}{\bar{D}_0}\frac{U^*}{\sigma^2}\int_0^\infty \frac{\tau}{\tau + \frac{U^*}{\sigma^2}} g(\tau)\,\mathrm{d}\tau \\
&= \frac{1}{\bar{D}_0}\frac{U^*}{\sigma^2}\bar{\kappa}_0
\end{aligned} \tag{76}$$

which implies that $\bar{D}_0 \frac{1-\bar{\kappa}}{\bar{\kappa}_0} = \frac{U^*}{\sigma^2}$. Substituting this into Equation 30 yields the result. □

A.6 Proof of Proposition 2

Proof. By Corollary 3, a firm's uncertainty h periods after changing its price is $U = U^* + \sigma^2 h \geq U^*$. Thus, $L(z) = \mathbb{P}[U \leq z] = \mathbb{P}\left[h \leq \frac{z-U^*}{\sigma^2}\right] = F\left(\frac{z-U^*}{\sigma^2}\right)$. Differentiating this expression yields the claimed formula for $l(z)$. □

A.7 Proof of Proposition 3

Proof. To derive the distribution of price changes, we start by finding the conditional distribution of price changes for firms who had a given duration of h periods who had a fixed information set at their last price change opportunity. We then marginalize over the distribution of price durations and information sets to obtain the price change distribution. To this end, consider a firm i that is changing its price at time t that changed its price h periods ago and define:

$$\Delta^h p_{i,t} \equiv p_{i,t} - p_{i,t-h} = \sigma\left(\mathbb{E}_{i,t}[W_{i,t}] - \mathbb{E}_{i,t-h}[W_{i,t-h}]\right) \tag{77}$$

Moreover, we have that:

$$\mathbb{E}[W_{i,t}] = \kappa_h s_{i,t} + (1-\kappa_h)\mathbb{E}_{i,t-h}[W_{i,t-h}] \tag{78}$$

where:

$$s_{i,t} = W_{i,t} + \tilde{\sigma}_h \varepsilon_{i,t} \tag{79}$$

Combining these equations, we can write:

$$\Delta^h p_{i,t} = \sigma\kappa_h\left(W_{i,t} + \tilde{\sigma}_h \varepsilon_{i,t} - \mathbb{E}_{i,t-h}[W_{i,t-h}]\right) \tag{80}$$

Therefore, we have that:

$$\Delta^h p_{i,t} | S_i^{t-h} \sim N(0, \check{\sigma}^2(S_i^{t-h})) \tag{81}$$

where:

$$\check{\sigma}^2(S_i^{t-h}) = \kappa_h^2 \mathbb{V}[\sigma W_{i,t} + \sigma\tilde{\sigma}_h \varepsilon_{i,t} | S_i^{t-h}] \tag{82}$$

where we know that:

$$\mathbb{V}[\sigma W_{i,t} | S_i^{t-h}] = \mathbb{V}[\sigma(W_{i,t} - W_{i,t-h}) + \sigma W_{i,t-h} | S_i^{t-h}] = \sigma^2 h + U^* \tag{83}$$

as, by Theorem 1, we have that at a time of price reset (which $t-h$ is by assumption) the firm's posterior uncertainty is always equal to $\mathbb{V}[\sigma W_{i,t-h} | S_i^{t-h}] = U^*$. Thus, we have that:

$$\check{\sigma}^2(S_i^{t-h}) = \kappa_h^2\left(\sigma^2 h + U^* + \sigma^2 \tilde{\sigma}_h^2\right) \tag{84}$$

Moreover, the signal noise $\tilde{\sigma}_h^2$ that achieves the Kalman gain κ_h solves:

$$\sigma^2 \tilde{\sigma}_h^2 = (U^* + \sigma^2 h) \frac{1 - \kappa_h}{\kappa_h} \tag{85}$$

and so we have that:

$$\check{\sigma}^2(S_i^{t-h}) = \kappa_h^2 (U^* + \sigma^2 h) \left(1 + \frac{1 - \kappa_h}{\kappa_h}\right) = \kappa_h (U^* + \sigma^2 h) = \sigma^2 h \tag{86}$$

Thus, we have that conditioning on the firm's information set is irrelevant and the conditional distribution of price changes is the marginal distribution of price changes:

$$\Delta^h p_{i,t} | S_i^{t-h} \sim N(0, \sigma^2 h) \implies \Delta^h p_{i,t} \sim N(0, \sigma^2 h) \tag{87}$$

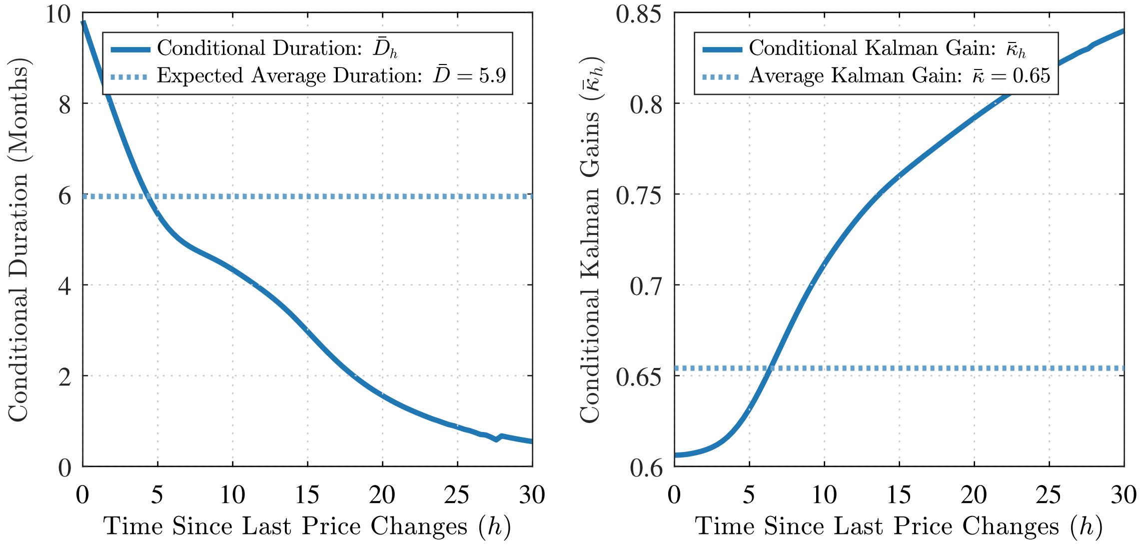
Finally, integrating over the distribution of price durations, G, we obtain that the distribution of price changes is:

$$\begin{aligned} H(\Delta p) = \mathbb{P}[\Delta p_{i,t} \leq \Delta p | \Delta p_{i,t} \neq 0] &= \mathbb{E}_g^h \left[\mathbb{P}[\Delta^h p_{i,t} \leq \Delta p | \Delta p_{i,t} \neq 0] \right] \\ &= \mathbb{E}_g^h \left[\Phi\left(\frac{\Delta p}{\sigma \sqrt{h}} \right) \right] = \int_0^\infty \Phi\left(\frac{\Delta p}{\sigma \sqrt{h}} \right) \mathrm{d}G(h) \end{aligned} \tag{88}$$

which depends on σ and G but does not depend on U^*. □

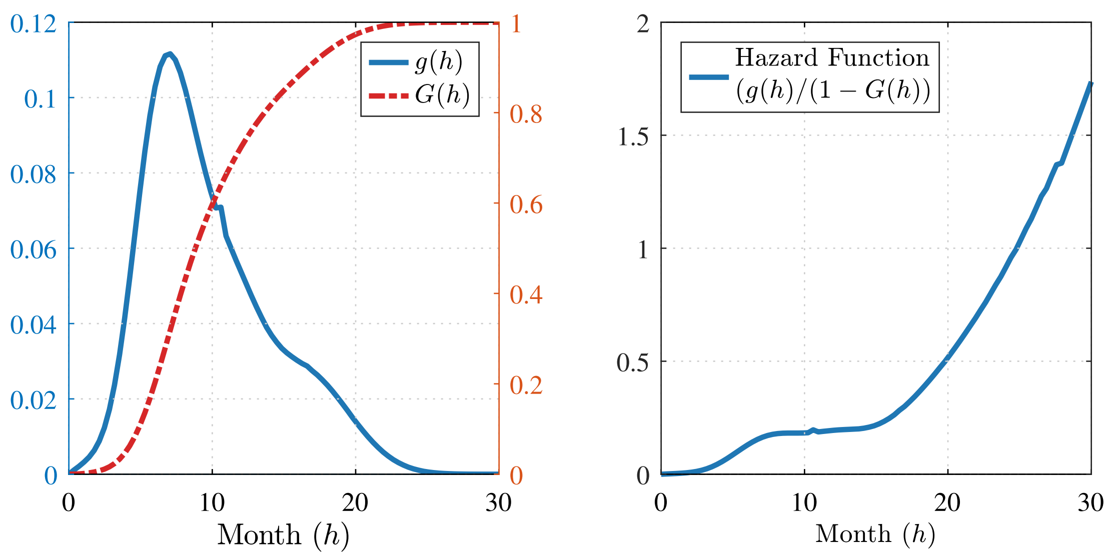
B Additional Figures

Appendix Figure 1: Expected Duration of Next Price Changes and Kalman Gains



Notes: The left panel shows the average conditional duration, $\bar{D}_h = \mathbb{E}_g^{h'}[h'|h]$, which is how long a firm that reset its price h periods ago expects to wait before resetting its price (blue solid line), as well as the average duration, $\bar{D} = \mathbb{E}_f^h[\bar{D}_h]$, which is how long the firms expect to wait *on average* before resetting their prices (blue dashed line). The right panel shows the average conditional Kalman gain, $\bar{\kappa}_h = \mathbb{E}_g^{h'}[\kappa_{h'+h}|h]$, which is the expected Kalman gain at the next price reset opportunity for a firm that last reset its price h periods ago (blue solid line), as well as the average Kalman gain, $\bar{\kappa} = \mathbb{E}_f^h[\bar{\kappa}_h]$, which is the average across all firms of the expected Kalman gain when they next reset their prices (blue dashed line)

Appendix Figure 2: Distribution of Price Reset Opportunities and the Hazard Rate



Notes: The left panel shows the empirically estimated distribution of price reset opportunities G, given by $G(h) = 1 - \hat{f}(h)/\hat{f}(0)$ where $\hat{f}$ is the empirical distribution of time since firms' last price changes. g is the density function. The right panel shows the hazard rate, $\theta(h) = g(h)/(1 - G(h))$.