Pricing and Production Without The Invisible Hand

Joel P. Flynn* Yale

George Nikolakoudis† Chicago

Karthik A. Sastry‡ Princeton

February 22, 2026

Abstract

Modern theories of the business cycle do not allow for the simultaneous rational choice of both prices and quantities, instead assuming that an "invisible hand" determines one of these variables to clear markets. In this paper, we develop a macroeconomic framework in which *both* prices and quantities are chosen directly by firms, and exchange is both voluntary and efficient. Because of uncertainty about demand and productivity, individual product markets can be in excess supply or rationed. The absence of market-clearing changes pricing and production in qualitatively important ways: markups are no longer determined solely by the elasticity of demand, and higher uncertainty reduces production and increases markups. In equilibrium, production in rationed markets has a negative aggregate demand externality on demand in slack markets. Differently from New Keynesian economies, monetary shocks propagate by reducing economic slack, raising aggregate labor productivity and consumption, while uncertainty shocks act as stagflationary cost-push shocks. We integrate our theory of disequilibrium in a dynamic, rational-expectations "New Old Keynesian Model" and demonstrate its implications for the business cycle.

---

*Yale University Department of Economics, 30 Hillhouse Avenue, New Haven, CT, 06511. Email: joel.flynn@yale.edu.

†Kenneth C. Griffin Department of Economics at the University of Chicago, 1126 E. 59th Street, Chicago, IL, 60637. Email: nikolakoudis@uchicago.edu.

‡Princeton University Department of Economics, Julis Romo Rabinowitz Building, Princeton, NJ, 08544, Email: ksastry@princeton.edu.

We are grateful to Susanto Basu, Christian Hellwig, Giuseppe Moscarini, Michael Peters, Pascual Restrepo, Aleh Tsyvinski, and seminar participants at Boston College for helpful comments. First posted version: February 22, 2026.

1 Introduction

We study the equilibrium determination of pricing *and* production in economies in which both of these decisions are made directly by economic agents. Although this may sound descriptive of standard general equilibrium models, that is far from the case. Indeed, Arrow (1959) describes the presence of "a logical gap in the usual formulations of the theory of the perfectly competitive economy, namely that there is no place for a rational decision with respect to prices as there is with respect to quantities." In modern models of imperfect competition in general equilibrium, such as the monetary model of Blanchard and Kiyotaki (1987) or the textbook New Keynesian model of Woodford (2003b), Arrow's critique can be inverted: there is a rational decision with respect to prices but not quantities. For example, the New Keynesian model requires firms to produce goods even when they would rather not do so (Huo and Ríos-Rull, 2020; Holden, 2025), violating the core economic principle of voluntary exchange. Thus, it is no new argument that the delegation of pricing or production to the invisible hand or the Walrasian auctioneer is potentially undesirable.

In this paper, to overcome the Arrow (1959) critique, we build a general equilibrium macroeconomic model in which firms make both pricing and production decisions to maximize expected profits under uncertainty, and exchange is both voluntary and efficient. The resulting model features markets in which goods are either rationed or over-supplied—as in classical analyses of disequilibrium (see *e.g.,* Barro and Grossman, 1971)—in a standard rational expectations equilibrium. On a technical level, the presence of rationing and slack creates cross-market spillovers that are challenging to characterize and have previously inhibited research on disequilibrium models (see *e.g.,* Bénassy, 1993).

Notwithstanding these complications, we fully characterize equilibrium outcomes in the model. Moreover, we find that severing the invisible hand has a number of non-standard implications: (i) the classic Lerner (1934) pricing formula breaks down and firms' markups are optimally given by the inverse probability that demand is rationed, (ii) increases in volatility have *first-order* effects on firms' decisions, driving up prices, reducing labor inputs, and increasing markups, (iii) aggregate demand externalities can turn negative because increasing production in rationed markets reduces demand for non-rationed markets, (iv) uncertainty shocks raise aggregate prices and depress aggregate consumption while leaving aggregate employment unchanged, (v) the extent of monetary non-neutrality is decreasing in volatility, discontinuous in the zero uncertainty limit, and ambiguously affected by market power, and (vi) unanticipated monetary shocks raise labor productivity. While our baseline model is intentionally as simple as possible, we also describe how it can be augmented to feature inventories, disequilibrium in product and labor markets, and dynamics with sticky decisions.

The Pricing and Production Problem. We first examine a single firm that must choose prices and inputs for production before it knows its demand and productivity. To focus on the novel economics of this problem, we consider the simplest possible model of this setting: the firm operates a constant returns-to-scale production function and faces a demand curve with constant price elasticity of demand, $\eta > 1$. The firm is uncertain about the strength of its demand and productivity, which are jointly log-normally distributed. Problems of this variety lie at the core of virtually all standard macroeconomic models, including the New Keynesian model and the Real Business Cycle model with monopolistic firms. The standard practice is to suppose that the firm picks either its price or its quantity, and then that the remaining variable is determined by market-clearing. To overcome the Arrow (1959) critique of this assumption, we instead adopt an axiomatic approach (Bénassy, 1993). First, we assume that exchange must be voluntary: no one can be forced to trade more than they would be willing. Second, we assume that exchange is efficient: there are no trades that fail to take place that both sides of the market would be willing and able to make. These axioms of exchange imply that the transacted quantity is the minimum of what the firm produces and what the household demands at the price set by the firm. For this reason, the firm faces sharply different pricing and production incentives *ex post* depending on which of two states occurs: demand exceeds supply and the market is *rationed*, or supply exceeds demand and the market is *slack*. The firm's problem is to balance these trade-offs *ex ante*.

We first characterize the optimal pricing and input decisions in terms of a single fixed-point equation for the firm's optimal *tightness*, defined as the ratio of the firm's price raised to the power of its negative price elasticity of demand to the firm's labor input. This equation requires that the firm's tightness is proportional to the ratio of the firm's expected productivity when the firm rations demand to the firm's expected demand when the firm over-produces. Intuitively, consider a firm raising its prices by 1 percent, a marginal amount. If the market ends up rationed, they just sell all of their produced goods at the now higher price and make 1% higher revenues. If the market is slack and output is demand determined, then the firm continues to earn 1% more per unit sold but also sells η% fewer units, where η is the elasticity of demand; the overall effect on revenues is a $(\eta - 1)$% reduction. The optimal choice of tightness balances these forces, weighed by their relative likelihood of occurring.

Once the optimal degree of tightness is determined by resolving this trade-off, the firm's optimal price is pinned down by setting a markup equal to the inverse probability that demand is rationed on the expected marginal cost of production. Without market clearing, hiring an additional unit of labor is only beneficial whenever the market is rationed, while costs are always paid. The optimal price must therefore be a markup on marginal costs equal to the inverse rationing probability to ensure that hiring labor is profitable on the margin. Thus, optimal pricing deviates from the classical Lerner (1934) formula, in which the optimal markup is solely determined by the elasticity of firms' residual demand. Instead, uncertainty about supply and demand forces now drives the optimal markup.

Indeed, we show that uncertainty has unusually strong effects on prices and inputs in this model: the perceived standard deviations of supply and demand shocks have *first-order* effects on decisions. This is in contrast to most widely-used business cycle frameworks, in which the standard deviations of these factors are only relevant to second-order or third-order (Schmitt-Grohé and Uribe, 2004). This feature arises from the discontinuity in firms' marginal revenues as a firm's product switches from being oversupplied to rationed. Moreover, the discontinuity of marginal revenues is a fundamental property of voluntary and efficient exchange. To make this point precise, we contrast our model to one with a matching function that violates the principle of efficient exchange, as in the literature on matching in goods markets (see Petrongolo and Pissarides, 2001, for a review), and show that uncertainty only matters for firm decision-making to second-order.

We characterize the effects of uncertainty and show that higher uncertainty increases firms' optimal prices and depresses their optimal usage of labor inputs, a feature that is unique to the fact that firms are choosing prices and quantities jointly. Intuitively, firms reduce their labor to insure themselves against low demand realizations in which production is wasted, and they raise prices so that purchasing labor continues to be profitable. The role of uncertainty in determining markups can help account for the disconnect in the magnitude of markups found in macroeconomic studies that use production function estimation approaches (*e.g.*, De Loecker et al., 2020), with those that directly estimate the elasticity of demand (*e.g.*, Broda and Weinstein, 2006). Our framework also implies that cyclical fluctuations in uncertainty can drive cyclical fluctuations in markups. Finally, we show that these qualitative insights translate to more general environments, such as those that feature inventories or disequilibrium in both input and output markets.

Pricing and Production in General Equilibrium. We next embed this firm problem in an otherwise canonical general equilibrium macroeconomic model. Following Blanchard and Kiyotaki (1987), households have constant elasticity of substitution preferences over a continuum of varieties with idiosyncratic taste shocks. Households further have Golosov and Lucas (2007) preferences over the consumption aggregate, money, and labor supply.

The household's problem in this model differs from that in standard analyses of monopolistic competition because the household cannot consume more of any good than has been produced. The resulting failure of market-clearing in any one market modifies effective demand in other markets. We characterize these spillovers analytically and show that the presence of rationed markets increases effective demand in slack markets. Intuitively, consumers substitute to other varieties once a particular variety is out of stock. Thus, the standard logic that production decisions are strategic complements (Blanchard and Kiyotaki, 1987) is no longer universally true: expanding production of rationed varieties *reduces* the demand for non-rationed varieties, generating negative aggregate demand externalities.

Next, we study how these market interactions shape prices, production, and aggregate consumption in general equilibrium. As in our partial equilibrium framework, we show that these variables respond to first-order in uncertainty. We show that an increase in uncertainty unambiguously raises firms' prices and reduces aggregate consumption. For this reason, uncertainty shocks have similar effects to markup shocks in general equilibrium. We contrast this case to the Walrasian benchmark, in which demand uncertainty has no impact on firms' pricing decisions. Surprisingly, and different from standard analysis of markup shocks, we find that uncertainty has no first-order impact on the choices of firms' inputs because of the aforementioned positive spillovers between slack and rationed markets. Thus, uncertainty reduces aggregate labor productivity (GDP per labor hour) while leaving *physical* productivity unchanged. This occurs because an increase in uncertainty raises the economy's "excess capacity"—that is, certain varieties are not consumed because there is no invisible hand to incentivize consumers to purchase those goods which are in excess supply.

We then revisit the monetary transmission mechanism when markets do not clear. We find that the passthrough of an unanticipated monetary policy shock to aggregate consumption is one-for-one in the zero uncertainty limit, but that it is declining in the degree of uncertainty that firms face. Crucially, monetary passthrough is positive even though aggregate production in the economy remains unchanged. Rather, unanticipated changes in aggregate demand give households resources to consume goods in previously overproduced markets. This mechanism is therefore distinct from that of standard "New Keynesian" frameworks in which changes in consumption arise entirely from firms' market-clearing commitments to produce whatever the market is demanding (for a discussion, see Flynn et al., 2026) and is more reminiscent of the "Old Keynesian" notion that consumption increases by picking up slack in the economy (for a discussion, see Barro, 2025). To see this formally, we find that monetary passthrough—while *decreasing* in uncertainty—is discontinuous in the zero-uncertainty limit: when there is no uncertainty in the economy, there is no slack in any market, and unanticipated monetary shocks have no effect on output.

Dynamics: A New Old Keynesian (NOK) Model. Finally, we study the dynamic implications of our framework when pricing and production decisions are also "sticky" in the sense of Calvo (1983): only a fraction of firms can adjust pricing and production in any given period. This allows for empirically realistic sluggishness in decision making and allows us to transparently compare our approach to that of the textbook New Keynesian model.

We first characterize the properties of our New Old Keynesian (NOK) model. First, as in the static analysis, uncertainty has first-order effects on allocations. These are in some ways similar to "cost-push" shocks in standard New Keynesian analyses (*e.g.*, Smets and Wouters, 2007) but, as we will see, with sharply different labor-market implications. Second, we show that the first-order propagation of shocks for the price-level and real consumption are the same as their New Keynesian counterparts: that is, conditional on the initial shock to the price level or consumption, these variables behave identically in the NK and NOK models. Third, we show that propagation for output, employment, and labor productivity is sharply different. The intuition is the dynamic analogue to what we found in the static analysis: consumption and sales can fluctuate due to absorbing and releasing product-market slack, even without corresponding movements in inputs.

We finally solve a calibrated version of this model nonlinearly to illustrate two key economic lessons. First, monetary expansions lead to consumption booms fueled by tightening product markets, with a more limited response of production and employment. This non-standard transmission mechanism is consistent with high-frequency empirical evidence of monetary transmission from Buda et al. (2025), who document rapid (week-to-week) responses of *sales* to monetary expansions without corresponding responses of production or inputs (including labor). Moreover, the prediction that monetary expansions increase labor productivity (GDP per worker)—despite fixed *physical* productivity—is consistent with the classic empirical findings of Christiano et al. (2005), who show that GDP expands more quickly and sharply than employment in response to shock monetary expansions.

Second, uncertainty shocks lead to sharp declines in consumption, increases in prices, and an *increase* in product-market tightness (decrease in slackness). As a result, they manifest in sharp declines in labor productivity: for close to the same level of physical production, high prices dissuade consumers and lead to lower transaction volumes. These effects are completely absent in the nested NK model, in which volatility has no effect on allocations even in the nonlinear solution. In this way, our findings contribute to the literature studying how and why spikes in uncertainty, known to be concurrent with recessions (see, *e.g.*, Bloom et al., 2018), might translate into economic contractions (as shown empirically by Bloom, 2009) and inflation (as shown empirically by Mumtaz and Ruch, 2025). In aggregate data, our uncertainty shocks would manifest as contractionary and inflationary reductions in the Solow residual with ambiguous effects on labor inputs, consistent with the classic findings of Basu et al. (2006). Thus, through the lens of our model, microeconomic uncertainty shocks affect the aggregate "productivity" of guiding consumers to slack markets, holding fixed the actual physical productivity of producers.

Related Literature. The prevailing tradition in modern macroeconomics is to study models in which firms set prices (see *e.g.,* Woodford, 2003b) or quantities (see *e.g.,* Angeletos and La'O, 2010) and that all markets clear *ex post.*[1] Our work is most related to a literature that explores equilibrium in contexts in which markets do not clear (see Bénassy, 1993, for a review). For example, Barro and Grossman (1971) and Michaillat and Saez (2015) study models in which there can be rationing and excess supply in the product and labor markets. A key challenge in this literature is the combinatorial nature of clearing across markets and accounting for all possible combinations of cross-market demand spillovers. Methodologically, we demonstrate that allowing for uncertainty and monopolistic competition (as in Dixit and Stiglitz, 1977) smooths over any discontinuities in the firm decision problem and gives rise to a well-defined distribution of markets that are below and above capacity in general equilibrium, allowing for an analytical characterization of these spillovers. This is a different approach than those based on tâtonnement, which postulate (Jaffé, 1967) or derive (Lorenzoni and Werning, 2026) an explicit process of price adjustment and study whether and how fast outcomes converge to Walrasian equilibrium. For example, in Lorenzoni and Werning (2026), while firms' prices are sticky, their quantities are not: they always produce the quantity that clears the market at their out-of-equilibrium price.

Most related to our analysis are Huo and Ríos-Rull (2020) and Holden (2025),[2] which respectively augment New Keynesian models with the ability of workers to not supply labor and firms to not produce goods if they do not wish to do so. In both instances, this leads to the potential for labor or goods demand to be rationed. Our analysis differs from these in the critical respect that firms must make both pricing and production decisions before the realization of uncertainty. This leads to the important economic difference that markets can end up in a state of either excess supply or rationing, and is the feature that gives rise to the novel implications of our analysis for the propagation of monetary and uncertainty shocks.

Outline. The rest of the paper proceeds as follows. Section 2 introduces the decision problem of a monopolist that must choose both prices and inputs under uncertainty and characterizes its properties. Section 3 embeds this decision in a general equilibrium model and characterizes household behavior. Section 4 characterizes equilibrium prices and production and leverages this to study the propagation of uncertainty shocks and monetary shocks. Section 5 extends the static model to a dynamic setting and quantitatively illustrates the dynamic implications of the theory. Section 6 concludes.

---

[1] Reis (2006) and Flynn et al. (2024) consider settings in which firms can choose to set either a price or a quantity. Flynn et al. (2026) allow firms to choose supply functions, but maintain market clearing.

[2] Liu (2026) embeds a similar rationing mechanism to that of Holden (2025) in a search-and-matching framework, allowing him to study the interaction of rationing with real rigidities.

2 Pricing and Production in Partial Equilibrium

We begin by analyzing the core new element of our paper: the decision problem of a monopolistic firm that must make pricing and production decisions before their demand ultimately realizes. This is the key building block of our subsequent general equilibrium model.

2.1 Model: The Firm's Problem

A firm sets a price $p \in \mathbb{R}_{++}$ and purchases labor $L \in \mathbb{R}_{+}$ at a cost $w \in \mathbb{R}_{++}$. Given these choices, the firm produces quantity $q^s \in \mathbb{R}_{+}$ according to a constant returns-to-scale production function given by:

$$q^s = AL \tag{1}$$

where $A \in \mathbb{R}_{++}$ is the firm's productivity. Moreover, the firm faces demand q^d for its product:

$$q^d = zp^{-\eta} \tag{2}$$

where $z \in \mathbb{R}_{++}$ is a demand shifter and $\eta > 1$ is the elasticity of demand. The firm values its profits according to a nominal stochastic discount factor $\Lambda \in \mathbb{R}_{++}$. We assume that the variables (Λ, A, z, w) are jointly log-normally distributed with mean μ, variance-covariance matrix Σ, and cumulative distribution function G. We use μ_X and σ_X^2 to represent, respectively, the mean and the variance of the logarithm of an arbitrary random variable X.

The amount of the good that is ultimately *sold* to consumers, q, derives from two basic and classic principles (see Bénassy, 1993, for a review). The first is voluntary exchange: no agent can be forced to trade more than that which they are able and/or willing, *i.e.,* $q \leq q^s$ and $q \leq q^d$. The second is efficient exchange: there are no trades that fail to take place that both sides of the market are willing and able to make, *i.e.,* it is never the case that both $q < q^s$ and $q < q^d$. These two principles imply that the ultimate transacted quantity is the minimum of what is supplied by the firm and what is demanded by consumers:

$$q = \min\{q^s, q^d\} \tag{3}$$

There are three possible outcomes in this market. First, the firm may supply more than what is demanded, $q^s > q^d$. In this case, we say that the market is *slack*. Second, the firm may supply less than what is demanded, $q^s < q^d$. In this case, we say that the market is *rationed*. Finally, the firm may supply exactly what is demanded, or $q^s = q^d$. In this case, we say that the market is *in Walrasian equilibrium*.

The firm makes *both* its pricing and input choices under multi-dimensional uncertainty about the state of demand, its own productivity, factor costs, and the stochastic discount factor. This implies, crucially, that the firm is also unsure of the eventual state of the market: slack, rationed, or in equilibrium. That is, the firm solves the following problem:

$$\begin{aligned} &\sup_{p,L} \int_{\mathbb{R}_4^{++}} \Lambda(pq - wL)\,\mathrm{d}G(\Lambda, A, z, w) \\ &\text{s.t. } q = \min\{q^s, q^d\},\ q^s = AL,\ q^d = zp^{-\eta} \end{aligned} \tag{4}$$

Substituting the constraints into the objective yields a more explicit formulation:

$$\sup_{p,L} \int_{\mathbb{R}^4_{++}} \Lambda\left(\min\{zp^{1-\eta}, pAL\} - wL\right) \mathrm{d}G\left(\Lambda, A, z, w\right) \tag{5}$$

This reveals the technically non-standard and economically important feature of this problem: the firm's revenue function has a kink at the point at which demand equals supply.

The Prevalence of Walrasian Disequilibrium. Only a small fraction of real-world markets—such as the stock market and other highly centralized exchanges in which the equality of supply and demand is ensured institutionally—are well-described by Walrasian equilibrium. In practice, most markets are in a fluctuating state of either slack or rationing.

Consider, by way of metaphor, the economic decisions of a baker. The baker produces bread, pastries, and other perishable goods. Before knowing demand for these goods with certainty, they must decide upon prices and purchase flour, other raw materials, and labor. On the one hand, they always run the risk that their goods might sell out, and a customer who wants a pastry is turned away (a rationed market). In practice, stockouts are prevalent in micro data and correlate with business cycle events (see *e.g.,* Cavallo and Kryvtsov, 2024; Holden, 2025). On the other hand, they also run the risk that pastries are left over at the end of the day and need to be thrown out (a slack market). This is also, of course, common in practice. In fact, the US Department of Agriculture estimates that 31% of the US food supply is wasted at the retail and consumer levels (USDA Economic Research Service, 2026).

Our analysis is particularly descriptive of *services*, where output is perishable and cannot be inventoried. For instance, in staffed service operations—such as those of management consultants, restaurants, barber shops, law firms, call services, *etc.*—when demand exceeds staffed capacity, customers must either wait or purchase an alternative service (the rationed state). Conversely, if demand is low, staff and employees are idle (the slack state). Moreover, a large literature documents substantial variation in capacity utilization, which reflects the fact that productive resources often sit idle rather than being instantaneously reallocated to other purposes in order to clear markets (Corrado and Mattey, 1997). Direct survey evidence from establishments also documents that firms often employ excess labor over the cycle (Fay and Medoff, 1985) and that employed workers often spend a substantial fraction of their work day idle, on average (Michaillat and Saez, 2015). Similarly, Liu (2026) provides evidence that a substantial fraction of products are produced below capacity. Thus, states of slack and rationing are descriptive of many markets in the economy.

It stands to reason that the mere possibility of slack and rationed market states—which is assumed away in the presence of the fictitious auctioneer— can shape firm decision-making in qualitatively important ways. It is to these decisions that we next turn.

2.2 Optimal Firm Behavior

We begin by characterizing the solution to the firm's problem of optimally choosing prices and inputs under uncertainty.

Theorem 1. *There exists a unique optimal price* p^* *and labor input* L^**. Moreover, optimal tightness* $t^* = \frac{p^{*-\eta}}{L^*}$ *is the unique solution to:*

$$t = \frac{1}{\eta - 1} \frac{\mathbb{E}[\Lambda A \mathbb{I}[t \geq U]]}{\mathbb{E}[\Lambda z \mathbb{I}[t \leq U]]} \tag{6}$$

where $U = A/z$*. Moreover, the optimal price is given by:*

$$p^* = \frac{\mathbb{E}[\Lambda w]}{\mathbb{E}[\Lambda A \mathbb{I}[t^* \geq U]]} \tag{7}$$

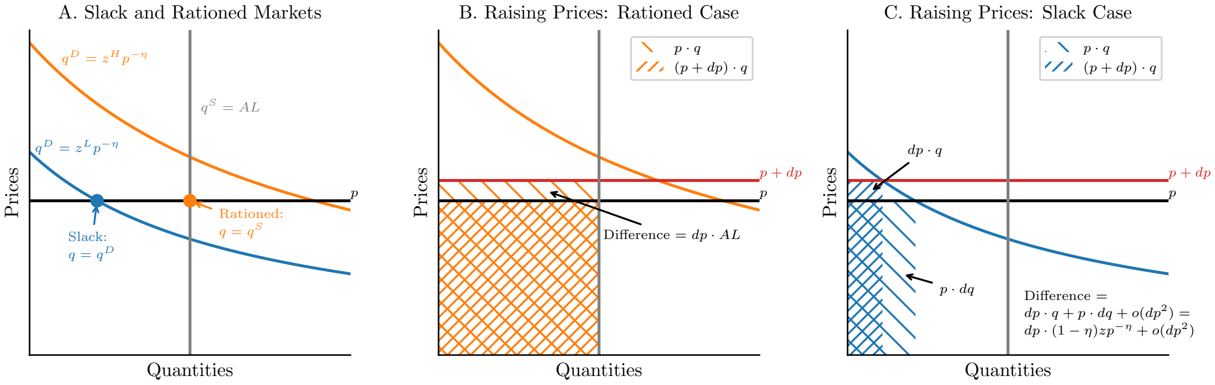
and the optimal labor input is then given by $L^* = \frac{p^{*-\eta}}{t^*}$

Proof. See Appendix A.1. □

Optimal Tightness. The key intuition for firms' optimal tightness (Equation 6) comes from firms' underlying pricing incentives and how these depend on the state of the market. This argument is visualized in Figure 1. We first observe that, given a chosen price p and labor input L, the market may be rationed or slack depending on the realizations of demand and productivity. In particular, demand is greater than supply whenever $q^d = zp^{-\eta} \geq AL = q^s$. Using our definitions of tightness, $t := p^{-\eta}/L$, and the shock to supply-demand imbalances, $U := A/z$, we can re-write the condition for rationing as $t \geq U$. Panel A of Figure 1 shows this by varying the demand state z, fixing (for illustrative purposes) productivity A: markets are rationed when z is sufficiently high, and slack when z is sufficiently low.

How does a higher price affect the firms' profits if the market ends up rationed? Firms obtain an additional revenue dp for all units produced, or $AL \cdot$ dp. The firm values these

Figure 1: The Incentives to Raise Prices



Note: This figure illustrates the incentives for price setting in the model. Panel A illustrates cases where the market is rationed or slack, depending on the realization of demand. Panels B and C show the marginal revenue effect of increasing prices from p to $p + \mathrm{d}p$ in states that are slack versus rationed, respectively. Equation 8 is obtained by setting the expected (risk-adjusted) marginal revenue effect to zero, averaging over slack and rationed cases under the stochastic discount factor, and Equation 6 of Theorem 1 is obtained by rearranging this condition.

revenues in risk-adjusted terms, conditional on the event that the market is rationed. Thus, conditional on the rationed state, expected marginal benefits are $\mathbb{E}[\Lambda AL | t \geq U] \cdot \mathrm{d}p$.

Suppose now that the market is slack. In this case, the firm operates on its demand curve, and so its revenue is given by $p \times zp^{-\eta}$. By raising the price, the firm's first-order change in revenue is $\mathrm{d}p \cdot q^D + p \cdot \mathrm{d}q^D$, which is readily calculated as $(1-\eta)zp^{-\eta} \cdot \mathrm{d}p$ (Panel C of Figure 1). Moreover, this state occurs when $t \leq U$. Thus, conditional on the slack state, expected marginal losses are $(\eta - 1)\mathbb{E}[\Lambda z p^{-\eta} | t \leq U] \cdot \mathrm{d}p$.

We finally observe that the knife-edge case of Walrasian equilibrium, or perfect balance of demand and supply, has probability zero since all random variables are drawn from a smooth distribution. Thus, at the optimal price, the expected losses from raising prices when the market is slack must be equal to the firm's expected gains from raising prices whenever the market is rationed, multiplied by the respective probability of being in each state:

$$\underbrace{(\eta - 1)\mathbb{E}[\Lambda z p^{-\eta} | t < U]}_{\text{Revenue losses when slack}} \times \underbrace{(1 - \mathbb{P}[t \geq U])}_{\text{Slack probability}} = \underbrace{\mathbb{E}[\Lambda AL | t \geq U]}_{\text{Revenue gains when rationed}} \times \underbrace{\mathbb{P}[t \geq U]}_{\text{Rationing probability}} \tag{8}$$

Rearranging this equation then yields Equation 6 exactly.

Prices and Markups without the Invisible Hand. We now turn to understanding Equation 7, which characterizes the firm's optimal price. This pricing condition is, paradoxically, obtained from the firm's first-order condition with respect to *inputs*. To see this, note that increasing inputs always has a cost, proportional to wages, but only sometimes has a benefit. If the market is slack (*i.e.,* goods are oversupplied), then extra production has no benefit. If the market is rationed (*i.e.,* goods are undersupplied), then the firm can sell additional units at its chosen price. Thus, the expected gains from marginally increasing inputs are given by $\mathbb{E}[\Lambda A p | t \geq U] \cdot \mathbb{P}[t \geq U] + 0 \cdot \mathbb{P}[t < U]$, while the expected losses are given by $\mathbb{E}[\Lambda w]$. At the optimum, these expected losses and gains must be equated. Re-arranging this expression yields Equation 7: the price is chosen such that the firm is locally indifferent about employing more or less inputs.

This pricing condition suggests a direct connection between the rationing probability and the markup. In particular, the condition can be rewritten in terms of the probability that demand is rationed, $r = \mathbb{P}\left[t \geq U\right]$:

$$p^* = \frac{1}{r} \frac{\mathbb{E}[\Lambda w]}{\mathbb{E}[\Lambda A | t^* \geq U]} \tag{9}$$

Thus, optimal pricing corresponds to placing a markup of the inverse rationing probability on expected (risk-adjusted) marginal costs, where the expectation must additionally correct for the inference that when demand is rationed, productivity is likely to be low and marginal costs are likely to be high. In the natural but special case in which the stochastic discount factor is uncorrelated with demand and the firm's productivity is known, the firm's price is simply a markup of $1/r$ on its expected marginal costs: $p^* = \frac{1}{r}\text{EMC}$, where $\text{EMC} = \frac{\mathbb{E}[w]}{A}$.

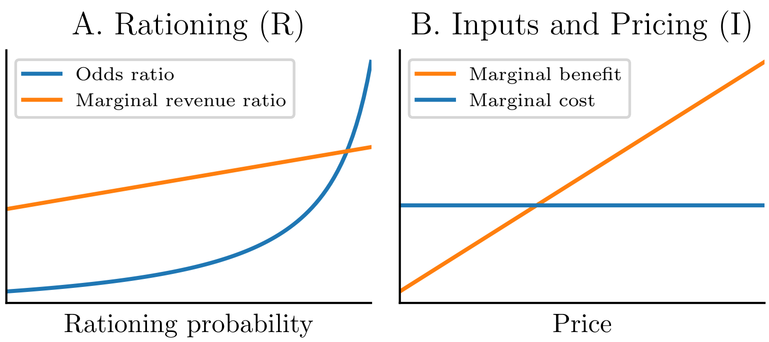
Conspicuously missing is any *direct* role for the elasticity of demand. That is, when pricing occurs outside Walrasian equilibrium, pricing decisions are decoupled from the well-known Lerner (1934) formula in which markups are solely determined by the elasticity of demand at the ultimate market-clearing quantities. We will shortly discover that a version of the Lerner (1934) formula is recovered in the limit with no uncertainty (*i.e.,* under which the market will indeed be in Walrasian equilibrium) and characterize how the presence of uncertainty, which creates the potential for rationing and slack, affects prices and markups.

Using the Model: Block-Recursivity. An important property of this model revealed by Theorem 1 is the following *block recursivity.* First, one can solve for optimal market tightness *or* the probability of rationing, as each is a monotone transformation of the other. In particular, we can write tightness as $T(r)$ where $T : (0, 1) \to \mathbb{R}$ is an increasing function.[3] Next, one can solve for the optimal price (and, moreover, labor input). This can be

---

[3] Since rationing occurs if and only if $t \geq U$, the probability of rationing is $r = R(t) := \Phi((\ln t - \mu_U)/\sigma_U)$. Moreover, under our maintained assumptions, R is a smooth, increasing function such that $\lim_{t \to -\infty} R(t) = 0$ and $\lim_{t \to \infty} R(t) = 1$; and it has a well-defined inverse function which we define as $t = T(r)$.

Figure 2: The Block Recursive Structure



Note: This figure shows an illustrative version of the system of equations (R) and (I). Panel A visualizes the determination of the rationing probability, where the blue curve is the left-hand side of Equation R and the orange curve is the right-hand side. Panel B visualizes the determination of prices, where the blue curve is the left-hand side of Equation I (marginal cost of expanding inputs) and the orange curve is the right-hand side (marginal benefit of expanding inputs).

represented in the following system of equations:

$$\frac{r}{1-r} = (\eta - 1)T(r)\frac{\mathbb{E}[\Lambda z | T(r) \leq U]}{\mathbb{E}[\Lambda A | T(r) \geq U]} \tag{R}$$

$$\mathbb{E}[\Lambda w] = r\mathbb{E}[\Lambda A \mid T(r) \geq U]p \tag{I}$$

Condition (R), simply a transformation of Equation 8, can be understood as equating the odds ratio of rationing to the (absolute value) of the firms' marginal revenues in each state. We will refer to the right-hand side of this condition as the *marginal revenue ratio*. Condition (I), simply a re-arrangement of Equation 9, represents the optimality of the firm's input choice. Crucially, the left-hand side (the marginal cost of hiring workers) is invariant to the rationing probability. We graphically illustrate this representation in Figure 2; later, we will show its utility for understanding comparative statics.

Closed-Form Characterization. We finally observe that the expectations in Theorem 1 can be evaluated analytically given our assumption that the underlying random variables are jointly log-normal. Doing this reveals that the logarithm of the optimal supply-demand imbalance is the solution to a specific fixed-point equation. In the proof of Theorem 1, we show that this fixed-point equation admits a unique solution. Moreover, the optimal price can be recovered easily once t^* is known. We summarize this below in the following corollary.

Corollary 1. *The optimal tightness t^* is the unique solution of:*

$$\ln t = \ln\left(\frac{1}{\eta-1}\right)+\mu_A-\mu_z+\frac{1}{2}\left(\sigma_A^2-\sigma_z^2+2\sigma_{\Lambda,A}-2\sigma_{\Lambda,z}\right)+\ln\frac{\Phi\left(\frac{\ln t-\mu_U}{\sigma_U}-\beta^S\sigma_U\right)}{1-\Phi\left(\frac{\ln t-\mu_U}{\sigma_U}-\beta^D\sigma_U\right)} \tag{10}$$

where $\beta^S=\frac{\mathbb{C}[\ln\Lambda+\ln A,\ln U]}{\mathbb{V}[\ln U]}$, *and* $\beta^D=\frac{\mathbb{C}[\ln\Lambda+\ln z,\ln U]}{\mathbb{V}[\ln U]}$. *Moreover, the optimal price is given by:*

$$\ln p^* = \mu_w-\mu_A+\frac{1}{2}\left(\sigma_w^2-\sigma_A^2+2\sigma_{\Lambda,w}-2\sigma_{\Lambda,A}\right)-\ln\Phi\left(\frac{\ln t^*-\mu_U}{\sigma_U}-\beta^S\sigma_U\right) \tag{11}$$

Proof. These expressions follow from derivations in Appendix A.1. ☐

Using the log-normality assumption, the inferences that the firm draws about demand when the market is slack and the firm draws about productivity when the market is rationed are compactly summarized by the ordinary least squares regression coefficients β^D and β^S, respectively. Concretely, β^D captures how much lower demand is when the market is in a state of slack, while β^S captures how much lower productivity is when the market is rationed. Finally, observe that both of these coefficients cannot be identically zero because of the identity that $\beta^S-\beta^D=1$.

Moreover, the general logic that the firm's price is a markup of the inverse rationing probability over the inference-corrected expected marginal cost is exactly captured by the final term in Equation 11. The logic of the previous discussion allows us to express the resulting optimal markup as:

$$\mathcal{M}=\frac{1}{\Phi\left(\frac{\ln t^*-\mu_U}{\sigma_U}-\beta^S\sigma_U\right)}=\frac{1}{\Phi\left(\Phi^{-1}(r)-\beta^S\sigma_U\right)} \tag{12}$$

Which simply collapses to the inverse rationing probability when $\beta^S=0$, *i.e.,* when being in a state of rationing is uninformative about SDF-weighted productivity. More generally, in the natural case that $\beta^S>0$, we can see that the optimal markup is larger than the inverse rationing probability. Intuitively, rationing is associated with high marginal costs, and so the markup must be adjusted to account for the possibility of higher marginal costs in states in which rationing occurs.

2.3 The Effects of Uncertainty

We now study the effects of uncertainty on firm behavior in this setting. To do this, we consider the random vector and we rescale the volatility of each variable about which firms are uncertain $(\ln \Lambda, \ln A, \ln z, \ln w)$ by $\delta \geq 0$.[4] Thus, δ scales the total uncertainty faced by the firm while preserving the covariance structure of the shocks.

The Complete Information Limit. The next result shows that the complete information limit corresponds to nothing more than the benchmark model of conduct by a monopolistic firm: set a price equal to a taste-based markup of $\frac{\eta}{\eta-1}$ over nominal marginal costs.

Proposition 1. *In the complete information limit, we have that:*

1. *Prices are a constant taste-based markup over nominal marginal costs:*

$$p^{FI} \equiv \lim_{\delta \to 0} p^* = \frac{\eta}{\eta - 1} \exp\{\mu_w - \mu_A\} \tag{13}$$

2. *Labor converges to its full information value:*

$$L^{FI} \equiv \lim_{\delta \to 0} L^* = \left(\frac{\eta}{\eta - 1}\right)^{-\eta} \exp\{\mu_z + (\eta - 1)\mu_A - \eta\mu_w\} \tag{14}$$

3. *The supply-demand imbalance in choices is the fundamental supply-demand imbalance:*

$$t^{FI} \equiv \lim_{\delta \to 0} t^* = \exp\{\mu_A - \mu_z\} \tag{15}$$

4. *The rationing probability $r = \mathbb{P}[q^d \geq q^s]$ is the inverse taste-based markup:*

$$\lim_{\delta \to 0} r^* = \frac{\eta - 1}{\eta} \tag{16}$$

Proof. See Appendix A.2 □

Statements 1 and 2 imply that, as uncertainty approaches zero, the resulting prices and quantities coincide with those in a standard monopolistic benchmark without uncertainty, where firms set prices and then produce to satisfy demand. Specifically, Statement 1 indicates that prices reduce to the usual taste-based markup over nominal marginal costs (while Statement 2 indicates that labor converges to the corresponding full information value). Statement 3 establishes that any gaps between demand and supply vanish in this limit. Consequently, in the zero-uncertainty limit, the framework yields an underpinning for markets in Walrasian equilibrium: one can simply view firms as choosing prices and having quantities determined by market clearing (and vice versa). Moreover, this claim makes clear

---

[4] That is, collapsing $\theta = (\ln \Lambda, \ln A, \ln z, \ln w)$, we consider a family of random vectors $\theta_\delta = \mu_\theta + \delta \Sigma^{\frac{1}{2}} Z$, where $Z \sim N(0, I)$ is a four-dimensional standard Gaussian noise vector.

that any deviations of prices or quantities from their Walrasian levels must stem from the existence of uncertainty.

While Statements 1 and 2 show that *allocations* converge to the standard monopolistic benchmark, Statement 4 shows that the rationing probability is non-zero and converges to the inverse of the taste-based markup. Intuitively, small (but non-zero) uncertainty implies that demand may still be higher than what the firm anticipates *ex ante*, leading to rationed markets. This result also shows that even small levels of uncertainty lead to different consequences for rationing probabilities compared to the no-uncertainty case, where markets are either always rationed or always slack (Barro and Grossman, 1971).

The following derivation directly shows why the rationing probability equals the inverse taste-based markup. Consider again the firm's marginal incentives to raise prices (Equation 8 and Figure 1). When uncertainty is vanishingly small, sales in slack and rationed markets are known and equal to each other, canceling out on both sides of Equation 8. Therefore, the firm equalizes the revenue elasticity in each state weighted by the rationing probabilities:

$$\underbrace{(\eta - 1)}_{\text{Revenue elast. when slack}} \times \underbrace{(1 - \mathbb{P}[t \geq U])}_{\text{Slack probability}} = \underbrace{1}_{\text{Revenue elast. when rationed}} \times \underbrace{\mathbb{P}[t \geq U]}_{\text{Rationing probability}} \tag{17}$$

Re-arranging this and defining $r = \mathbb{P}[t \geq U]$ gives the result. Moreover, as the market becomes perfectly competitive ($\eta \to \infty$), the rationing probability converges to one. Intuitively, more severe competition raises the losses from higher prices in the slack state while leaving constant the gains from higher prices in the rationed state; to balance incentives, the firm drives the rationing probability to one. The opposite logic implies that as monopoly power becomes very strong ($\eta \to 1$), the rationing probability converges to zero.

Thus, the general logic of our model collapses to the standard logic of the Lerner (1934) markup formula as uncertainty vanishes. However, in the presence of uncertainty, our analysis has demonstrated a more general principle: markups correspond (roughly) to the inverse of the probability that the firm rations demand. Moreover, while the demand elasticity is one component of this probability, it does not tell the whole story. For that, we must consider the effects of uncertainty.

The First-Order Effects of Volatility. Having understood that uncertainty is the core element of this theory's departure from standard Walrasian equilibrium, we now characterize how the presence of volatility affects optimal behavior to first-order. Mathematically, we do this by characterizing the first-order effects of scaling volatility by $\delta \geq 0$ on choices around the point at which there is no uncertainty ($\delta = 0$). We use the language of volatility to emphasize that changes in standard deviations can have first-order effects on choices (as opposed to variances having first-order effects on choices).

The following proposition derives the first-order effects of volatility on the firm's optimal price, labor input, and rationing probability.

Proposition 2. *Up to a first-order approximation in volatility, the firm's optimal price and labor inputs are given by:*

$$\begin{aligned} \ln p^* &= \ln p^{FI} + \left(\phi\left(\kappa\right) - \frac{\kappa}{\eta}\right)\sigma_U \\ \ln L^* &= \ln L^{FI} - \eta\phi(\kappa)\sigma_U \end{aligned} \tag{18}$$

where $\kappa = \Phi^{-1}\left(\frac{\eta-1}{\eta}\right)$*. Moreover, the probability with which demand is rationed is given, up to a first-order approximation in volatility, as:*

$$r = \frac{\eta-1}{\eta}\left[1 + \left(\frac{\kappa}{\eta} + \left(\frac{\beta^S}{\eta-1} + \beta^D\right)\phi(\kappa)\right)\sigma_U\right] \tag{19}$$

Proof. See Appendix A.3. □

The basic reason why uncertainty has first-order effects was highlighted earlier: the firm's payoff has a kink at the point where supply meets demand. We next explain the intuition for the exact form of this effect.

How Uncertainty Affects Rationing. To build intuition, we assume momentarily that productivity and the stochastic discount factor are constant, which implies that supply-demand imbalances are driven solely by demand z (*i.e.*, $\sigma_A = 0$, $\sigma_\Lambda = 0$, and $\sigma_U = \sigma_z$). Let us begin by examining the first-order effects on the rationing probability given in Equation 19. This effect consists of two terms, which we discuss in sequence.

The first term, κ/η is the mechanical effect by which increasing volatility affects the marginal revenue ratio. Absent a response in firm tightness, increasing volatility is a force that makes rationing and slack states equiprobable. Hence, if the rationing probability in the zero-uncertainty limit is less than one-half (which corresponds to $\kappa < 0$), then a slight increase in volatility lowers the marginal revenue ratio because it makes slack states less likely. From equation R, firm optimality requires that the rationing probability decrease. Conversely, if the rationing probability in the zero-uncertainty is greater than one-half, then this term is positive.

The second term captures the previously discussed inferences that the firm draws about (risk-weighted) demand and productivity when the market is slack and rationed, respectively. Under our temporary simplification that productivity and the SDF are constant, this implies that $\beta^S = 0$ and $\beta^D = -1$. In other words, the firm understands that demand is low in states in which the market is slack. This incentivizes the firm to lower its tightness in order to insure itself against these low-demand realizations. In turn, this lowers the probability of rationing: the change in tightness t due to these inferences moves the rationing probability by $-\phi(\kappa)$, which represents the prior probability of being at the slack-rationing threshold. The total effect on the rationing probability is thus proportional to $\kappa/\eta - \phi(\kappa)$.

From Rationing to Prices and Labor. Having understood changes in the rationing probability, interpreting Equation 18 is straightforward. Recall from Equation 9 that the optimal price is a markup of the inverse rationing probability on marginal costs. This is because the benefit of hiring an additional unit of labor only accrues when the market is rationed (and depends on the price charged, while costs are always paid). It follows that, absent additional effects on inference about productivity, the price charged moves in opposite directions to the rationing probability, with a first-order effect given by $\phi(\kappa) - \kappa/\eta$. Finally, the term $\eta\phi(\kappa)$ in the labor Equation can then be understood as the "residual" that is needed to ensure that Equation 19 holds. Concretely, note that for a 1% change in prices, the corresponding market tightness moves by $\eta > 1\%$. Thus, labor must also adjust to ensure that the change in prices is consistent with the change in the rationing probability. This intuition remains unchanged when productivity and the SDF are also allowed to vary, with the exception that the firm understands that states in which the market is rationed are associated with relatively low productivity. This inference effect is captured by β^S in Equation 19.

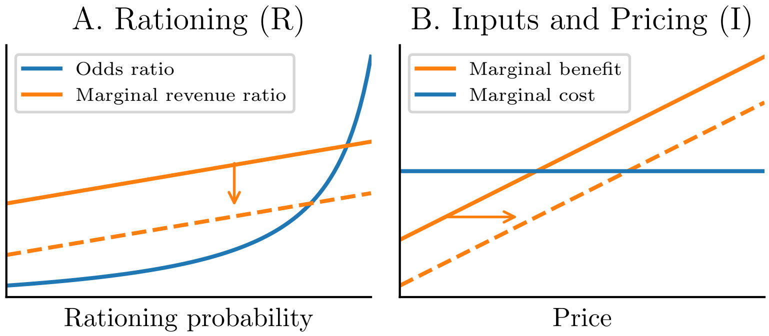
Firms Shrink When They are Uncertain. Although the first-order impact of volatility on rationing can be either positive or negative—depending on the elasticity of demand and the strength of the inference effects just described—the sign of its impact on prices and labor is unambiguous. Concretely, in response to an increase in volatility, the firm always raises its prices and reduces the amount of labor it hires. We formalize this with the following.

Corollary 2. *The presence of imperfect information reduces hiring,* $\ln L < \ln L^{FI}$*, and increases prices,* $\ln p > \ln p^{FI}$*.*

Proof. See Appendix A.4. □

Why does volatility make the firm "shrink"? Observe that the firm can increase tightness either by reducing prices or by reducing its input quantities. Consider again the case in which productivity and the SDF are constant. In this case, the firm wants to unambiguously reduce the probability that the market is rationed. Intuitively, demand matters for the firm only insofar as the market is slack. If demand becomes more "spread out," holding fixed tightness,

Figure 3: Visualizing the Response to an Uncertainty Shock



Note: This figure illustrates the comparative statics of the firm's response to an uncertainty shock. In particular, we consider a case in which demand uncertainty increases (given zero uncertainty about productivity or the stochastic discount factor), in the neighborhood of zero uncertainty Panel A visualizes the determination of the rationing probability, where the blue curve is the left-hand side of Equation R and the orange curve is the right-hand side. Panel B visualizes the determination of prices, where the blue curve is the left-hand side of Equation I (marginal cost of expanding inputs) and the orange curve is the right-hand side (marginal benefit of expanding inputs). In both Panels, the dashed line indicates the effect of the shock.

then the firm is only exposed to the associated *downside* risk of having a slack market when demand is low. The firm instead wishes to expose itself to the *upside* of these demand movements by increasing the probability that the market is slack.

The corresponding change in tightness can be mediated either by increasing prices or by increasing labor. Suppose, counterfactually, that this change were mediated by increasing labor and that prices remained constant. Given this new tightness, this would imply that the marginal profit in hiring a unit of labor would decline: the firm always pays the wage, while an additional unit of labor only increases revenues whenever the market is rationed. Since prices are assumed constant and the rationing probability declines, hiring labor at the margin is unprofitable. The firm must therefore choose to raise prices. But any change in prices moves the rationing probability more than one-to-one, precisely because the elasticity of demand is larger than unity. It follows that the firm "shrinks" in response to volatility: labor purchases decline so that the firm is insured from low demand realizations, and prices increase so that labor purchases remain profitable.

We graphically illustrate the effects of volatility on firm decisions using the block-recursive structure in Figure 3. In particular, we consider an increase in demand uncertainty in the simplified scenario, which shuts down uncertainty about the stochastic discount factor and about productivity. The shock reduces the marginal revenue ratio, leading to a decline in the desired rationing probability (Panel A). This translates to a global decline in the marginal benefit of hiring labor for any fixed price; equalizing this marginal benefit to the fixed marginal costs yields a higher price (Panel B).

Uncertainty-Based Markups. Earlier, we remarked that markups are decoupled from the canonical Lerner (1934) formula in the disequilibrium model (see Equation 9). We can now more precisely describe how uncertainty affects this phenomenon, using our first-order characterization of how the firm's decisions respond to uncertainty. In this case, markups $\mathcal{M}$ can be decomposed into a taste-based component and their uncertainty-based component, which we have shown to be strictly positive:

$$\ln \mathcal{M} = \underbrace{\ln\left(\frac{\eta}{\eta - 1}\right)}_{\text{Taste-Based Markup}} + \underbrace{\left(\phi(\kappa) - \frac{\kappa}{\eta}\right)\sigma_U}_{\text{Uncertainty-Based Markup}} \tag{20}$$

This has at least two concrete implications. First, our model can rationalize differences between markups estimated via the two most standard approaches in economics: measuring the elasticity of demand on the "demand side" and combining this with assumptions about how firms compete (*e.g.*, Bresnahan, 1989; Berry et al., 1995), or measuring marginal costs on the "supply side" given suitable assumptions about production technology and/or input markets (*e.g.*, De Loecker and Warzynski, 2012; De Loecker et al., 2020). For example, studies applying the production approach to public firms in the US typically find markups in the range of 1.2-1.6 (*e.g.*, De Loecker et al., 2020; Demirer, 2025). By contrast, studies applying the demand approach to narrowly defined sectors or retail goods in the US find that the majority of markets have demand elasticities above 8 (*e.g.*, Broda and Weinstein, 2006; Hottman et al., 2016). Under our model, the standard translation from demand elasticity to markup would be incorrect *even* under correctly specifying the model of monopolistic competition. The bias would be in the direction of under-stating markups with the demand approach due to neglecting the uncertainty-based markup.

Second, this framework can provide a rationale for the contribution of markup shocks to business cycle dynamics. For instance, Smets and Wouters (2007) argue that the majority of short-run variation in inflation, as well as a significant fraction of variation in GDP, is driven by shocks to markups. It may be hard to rationalize these fluctuations via changes in the elasticity of demand, which is presumably a slow-moving object related to consumer preferences. On the other hand, idiosyncratic risk at the firm level is known to be highly cyclical (Bloom et al., 2018). In our framework, movements in uncertainty generate markup fluctuations without requiring implausibly fast changes in underlying demand elasticities.

2.4 Extensions: Matching, Inventories, and Multiple Markets

Before embarking on our general equilibrium analysis, we first highlight three extensions of our framework to richer environments.

Matching: The Qualitative Importance of Efficiency. Uncertainty in our model has large effects, in the sense that decision problems depend on volatility to first-order. To understand what drives this result, it is instructive to extend the model to imagine that realized sales are determined by a reduced-form matching function $m : \mathbb{R}_+^2 \to \mathbb{R}_+$ such that $q = m(q^s, q^d)$ (for a review of the matching literature in goods markets, see Petrongolo and Pissarides, 2001). Some macroeconomic studies have explored the implications of such matching technologies for goods markets (*e.g.*, Michaillat and Saez, 2022; Bianchi et al., 2024; Ghassibe and Zanetti, 2022; Bai et al., 2025).[5] The principle of voluntary exchange requires that any matching function satisfy $m(x, y) \leq \min\{x, y\}$. However, if exchange is inefficient, it can be the case that $m(x, y) < \min\{x, y\}$. Thus, the qualitative property of efficiency generates a kink in the matching function and thereby the firm's profit function.

We now consider a "smooth" matching function to show that our model differs qualitatively from one in which there is *inefficient* matching. As before, we will scale the volatility that the firm perceives by δ. To define what we mean by smooth, observe that the firm's objective is now:

$$\tilde{J}(p, L; \delta) = \mathbb{E}[\Lambda \left(pm(AL, zp^{-\eta}) - wL \right)] \tag{21}$$

We say that the matching function is smooth if $\tilde{J}$ is three times continuously differentiable and for all $\delta \geq 0$ there exists a unique optimal $(p^*, L^*) \gg 0$ that solves the first-order conditions $0 = \tilde{J}_p(p^*, L^*; \delta) = \tilde{J}_L(p^*, L^*; \delta)$ and the second-order condition that the Hessian of $\tilde{J}$ is negative definite at this solution.

Proposition 3. *If the matching function is smooth, then, up to a first-order approximation in volatility:*

$$p^* = p^{FI} \quad \text{and} \quad L^* = L^{FI} \tag{22}$$

Proof. See Appendix A.5 $\square$

As efficient exchange rules out the smoothness of the matching function, this result highlights that the efficiency of exchange is the qualitative economic property that generates the first-order effects of volatility. At a more technical level, efficiency generates a kink in the firm's profit function, which gives rise to a failure of first-order conditions at $\delta = 0$ to characterize the optimum. Because first-order conditions do not characterize the limit

---

[5] Bianchi et al. (2024) discuss how smooth matching functions relate to efficient matching functions in Section 2.3 of their paper, but use a smooth matching function for their main analysis.

optimum, the standard principle that volatility has second-order effects on behavior does not hold.

Inventories. In this simple model, unsold production has no value to the firm. For services, this is a reasonable assumption: if a hairdresser sits in the salon and nobody shows up, there is no way to recoup the lost time. Moreover, for perishable goods, such as food, a similar principle applies. However, for durable goods, there is naturally a non-zero value of unsold production. To model this value, let us suppose abstractly that each unsold unit of production becomes inventory $I = \max\{AL - zp^{-\eta}, 0\}$, each unit of which has future value $\Lambda' \in \mathbb{R}_{++}$, where Λ' is jointly log-normal with the other random variables that enter the producer's problem. We assume that the retention value of inventory is such that it would be better not to produce today than to produce fully for inventory (in expectation), *i.e.,* $\mathbb{E}[\Lambda' A] < \mathbb{E}[\Lambda w]$. The following proposition characterizes the salient properties of the model when augmented with inventories:

Proposition 4. *If the firm's behavior is optimal, then tightness must solve the following fixed-point equation:*

$$t = \frac{\mathbb{E}[\Lambda A \mathbb{I}[t \geq U]]}{(\eta - 1)\mathbb{E}[\Lambda z \mathbb{I}[t \leq U]] - \eta \mathbb{E}[\Lambda' z \mathbb{I}[t \leq U]] \frac{\mathbb{E}[\Lambda A \mathbb{I}[t \geq U]]}{\mathbb{E}[\Lambda w] - \mathbb{E}[\Lambda' A \mathbb{I}[t \leq U]]}} \tag{23}$$

Moreover, given a solution for tightness t^*, any optimal price must be given by:

$$p^* = \frac{\mathbb{E}[\Lambda w] - \mathbb{E}[\Lambda' A \mathbb{I}[t^* \leq U]]}{\mathbb{E}[\Lambda A \mathbb{I}[t^* \geq U]]} \tag{24}$$

and any optimal labor input quantity must be given by $L^* = p^{*-\eta}/t^*$. Moreover, we have that the rationing probability converges in the zero uncertainty limit to:

$$r_I = \frac{\eta - 1}{\eta + \frac{1}{\exp\{(\mu_\Lambda + \mu_w) - (\mu_{\Lambda'} + \mu_A)\} - 1}} \tag{25}$$

and, up to a first-order approximation in volatility, that:

$$\ln t = \mu_U + \kappa_I \sigma_U \tag{26}$$

where $\kappa_I = \Phi^{-1}(r_I)$.

Proof. See Appendix A.6. □

Intuitively, the presence of inventories leads the firm to be less averse to over-production. For this reason, the presence of inventories decreases the rationing probability $r^I < \frac{\eta-1}{\eta}$ and also reduces the responsiveness of tightness to volatility $\kappa_I < \kappa$. However, the qualitative properties of the model remain much the same: rationing probabilities govern markups, and volatility continues to have first-order effects.[6]

Walrasian Disequilibrium in Multiple Markets. In our baseline model, we relaxed the imposition of market-clearing in the goods market. However, for analytical simplicity, we maintained market-clearing in the labor market. In this extension, we show how to allow for disequilibrium in both the product and labor markets within our framework, in the spirit of Barro and Grossman (1971) and Michaillat and Saez (2015).

We augment our baseline model, in which the firm took the wage as given, by supposing that the firm must also set wages $w \in \mathbb{R}_+$ alongside the price p and labor demand L^d. When the firm sets a wage w, it faces the labor supply curve $L^s = vw^\chi$, where $\chi \geq 0$ is the elasticity of labor supply and $v \in \mathbb{R}_{++}$ is a labor supply shifter. We assume that the variables (Λ, A, z, v) are jointly log-normally distributed. We impose the same voluntary and efficient exchange assumptions on the labor market, so the realized labor quantity is $L = \min\{L^s, L^d\}$. The firm now solves the following problem:

$$\sup_{p,w,L^d} \mathbb{E}\left[\Lambda(pq - wL)\right] \tag{27}$$

subject to constraints that describe the state of both the product and labor markets.

The two linked markets could be in one of four states, ignoring the knife-edge cases of Walrasian equilibrium. The labor market is rationed if and only if $t^L > v$, where we introduce the new variable $t^L = \frac{L^d}{w^\chi}$ to represent labor-market tightness or the imbalance between labor demand and supply. The product market is rationed if demand exceeds production, but the precise condition under which this occurs depends on the state of the labor market. If the labor market is slack, the condition remains $t = \frac{p^{-\eta}}{L^d} > A/z$. If the labor market is rationed, then labor is supply-determined, and the relevant condition is $\frac{p^{-\eta}}{L^s} > A/z$, or $t \cdot t^L > Av/z$. We summarize this by defining the indicator functions

$$R^L = \mathbb{I}[\{t^L > v\}],\ R^P = \mathbb{I}[\{t^L < v, t > U\} \cup \{t^L > v, t \cdot t^L > Uv\}] \tag{28}$$

to denote the states in which the labor and product markets, respectively, are rationed.

The solution to the firm's problem can be characterized by a pair of fixed-point equations that describe the firm's optimal desired tightness in both markets:

---

[6] The first-order expansions for prices and labor inputs, although not reported for brevity, can be readily computed by combining the arguments in Propositions 2 and 4.

Proposition 5. *Any optimal desired tightness in the product and labor markets solves the following system of equations:*

$$\begin{aligned} t &= \frac{1}{\eta - 1}\frac{\mathbb{E}[\Lambda A(1-R^L)R^P] + \frac{1}{t^L}\mathbb{E}[\Lambda A v R^L R^P]}{\mathbb{E}[\Lambda z(1-R^P)]} \\ t^L &= \chi \frac{\mathbb{E}[\Lambda A v R^L R^P]}{\mathbb{E}[\Lambda A(1-R^L)R^P]} - (1+\chi)\frac{\mathbb{E}[\Lambda v R^L]}{\mathbb{E}[\Lambda(1-R^L)]} \end{aligned} \tag{29}$$

Moreover, the firm's optimal price is

$$p = w\frac{\mathbb{E}[\Lambda(1-R^L)]}{\mathbb{E}[\Lambda A(1-R^L)R^P]} \tag{30}$$

Proof. See Appendix A.7. □

The core economics of Equation 29 come from the producer's local indifference about changing prices and posted wages. The logic of the former is unchanged from before (see Theorem 1 and Figure 1), with only one twist: when the product market is rationed, the marginal revenue from raising prices could be proportional either to labor demand (slack labor markets) or labor supply (rationed labor markets). The logic of the latter is new to the two-market analysis and reflects three forces. If labor markets are slack, raising wages increases costs in proportion to labor demand. If labor markets are rationed, they increase costs by both raising inframarginal wages and moving up the labor supply curve. And if product markets are *also* rationed, then increasing wages leads to additional marginal production and sales.

Like its twin in Theorem 1, this characterization opens the door for uncertainty to have first-order effects on outcomes. A new implication, relative to the earlier analysis, is that uncertainty may have first-order effects on the excess supply of labor, which can be interpreted as involuntary unemployment.

We close by describing the condition relating wages and prices (Equation 30) and describing its relationship in terms of markups and markdowns. This condition is derived from the firm's indifference condition for labor demand, observing that this choice affects payoffs *only* if the labor market is slack. For intuition, it is easiest to consider the case in which Λ and A are constant, in which case

$$p = \frac{1}{r^C}\frac{w}{A} \geq \frac{w}{A}, \qquad w = r^C pA \leq pA \tag{31}$$

where $r^C \in [0,1]$ is the *conditional* probability that the product market is rationed given a slack labor market. These formulas imply that both markups and markdowns depend on the probability of rationing in both markets. This, in turn, opens the possibility that both depend (to the first-order) on firms' uncertainty.

3 A Static General Equilibrium Model

We now embed this partial equilibrium model in a general equilibrium setting to understand how uncertainty shocks and monetary shocks propagate. The goal of this analysis is to consider the simplest possible and most standard general equilibrium closure to illustrate the new economic mechanisms that emerge from this model of firm decision-making. For this reason, we largely follow the microfoundations of Blanchard and Kiyotaki (1987).

3.1 Model: The Household's Problem

A representative household has constant elasticity of substitution preferences over a continuum of consumption varieties, indexed by $i \in [0,1]$:

$$C = \left(\int \theta_i^{\frac{1}{\eta}} c_i^{\frac{\eta-1}{\eta}} \, \mathrm{d}i \right)^{\frac{\eta}{\eta-1}} \tag{32}$$

where $\eta > 1$ is the elasticity of substitution between varieties and θ_i is a variety-specific taste shock, which we assume to be IID log-normal with mean μ_θ and variance σ_θ^2. The household has Golosov and Lucas (2007) preferences over the consumption aggregate, money, and labor supply given by:

$$\ln C + \ln \left(\frac{M}{P} \right) - \alpha N \tag{33}$$

where $\alpha > 0$ indexes disutility from labor, and $P = (\int p_i c_i \, \mathrm{d}i)/C$ is the ideal price index. The household supplies labor to the firms, receives the profits from the firms, and receives transfers from the government. The household faces the following budget constraint:

$$PC + M = wN + \Pi + T \tag{34}$$

where T is a monetary transfer from the government and Π are aggregate profits. We assume that the household chooses its labor supply at the same time that firms choose their labor inputs. The household chooses its level of consumption after the firms have produced.

3.2 Optimal Household Behavior

We first consider the household's problem of how much to consume of each consumption variety, given a fixed level of expenditure E. Critically, the household cannot consume more of any variety than whatever level of the variety has actually been produced, $\bar{c}_i$. Thus, the household faces the following optimization problem:

$$\max_{\{c_i\}_{i\in[0,1]}} \left(\int \theta_i^{\frac{1}{\eta}} c_i^{\frac{\eta-1}{\eta}}\,\mathrm{d}i\right)^{\frac{\eta}{\eta-1}} \quad \text{s.t.} \quad \int p_i c_i\,\mathrm{d}i = E, \quad c_i \leq \bar{c}_i \ \text{ for all}\, i \in [0,1] \tag{35}$$

To solve this problem, we define for each good i the statistic:

$$\iota_i = \frac{\frac{p_i^{-\eta}}{L_i}}{\frac{A_i}{\theta_i}} = \frac{t_i}{U_i} \tag{36}$$

Intuitively, this statistic represents the ultimate imbalance between supply and demand for each variety. Without loss of generality, we re-index i such that i is ordered by ι_i. That is, $i < j$ if and only if $\iota_i < \iota_j$. The following result characterizes household demand for varieties in this setting.

Proposition 6. *There exists a unique index threshold $k \in [0,1]$ such that:*

1. *For all varieties $j \in [0,k)$, the household's demand is given by:*

$$c_j = \frac{E - \int_k^1 p_i\bar{c}_i \mathrm{d}i}{\int_0^k \theta_i p_i^{1-\eta}\mathrm{d}i}\theta_j p_j^{-\eta} \tag{37}$$

2. *For all varieties $j \in (k,1]$, the household demands $c_j = \bar{c}_j$.*

Moreover, the threshold k is the unique solution to Equation 37 when we replace j by k. The aggregate level of consumption is given by:

$$C = \left(\left(E - \int_k^1 p_i\bar{c}_i\,\mathrm{d}i\right)^{\frac{\eta-1}{\eta}} \left(\int_0^k \theta_i p_i^{1-\eta}\,\mathrm{d}i\right)^{\frac{1}{\eta}} + \int_k^1 \theta_i^{\frac{1}{\eta}} \bar{c}_i^{\frac{\eta-1}{\eta}}\,\mathrm{d}i\right)^{\frac{\eta}{\eta-1}} \tag{38}$$

Proof. See Appendix A.8. □

The solution to this problem thus has the household start from $i = 1$, which is the most under-supplied market, by consuming everything that is produced. Thus, outcomes in this market lie on the supply curve. The household then goes to all subsequent, better-supplied markets and consumes everything that is produced until they hit a threshold market k at which they are exactly indifferent between consuming everything and just a little bit less. At this point, we have:

$$\bar{c}_k = \frac{E - \int_k^1 p_i \bar{c}_i \mathrm{d}i}{\int_0^k \theta_i p_i^{1-\eta} \mathrm{d}i} \theta_k p_k^{-\eta} \tag{39}$$

Then, for all subsequent markets, the household's demand is of the iso-elastic form:

$$c_j = z_j p_j^{-\eta} \tag{40}$$

but where:

$$z_j = D\theta_j \quad \text{with} \quad D = \frac{E - \int_k^1 p_i \bar{c}_i \mathrm{d}i}{\int_0^k \theta_i p_i^{1-\eta} \mathrm{d}i} \tag{41}$$

Thus, for all markets in which the household is not rationed, outcomes lie on the demand curve. A critical upshot of this analysis is that it generates the exact demand curves for the firm that we assumed in the partial equilibrium analysis; the amount that the firm actually sells is given by:

$$c_j = \min\{z_j p_j^{-\eta}, \bar{c}_j\} \tag{42}$$

Moreover, as $z_j = D\theta_j$, D is deterministic, and θ_j is log-normal, we also obtain that z_j is endogenously log-normally distributed (as we assumed in Section 2).

Cross-Substitution Without the Invisible Hand. Observe that in the limiting case in which all markets are slack, we recover the demand system of Dixit and Stiglitz (1977):

$$c_j = \theta_j \frac{E}{P} \left(\frac{p_j}{P}\right)^{-\eta}, \quad P = \left(\int_i \theta_i p_i^{1-\eta} \mathrm{d}i\right)^{\frac{1}{1-\eta}} \tag{43}$$

In contrast, with rationing, we may write the demand system for all slack markets as

$$c_j = \theta_j \frac{E - \int_k^1 p_i \bar{c}_i \mathrm{d}i}{P_k} \left(\frac{p_j}{P_k}\right)^{-\eta}, \quad P_k = \left(\int_0^k \theta_i p_i^{1-\eta} \mathrm{d}i\right)^{\frac{1}{1-\eta}} \tag{44}$$

Hence, relative to the Dixit and Stiglitz (1977) benchmark, the demand shifter with rationing needs to allocate all income that is not spent on markets that stock out, $E - \int_k^1 p_i \bar{c}_i \mathrm{d}i$, on the remaining markets, which feature a price index of P_k. This also reveals that increasing the tightness of the economy by lowering $\bar{c}_i$ acts as a positive demand shifter for slack markets.[7] In other words, if consumers are unable to purchase certain goods because they are rationed, this will increase the demand for goods that are available. Thus, there are

---

[7] This can be proven formally by considering a small deviation $\bar{c}_i + \delta$ for a non-zero measure of goods $i \in [k, 1]$ and differentiating D with respect to δ.

negative aggregate demand externalities from production in rationed markets on demand in slack markets. These *spillovers* link the aggregate tightness of the economy with the effective demand experienced by producers (and thus their chosen prices and quantities). Other studies have shown that the absence of market clearing in one market can modify the supply or demand of other markets, most notably in the presence of a homogeneous good and the labor market (Barro and Grossman, 1971; Bénassy, 1993). However, to the best of our knowledge, the characterization of these spillovers in the presence of imperfectly substitutable goods is novel to our analysis.

Labor Supply Decisions. Finally, we consider the household's problem of how much labor to supply. Because we have assumed Golosov and Lucas (2007) preferences, the household's first-order conditions imply that:

$$w\frac{1}{PC} = \alpha \quad \text{and} \quad \frac{1}{PC} = \frac{1}{M} \tag{45}$$

Together, these conditions imply that wages are pinned down by, and scale linearly with, money holdings:

$$w = \alpha M \tag{46}$$

Moreover, these conditions imply that nominal consumption expenditure is simply equal to money holdings:

$$E = PC = M \tag{47}$$

3.3 Equilibrium

We now state the timing assumptions and equilibrium definition that we employ. The model is split into three periods: the morning, the afternoon, and the evening. In the morning, the government announces the money supply $\bar{M}$, the firms set their prices p_i and hire labor L_i, and the household chooses its labor supply N. In the afternoon, production takes place and so production of each variety is $\bar{c}_i = A_i L_i$. In the evening, the household chooses its monetary holdings M and its consumption of each variety of the good c_i. To focus solely on the failure of market clearing in the goods markets, for our baseline analysis, we will adopt the simplification that the labor market clears in the morning ($N = \int L_i \, \mathrm{d}i$) and the money market clears in the evening ($M = T = \bar{M}$). As we have seen, it is conceptually simple to relax this assumption; we maintain it only for the simplest exposition of our modeling departure. We define an equilibrium in this setting as follows.

Definition 1 (Equilibrium)**.** *An equilibrium is a collection of variables:*

$$\{\{t_i, p_i, L_i, c_i, A_i, \theta_i, z_i\}_{i\in[0,1]}, C, M, P, E, \Pi, T, N, k, w, \bar{M}, \Lambda\} \tag{48}$$

such that:

1. *Each firm $i \in [0,1]$ chooses prices, labor, and tightness (p_i, L_i, t_i) according to Equations 6 and 7*
2. *The household chooses consumption of each variety $\{c_i\}_{i\in[0,1]}$ according to Equation 42 where the demand shifter $\{z_i\}_{i\in[0,1]}$ is given by Equation 41 and the threshold k satisfies Equation 39.*
3. *The household chooses expenditure, consumption, money balances, and labor supply (E, C, M, N) according to equations 38, 47, 46.*
4. *The money market clears, the labor market clears, Π is the aggregate profit, the price index is $P = (\int p_i c_i \,\mathrm{d}i)/C$, and $\Lambda = 1/(PC)$ is the stochastic discount factor.*

When we study an unanticipated monetary shock, we consider a scenario in which the government changes the money supply from their announcement in the morning of $\bar{M}$ to some value $\bar{M}\Delta$ in the afternoon. Unanticipated monetary shocks differ from anticipated monetary shocks in the sole respect that all decisions made in the morning (production, pricing, and labor supply) cannot respond.

4 Pricing and Production in General Equilibrium

We now combine the firm and household problem to characterize equilibrium outcomes. We use this model to study the propagation of monetary shocks and uncertainty shocks.

4.1 Characterizing Equilibrium

In general equilibrium, firms' production and pricing choices generate spillovers in other markets through effective demand. These general equilibrium interactions are captured through firms' expectations of demand when they make decisions. With some abuse of notation, we write this as $\mu_z = \ln D + \mathbb{E}[\ln\theta] = \mathbb{E}[\ln z]$.[8] Solving for equilibrium thus requires solving for this single scalar. By combining the optimality conditions for firm behavior (Proposition 2) with the optimality conditions for household behavior (Proposition 6), we can solve for equilibrium mean demand, which then allows us to solve for prices, labor, and

---

[8] The average level of perceived demand in the morning can differ from the average level of *realized* demand to the extent that realized money balances differ from their expected level, $\ln\Delta \neq 0$.

aggregate consumption in general equilibrium. The following Theorem characterizes how the first-order response of these variables to uncertainty.

Theorem 2. *Up to a first-order approximation in volatility, the unique equilibrium level of mean demand in non-rationed markets is given by:*

$$\mu_z = \ln\left[\left(\frac{\eta}{\eta-1}\frac{\alpha}{e^{\mu_A}}\right)^{\eta-1}\bar{M}^{\eta}\right] + \eta\phi(\kappa)\sigma_U \tag{49}$$

Thus, the unique equilibrium prices, labor input, and aggregate consumption are given by:

$$\ln p = \ln\left(\frac{\eta}{\eta-1}\frac{\alpha M}{e^{\mu_A}}\right) + \left(\phi\left(\kappa\right) - \frac{\kappa}{\eta}\right)\sigma_U \tag{50}$$

$$\ln L = \ln\left(\frac{\eta-1}{\eta}\frac{1}{\alpha}\right) \tag{51}$$

$$\ln C = \ln\left(\frac{\eta-1}{\eta}\frac{e^{\mu_A+\frac{\mu_\theta}{\eta-1}}}{\alpha}\right) - \left(\phi(\kappa) - \frac{\kappa}{\eta}\right)\sigma_U \tag{52}$$

Proof. See Appendix A.9. ☐

Note that as $\sigma_U \to 0$, all variables coincide with those of a standard monopolistic benchmark with market clearing. Furthermore, observe that the first-order impact of uncertainty on prices coincides with the partial equilibrium impact, as described in Proposition 2. This is because the optimal level of prices must equate the marginal benefit of hiring an additional unit of labor with its marginal cost. As discussed previously, this level depends only on the rationing probability, which is independent of the mean level of demand. Hence, prices in general equilibrium and partial equilibrium coincide.

However, the amount of labor that firms purchase differs in general equilibrium relative to partial equilibrium due to spillovers. Concretely, we know from our partial equilibrium analysis that an increase in uncertainty induces firms to raise prices and reduce their labor to first-order (holding mean demand fixed). Since prices increase, this contributes to a positive spillover via an increase in the price level of all slack markets. This can be seen via the denominator of Equation 41. Depending on the sign of the first-order response of rationed sales ($p \times L$), this creates either a positive or negative spillover because households have less money to spend on slack markets. This can be seen from the numerator of Equation 41. Overall, one can prove that the sum of these two forces is unambiguously positive and equals $\eta\phi(\kappa)$. This shift in effective demand induces the firm to purchase more labor inputs. Thus, in general equilibrium, there is no first-order impact of uncertainty on labor because the partial equilibrium effect (due to insurance) and the general equilibrium effect (due to spillovers) exactly offset each other.

A surprising implication of Theorem 2 is that although total input purchases are unresponsive to uncertainty, aggregate consumption is always declining in uncertainty to first-order. How can aggregate consumption change if aggregate production remains fixed? Observe that household expenditure must equal the total circulation of money in the economy. As prices uniformly increase in response to uncertainty, it follows that aggregate consumption must decline. This rationalizes why the impact of uncertainty on aggregate consumption is simply the negative of that of prices.

One can also see how consumption declines more directly by considering slack and rationed markets separately. For simplicity, consider the case with only productivity uncertainty ($\sigma_U = \sigma_A$) to eliminate the mechanical effect of taste shocks on the consumption aggregator. In this case, by Theorem 2, the first-order impact of uncertainty in slack markets is equal to

$$\frac{\partial}{\partial \delta}\left[\ln c_i\right] = \frac{\partial}{\partial \delta}\left[\ln \mu_z - \eta \ln p\right] = \kappa \sigma_U \tag{53}$$

The share of slack markets in the zero uncertainty limit is equal to $1/\eta$. Hence, the total impact on consumption that arises from this effect is $\kappa/\eta \times \sigma_U$.[9] Note that this term can either be positive or negative. Intuitively, if spillovers are large relative to firms' increase in prices, uncertainty can raise consumption in slack markets. We may similarly consider the first-order impact of uncertainty on the consumption of the "average" rationed markets. This is equal to

$$\frac{\partial}{\partial \delta}\left[\ln c_i\right] = \frac{\partial}{\partial \delta}\left[\ln \mathbb{E}[A|t \geq U] + \ln L\right] = -\frac{\eta}{\eta - 1}\phi(\kappa)\sigma_U \tag{54}$$

Since the share of rationed markets is $(\eta - 1)/\eta$, the total effect is $-\phi(\kappa)\sigma_U$. Therefore, uncertainty unambiguously lowers consumption in rationed markets. Intuitively, these are exactly the markets characterized by relatively low productivity. As a result, an increase in uncertainty further lowers their average productivity and, by extension, the average level of consumption of these goods. In addition, firms that experience more favorable productivity shocks are unable to adjust prices to incentivize consumers to buy more of their products. Consequently, these positive productivity realizations are effectively "wasted," leading to a reduction in aggregate consumption in response to an increase in uncertainty.

---

[9] Because consumption is continuous as one crosses the rationing threshold, changes in the rationing probability do not affect aggregate consumption to first-order.

4.2 The Effects of Uncertainty Shocks

An implication of this analysis is that uncertainty shocks propagate as markup shocks to prices and consumption. Concretely, observe that we may write prices and consumption as

$$\ln p = \underbrace{\ln\left(\frac{\alpha M}{e^{\mu_A}}\right)}_{\text{efficient prices}} + \underbrace{\ln \mathcal{M}}_{\text{markup}} \quad \text{and} \quad \ln C = \underbrace{\ln\left(\frac{e^{\mu_A + \frac{\mu_\theta}{\eta-1}}}{\alpha}\right)}_{\text{efficient consumption}} - \underbrace{\ln \mathcal{M}}_{\text{markup}} \tag{55}$$

where recall the markup is the sum of the canonical taste-based markup and the uncertainty-based markup:

$$\ln \mathcal{M} = \underbrace{\ln\left(\frac{\eta}{\eta-1}\right)}_{\text{taste-based markup}} + \underbrace{\left(\phi(\kappa) - \frac{\kappa}{\eta}\right)\sigma_U}_{\text{uncertainty-based markup}} \tag{56}$$

Thus, a rise in uncertainty influences consumption and prices in the same qualitative manner as a shift in market power. This creates a connection between empirically observable measures of firm uncertainty and the otherwise opaque "black box" of changes in markups.

A second implication is that uncertainty shocks depress aggregate labor productivity, equivalent also in this model to the Solow residual. That is,

$$\ln \frac{C}{L} = \underbrace{\mu_A + \frac{\mu_\theta}{\eta-1}}_{\text{steady-state level}} - \underbrace{\left(\phi(\kappa) - \frac{\kappa}{\eta}\right)\sigma_U}_{\text{effect of uncertainty}} \tag{57}$$

Because there is no invisible hand to clear markets, the consequence of higher prices is that goods are produced but go unsold. Thus, the economy generates less consumption with the same level of inputs: a decline in "productivity" due to a purely non-technological source.

A Back-of-the-Envelope Calculation. Even small amounts of uncertainty can have a large effect on markups. For instance, Broda and Weinstein (2006) find an average elasticity of demand close to 8. This implies a taste-based markup of 13%. In Appendix B.1, we show how to use measurements of the volatility of traditional and revenue-based TFP (Foster et al., 2008; Bloom et al., 2018) to back out σ_U; we obtain a point estimate of 0.26, corresponding to a quarterly calibration for US manufacturing plants. Recall also that $\kappa = \Phi^{-1}((\eta-1)/\eta)$ only depends on the elasticity of demand. A simple calculation then reveals that approximately 11% of markups arise due to their uncertainty-based component, on average.

Shocks to uncertainty, by the same token, can have a large effect on the economy. Bloom et al. (2018) argue that firm-level volatility spikes in recessions. Increasing volatility by a factor of two, a *conservative* experiment relative to the estimates of Bloom et al. (2018) for

US recessions, raises the relative contribution of the uncertainty-based markup to 19.6%. This amounts to a 1.6% increase in the price level or a 1.6% reduction in labor productivity. In Section 5.4, we further explore these findings and their relationship to empirical evidence in a calibrated, dynamic, and nonlinear version of the model.

4.3 The Effects of Monetary Shocks

We now study the propagation of an unanticipated monetary shock. The key difference between such a shock and an anticipated shock is that prices, wages, and labor inputs into production cannot respond to an unanticipated shock. In standard models in which every goods market clears *ex post*, such shocks would have no impact on real aggregate quantities. However, in the present model, unanticipated monetary shocks have real effects. We define the extent of monetary non-neutrality of an unanticipated monetary shock as:

$$\mathscr{M} = \left. \frac{\partial \ln C}{\partial \ln \Delta} \right|_{\ln \Delta = 0} \tag{58}$$

We also define the right and left derivatives of the passthrough as $\mathscr{M}^+$ and $\mathscr{M}^-$. These terms capture how much output expands to a small positive and a small negative change in demand, respectively.

Monetary Non-Neutrality. We begin by characterizing the extent of monetary non-neutrality and its relationship with underlying firm-level uncertainty:

Theorem 3. *Up to a first-order approximation in volatility, the extent of monetary non-neutrality is given by:*

$$\mathscr{M} = 1 - \frac{1}{\eta} \left(\phi(\kappa) + \frac{\eta - 1}{\eta} \kappa \right) \sigma_U \tag{59}$$

Proof. See Appendix A.10. □

Theorem 3 is intimately related to how much slack there is in the economy. To see this, consider again the case with only productivity uncertainty. We can write the elasticity of consumption with respect to an unanticipated change in money balances as

$$\frac{\partial \ln C}{\partial \ln \Delta} = \frac{M \times C^{1/\eta} \times \int_0^k c_i^{1/\eta} \mathrm{d}i \times \left(\frac{\partial}{\partial M} z p^{-\eta} \right)}{C} \tag{60}$$

The last two terms reflect the fact that consumption can only increase in slack markets, where sales are demand-determined. Furthermore, by the definition of the demand shifter, an increase in one unit of money raises the quantity demanded in slack markets by $(1/\int_0^k p^{1-\eta}) \times$

$p^{-\eta} = \eta/p$. Thus, since $1/\eta$ fraction of markets are slack in equilibrium, households spend η more units of money in slack markets and increase their consumption of each variety in excess supply by η/p. Using the definition of the price index $P = \int p_i c_i \mathrm{d}i/C$ and the fact that nominal expenditures are equal to money holdings, $PC = M$, we can then rewrite Equation 60 as

$$\frac{\partial \ln C}{\partial \ln \Delta} = \underbrace{\left(\frac{\int_0^1 c_i \mathrm{d}i}{C}\right)^{\frac{\eta-1}{\eta}}}_{\text{consumption misallocation}} \times \underbrace{\eta \int_0^k \left(\frac{\int_0^1 c_i \mathrm{d}i}{c_i}\right)^{\frac{1}{\eta}} \mathrm{d}i}_{\text{average excess consumption in slack markets}} \tag{61}$$

This makes it clear that the passthrough of an aggregate demand shock is determined by two features. First, as reflected by the first term, it depends on the extent of misallocation in the economy (holding the average consumption of each variety fixed). This term is unity to first-order in uncertainty which reflects the well-known result that "small" distortions on consumption only affect welfare to second-order (Harberger, 1964). In contrast, the second term differs from unity if uncertainty is non-zero and reflects how much more consumption is present in slack markets relative to the average market. Since goods are imperfectly substitutable and households cannot increase consumption of rationed varieties, this term reduces the passthrough of an aggregate demand shock. The term $\phi(\kappa) + \frac{\eta-1}{\eta}\kappa$ reflects this average consumption gap and is strictly positive for all values of η.

The Role of Uncertainty. This argument also reveals that uncertainty and slack lie at the heart of monetary non-neutrality. Concretely, as uncertainty vanishes, so does the average consumption gap between the average market and slack markets. Thus, in the zero uncertainty limit, the passthrough of the money shock to consumption is equal to unity. Moreover, uncertainty is critical to ensure that the passthrough of monetary policy is well-defined (even in the limit). Indeed, as discussed previously, if there is no uncertainty, markets are in a state of Walrasian equilibrium and there is no slack. Thus, an unanticipated increase in the money supply renders money neutral. Conversely, an unanticipated contraction implies that money reduces consumption one-to-one, because households reduce the consumption of all goods uniformly. This discontinuity arises because of the kink in the aggregate supply curve: as uncertainty vanishes, optimal producer behavior ensures that output lies exactly on the kink. We now write $\mathscr{M}(\delta)$ as the extent of monetary non-neutrality when uncertainty about supply-demand imbalances is given by δ. We formalize this below in the following corollary.

Corollary 3. *The extent of monetary non-neutrality is discontinuous in the zero-uncertainty* *limit. That is, we have that:*

$$\lim_{\delta\to 0}\mathscr{M}^{+}(\delta)=1,\ \lim_{\delta\to 0}\mathscr{M}^{-}(\delta)=1 \quad \textit{and} \quad \mathscr{M}^{+}(0)=0,\ \mathscr{M}^{-}(0)=1 \tag{62}$$

Proof. See Appendix A.11. ☐

The Role of Market Power. Finally, given uncertainty σ_U, we observe that the degree of monetary neutrality is determined solely by market power, as captured by the demand elasticity η. The following proposition identifies the value of η for which monetary passthrough is dampened as much as possible.

Proposition 7. *The extent of monetary non-neutrality, $\mathscr{M}$, is non-monotonically related to market power. In particular, $\mathscr{M}$ is strictly decreasing in η if $\eta < \eta^*$ and strictly increasing in η if $\eta > \eta^*$, where $\eta^* = \frac{1}{1-\Phi(\kappa^*)} \approx 2.83$ and κ^* is the unique solution to the equation:*

$$1=\frac{\phi(x)}{1-\Phi(x)}\left(x+\frac{\phi(x)}{\Phi(x)}\right) \tag{63}$$

Proof. See Appendix A.12. ☐

The non-monotonic nature of this result can be explained as follows. As established in Theorem 3, the degree of monetary neutrality falls as the consumption gap between the average slack market and the average market in the entire economy widens. The magnitude of this mechanism weakens with higher firm market power: when market power is lower, the relative cost of overproduction rises and the likelihood that a market becomes rationed increases (Proposition 1). At the same time, a reduction in market power makes goods more substitutable, which diminishes the relative welfare losses from consuming "overproduced" varieties. The cutoff value η^* pins down the point at which these two opposing effects offset each other.

5 Dynamics: A New Old Keynesian Model

To most transparently discuss the novel economics of our setting, our baseline model was static (in the style of Blanchard and Kiyotaki, 1987). However, it is simple to extend our model of firm behavior to dynamic settings in which decisions may be sticky. To make the parallel to the New Keynesian model as tight as possible, we now extend our baseline model of the firm to a dynamic setting and subject firms to Calvo (1983) stickiness. The key departure of our model from the standard New Keynesian model is that firms' prices and quantities are both sticky, and so goods markets do not necessarily clear in each period. In this sense, this model marries elements of the New Keynesian model of price setting with an Old Keynesian model of Walrasian disequilibrium. Our model differs relative to the Old Keynesian tradition in the critical respect that Walrasian disequilibrium is not assumed; it is instead an outcome of a rational expectations equilibrium. For these reasons, we refer to this model as the New Old Keynesian (NOK) Model.

5.1 The NOK Model

Time is discrete and infinite, indexed by $\tau \in \mathbb{N}$ to avoid confusion with market tightness t.

Households. The household side of the model is identical to our baseline model except that they now act dynamically. That is, we continue to assume that:

$$C_\tau = \left(\int \theta_{i,\tau}^{\frac{1}{\eta}} c_{i,\tau}^{\frac{\eta-1}{\eta}} \, \mathrm{d}i \right)^{\frac{\eta}{\eta-1}} \tag{64}$$

where $\theta_{i,t}$ is lognormal with mean μ_θ and variance $\sigma^2_{\theta,\tau}$. The household has preferences over streams of aggregate consumption, real money balances, and labor supply given by:

$$\sum_{j=0}^{\infty} \beta^j \left(\ln C_{\tau+j} + \ln \left(\frac{M_{\tau+j}}{P_{\tau+j}} \right) - \alpha N_{\tau+j} \right) \tag{65}$$

where $\beta \in [0, 1)$ is the discount factor and the ideal price index is given by $P_\tau = (\int p_{i,\tau} c_{i,\tau} \, \mathrm{d}i)/C_\tau$. The household faces the flow budget constraint:

$$P_\tau C_\tau + M_\tau = w_\tau N_\tau + \Pi_\tau + T_\tau + M_{\tau-1} \tag{66}$$

Thus, the household's optimality conditions continue to imply that:

$$w_\tau = \alpha M_\tau \quad \text{and} \quad P_\tau C_\tau = M_\tau \tag{67}$$

These optimality conditions imply that monetary policy controls nominal GDP and wages, as in Golosov and Lucas (2007). It is not essential that the monetary authority does this through changing the money supply or interest rates. Moreover, if conceptually preferable to having money in utility, a cash-in-advance constraint would yield identical equilibrium conditions (see Woodford, 2003a, for a discussion of these points). We also emphasize that it is simple to supplement the household side of this model with a dynamic IS equation, as in the standard New Keynesian model.

Importantly, household demand over varieties continues to follow Proposition 6:

$$c_{i,\tau} = \min\{z_{i,\tau} p_{i,\tau}^{-\eta}, \bar{c}_{i,\tau}\} \tag{68}$$

where:

$$z_{i,\tau} = D_\tau \theta_{i,\tau} \quad \text{with} \quad D_\tau = \frac{M_\tau - \int_{k_\tau}^{1} p_{i,\tau}\bar{c}_{i,\tau}\,\mathrm{d}i}{\int_0^{k_\tau} \theta_{i,\tau} p_{i,\tau}^{1-\eta}\,\mathrm{d}i} \tag{69}$$

and k_τ solves $\bar{c}_{k_\tau,\tau} = z_{k_\tau,\tau} p_{k_\tau,\tau}^{-\eta}$, where we are re-indexing i in each period by $\iota_{i,\tau} = t_{i,\tau}/U_{i,\tau}$, as we did in the static model.

Firms. The departure of this model from our baseline framework is that firms' decisions are sticky: there is a constant probability $\omega \in [0,1)$ that the firm cannot reset its decision over both prices and labor in each period. Thus, this model extends the standard Calvo (1983) model to also incorporate stickiness along the input margin. The rest of our model is as before. The firm produces $A_{i,\tau}L_{i,\tau}$, where $A_{i,\tau}$ is IID log-normal with mean μ_A and variance $\sigma_{A,\tau}^2$. Moreover, the nominal SDF is $\Lambda_\tau = (P_\tau C_\tau)^{-1}$. Given these assumptions, the firm's problem at time τ is:

$$\sup_{p,L} \sum_{j=0}^{\infty} (\beta\omega)^j \mathbb{E}_{i,\tau}[\Lambda_{i,\tau+j}\left(\min\{z_{i,\tau+j}p^{1-\eta}, pA_{i,\tau+j}L\} - w_{\tau+j}L\right)] \tag{70}$$

where $\mathbb{E}_{i,\tau}$ denotes the conditional expectation given knowledge of all shocks that realized in periods $\hat{\tau} \leq \tau - 1$. Applying the same arguments as the proof of Theorem 1, firms' optimal tightness and prices must satisfy the following conditions:

$$\begin{aligned} t_{i,\tau} &= \frac{1}{\eta - 1} \frac{\sum_{j=0}^{\infty}(\beta\omega)^j \mathbb{E}_{i,\tau}[\Lambda_{\tau+j}A_{i,\tau+j}\mathbb{I}[t_{i,\tau} \geq U_{i,\tau+j}]]}{\sum_{j=0}^{\infty}(\beta\omega)^j \mathbb{E}_{i,\tau}[\Lambda_{\tau+j}z_{i,\tau+j}\mathbb{I}[t_{i,\tau} \leq U_{i,\tau+j}]]} \\ p_{i,\tau} &= \frac{\sum_{j=0}^{\infty}(\beta\omega)^j \mathbb{E}_{i,\tau}[\Lambda_{\tau+j}w_{\tau+j}]}{\sum_{j=0}^{\infty}(\beta\omega)^j \mathbb{E}_{i,\tau}[\Lambda_{\tau+j}A_{i,\tau+j}\mathbb{I}[t_{i,\tau} \geq U_{i,\tau+j}]]} \end{aligned} \tag{71}$$

and labor inputs are determined residually as $L_{i,\tau} = p_{i,\tau}^{-\eta}/t_{i,\tau}$. Critically, these equations just involve present discounted values of the same conditional expectations that characterized the firm's optimal behavior in the static setting. Moreover, they retain the block-recursive structure; once the fixed point for $t_{i,\tau}$ has been solved, the optimal choices for $p_{i,\tau}$ and $L_{i,\tau}$ can be determined residually.

Monetary Policy. We consider a situation in which money is held fixed at some steady-state level $\bar{M}$. Starting from this steady state, we will consider an unanticipated monetary shock at date $\tau = 0$. At date $\tau = 0$, it is announced that the monetary stock will follow

$M_\tau = \bar{M}\Delta_\tau$ for some sequence $\{\Delta_\tau\}_{\tau\in\mathbb{N}}$ such that $\Delta_\tau \to \Delta \in \mathbb{R}_{++}$.

5.2 The Linearized NOK Model

To analyze how this dynamic model behaves, we log-linearize it around its steady state. In the steady state, the previous equations imply that $P_\tau \equiv P$, $C_\tau \equiv C$, $t_{i,\tau} \equiv t$, $p_{i,\tau} \equiv p$. The solution to this model reduces to that of our static model. Indeed, the firm's optimality conditions reduce to:

$$tD = \frac{1}{\eta - 1}\frac{\mathbb{E}[A\mathbb{I}[tD \geq A/\theta]]}{\mathbb{E}[\theta\mathbb{I}[tD \leq A/\theta]]} \quad \text{and} \quad p = \frac{\alpha\bar{M}}{\mathbb{E}[A\mathbb{I}[tD \geq A/\theta]]} \tag{72}$$

Moreover, D solves:

$$D = \frac{\bar{M} - pL\int_k^1 A_i \,\mathrm{d}i}{p^{1-\eta}\int_0^k \theta_i \,\mathrm{d}i} \quad \text{where} \quad k = 1 - r = 1 - \Phi\left(\frac{\ln(Dt) - (\mu_A - \mu_\theta)}{\sqrt{\sigma_A^2 + \sigma_\theta^2}}\right) \tag{73}$$

completing the characterization of the steady state. Hence, all of our static results apply to the steady state of this dynamic model.

The shocks in this model are those to the money supply $\hat{M}_\tau$ and those to the volatility of the supply-demand imbalance $\hat{\sigma}_{U,\tau}$. For any variable Z with steady-state value $\bar{Z}$, we let $\hat{Z} = \ln\big(Z/\bar{Z}\big)$ denote the log-deviation from the steady state. We also define $\bar{t}_\tau$ and $\bar{p}_\tau$ as the average log-deviation from the steady state in firms' tightness and prices, and we denote price inflation by $\pi_\tau = \Delta\bar{p}_\tau$ and tightness inflation by $\upsilon_\tau = \Delta\bar{t}_\tau$. With these definitions in hand, we have that:

Proposition 8. *When log-linearized around the steady state and given a process for monetary shocks and volatility shocks $\{\hat{M}_\tau, \hat{\sigma}_{U,\tau}\}_{\tau\in\mathbb{N}}$, the dynamic equilibria of the model solve the following three equations:*

1. *The New Keynesian Phillips Curve:*

$$\pi_\tau = \Gamma(\hat{M}_\tau - \bar{p}_\tau) + \beta\mathbb{E}_\tau[\pi_{\tau+1}] + \zeta\hat{\sigma}_{U,\tau} \tag{74}$$

2. *The New Old Keynesian Tightness Curve:*

$$\upsilon_\tau = -\Gamma(\hat{D}_\tau + \bar{t}_\tau) + \beta\mathbb{E}_\tau[\upsilon_{\tau+1}] + \psi\hat{\sigma}_{U,\tau} \tag{75}$$

3. *The Demand Spillovers Curve:*

$$\hat{D}_\tau = \gamma_M\hat{M}_\tau + \gamma_p\bar{p}_\tau + \gamma_t\bar{t}_\tau + \gamma_U\hat{\sigma}_{U,\tau} \tag{76}$$

where $\Gamma = \frac{(1-\omega)(1-\beta\omega)}{\omega}$ *and the remaining constants* $(\zeta, \psi, \gamma_M, \gamma_p, \gamma_t, \gamma_U)$ *are derived in the proof of the result.*

Proof. See Appendix A.13. ☐

In this representation, we track the three key endogenous state variables in the economy: inflation, market tightness, and demand spillovers. Using these variables, we can calculate the equilibrium behavior of consumption, labor, and other aggregates.

Consistent with our results in static equilibrium (Theorem 2), uncertainty shocks continue to have first-order effects on allocations. In particular, a spike in uncertainty appears as if a "cost push shock" in the New Keynesian Phillips curve, increasing inflation holding fixed marginal cost. Uncertainty also has first-order effects on tightness inflation and demand spillovers for largely the same reasons as in the static analysis.

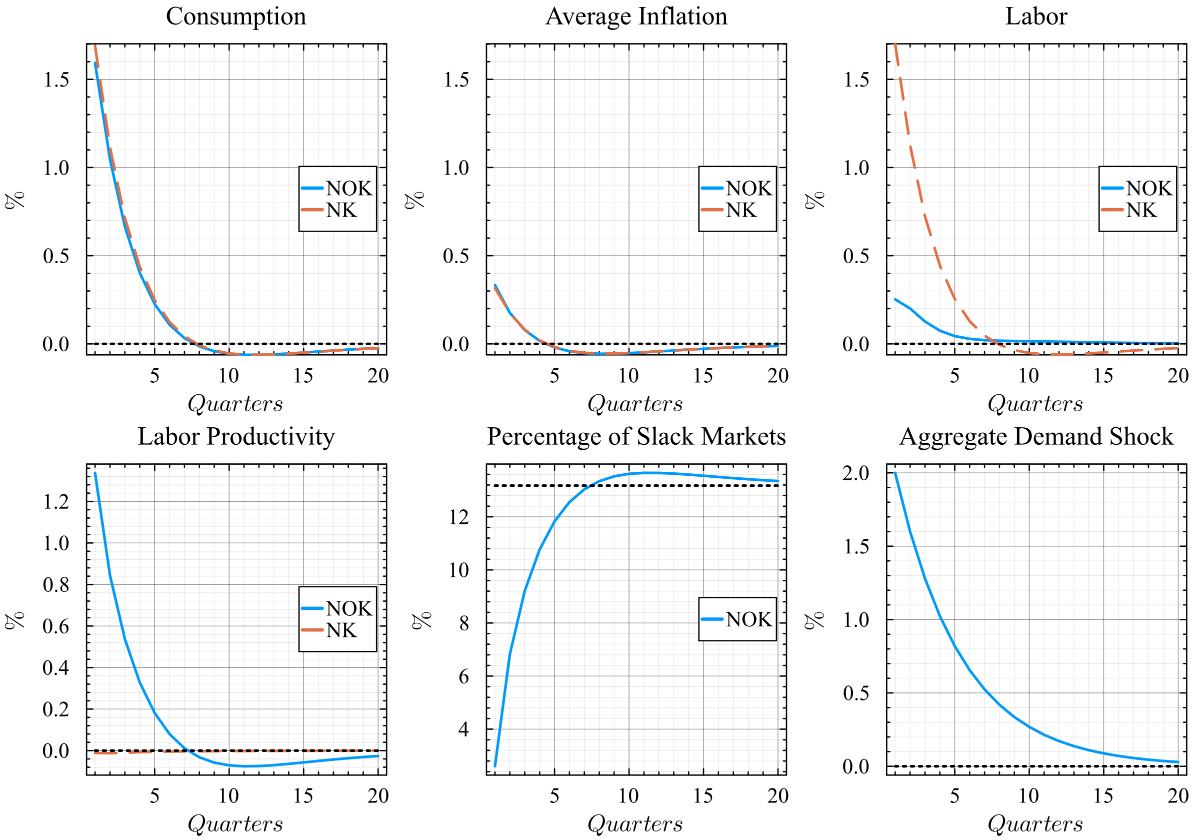
Very strikingly, the dynamics of the New Keynesian Phillips curve in Equation 74 show that the first-order *propagation* of shocks is just as in the standard New Keynesian model. Intuitively, this arises because the effects of (current and future) rationing on desired markups have no first-order effects *in deviations from the steady state*: that is, while they affect steady-state prices and markups themselves, they do not affect conditional price dynamics. This suggests, as we will soon verify in a (nonlinear) numerical example below, that the broad dynamic behavior of inflation—a variable held fixed in our intentionally stylized static analysis (Section 3)—is as in the standard benchmark analysis. We also remark that, due to the tight link between the price level, money supply, and consumption ($C_\tau = M_\tau / P_\tau$, for all time periods τ), this implies similar dynamics for consumption across the models.

Nonetheless, the *mechanics* of shock transmission will prove to be quite different. In particular, they are governed by the dynamic coevolution of market tightness and demand spillovers. Below, we show how this leads to a qualitatively different picture for *which* shocks are relevant for the business cycle and *why* shocks have the effect they do on inflation and consumption.

5.3 The Propagation of Monetary Shocks

Having described the intuition for the linearized dynamics, we now study the fully nonlinear shock responses in the New Old Keynesian Model. We compare these dynamics to their equivalent under the standard New Keynesian model under the same microfoundations and parameter values. Thus, concretely, the two models considered differ only in their assumptions about the markets for intermediate goods. In the NOK model, firms choose prices and inputs in advance, so individual markets may be slack or rationed; whereas in the comparable NK model, inputs are fully adjustable *ex post* to allow for market clearing.

Figure 4: Impulse Responses to an Aggregate Demand Shock



Note: This figure shows the response of macroeconomic variables to a temporary 2% increase in the money supply at $t = 0$, beginning from a steady state. Each panel contrasts the responses in the New Old Keynesian model (blue solid line) with those in the New Keynesian model (orange dashed line) with the same parametrization. The bottom right plot shows the path of the money supply M_t. "Consumption" is the consumption quantity index (Equation 64); "Average Inflation" is the growth in the ideal price index, $P_t = M_t/C_t$, where M_t is the money supply and C_t is the consumption quantity index; "Labor" is total labor hours; "Labor productivity" is GDP (Consumption) per labor hour; and the "Percentage of Slack Markets" is the percentage of markets for intermediate goods in which supply exceeds demand. All variables except the percentage of slack markets are expressed in percent deviations from the steady state.

We choose intentionally standard parameters for illustration: a quarterly discount rate of $\beta = 0.99$; an elasticity of substitution of $\eta = 8$ (consistent with firm-level evidence from Hottman et al., 2016); and a stickiness parameter (probability decisions are fixed quarter-to-quarter) of $\omega = 0.75$, implying an average duration of prices and labor of 4 quarters. In Appendix B.1, we show how the model can be used to compute microeconomic volatility using the reported volatilities of traditional TFP (TFTT) and revenue TFP (TFPR) from Foster et al. (2008) along with information from Bloom et al. (2018) about the unforecastability of firm-level shocks. Using this method, we find that $\sigma_A = 0.096$ and $\sigma_z = 0.24$.

We begin by studying a temporary monetary expansion, which is the dynamic analogue of the analysis in Section 4.3. We plot the results of an unexpected and temporary 2% increase in the money supply in Figure 4.

Consider first the standard New Keynesian (NK) framework, where both aggregate consumption and inflation rise. The increase in inflation is driven by changes in firms' marginal costs via wages, as summarized by the canonical Phillips Curve in Equation 74. Output then changes because firms commit to hiring the labor necessary to meet the corresponding changes in aggregate demand. Consequently, the change in aggregate labor closely tracks the change in aggregate consumption. Furthermore, labor productivity falls slightly, since price dispersion lowers the marginal utility derived from consuming each individual variety. Finally, note that in the NK model, there is no slack as all markets are in Walrasian equilibrium.

Next, consider the NOK model. The response of inflation is nearly identical in the NOK and NK models. As previously explained, this is because changes in firms' rationing probabilities are only second-order in determining firms' reset prices. Hence, to a very accurate first-order approximation, inflation dynamics are determined by the same New Keynesian Phillips Curve in both models. Since aggregate consumption is simply total money holdings divided by the price level, the path of aggregate consumption must also be similar in both models.

However, the monetary transmission mechanism in the NOK model is fundamentally different. In the NK setting, higher consumption is generated by an implausible commitment assumption in which firms hire more labor in input markets to meet demand while holding prices fixed. In contrast, in the NOK model, the rise in consumption is primarily due to a reduction in slack in the economy. For instance, on impact, the share of slack markets decreases from 12.5% to 1%. Notably, this does not mean that total production is unchanged in the NOK model. Instead, firms that can adjust their decisions find it optimal to raise prices and expand labor input in order to take advantage of the temporarily higher aggregate demand. Critically, the increase in consumption arises from both higher production *and* lower slack. This is reflected in an increase in measured labor productivity, as shown in the bottom-left panel.

This "slack-absorbing" transmission mechanism for monetary expansions (or "slack-creating" mechanism for contractions) is consistent with evidence from Buda et al. (2025) on the perhaps surprisingly immediate effects of monetary policy shocks that are visible in data that aggregate economic transactions at a high frequency. In particular, these authors show that a one-standard-deviation monetary policy shock in their data affects sales, consumption, and investment on the order of 1% within only 1 to 2 months, while employment is reduced by a tenth of this (0.1%). This is obviously inconsistent with the New Keynesian benchmark studied here, in which output, production, and employment must move one-for-one with one another; it is also likely inconsistent with any alternative view that incorporates capital in production, given reasonable values for the elasticity of the capital stock to investment and the elasticity of output to the capital stock. The findings would, however, be consistent with the hypothesis that monetary transmission operates through changes in product market slack. In the short run, as aggregate demand increases, consumers flood into previously slack markets; in the longer run, firms slowly begin to respond by also ramping up production.

We finally observe that both the high-frequency analysis of Buda et al. (2025) as well as more classical, business-cycle frequency analysis in the US (Christiano et al., 2005) imply a significant response of labor productivity to monetary policy shocks. In the New Keynesian benchmark we study, this is impossible as labor productivity is purely determined by (exogenous) technology. In the New Old Keynesian Model, as markets slacken and transactions increase, the economy produces more GDP for the same amount of labor inputs despite there being no change in underlying production technology.

5.4 The Propagation of Uncertainty Shocks

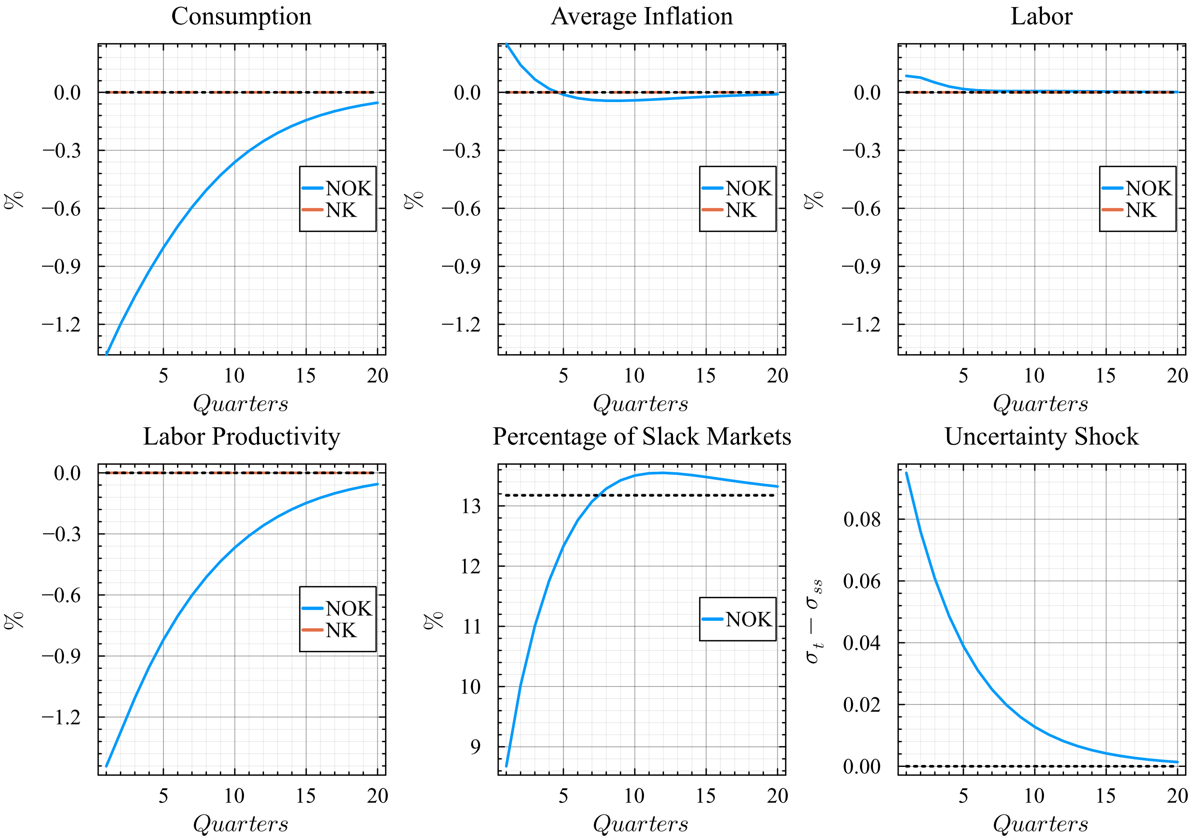
We next consider a temporary increase in the uncertainty that firms face about idiosyncratic demand and productivity, which is the dynamic analogue of the analysis in Section 4.2. We think of this as a stylized representation of the "uncertainty shocks" uncovered by previous work in firm-level microdata. For instance, Bloom et al. (2018) estimate that the standard deviation of firm-level shocks (specifically, to revenue-based TFP) can more than quadruple in a recession versus a normal state; we study a more conservative shock in which uncertainty regarding the relevant variable, the supply-demand shifter $U = A/z$, almost doubles. We plot the response of macroeconomic variables to this shock in Figure 5.

In the standard New Keynesian model, uncertainty shocks have zero effect on economic allocations. Firms' optimal reset prices are invariant to the level of product demand as well as uncertainty thereof. Because the "invisible hand" determines quantities produced, there is also no higher-order effect on aggregates via misallocation in the economy. As such, the shocks have no effects.[10]

---

[10] We note that our definition of an uncertainty shock, while consistent with the main empirical findings of Bloom et al. (2018), differs from the notion of an "uncertainty shock" studied by Basu and Bundick (2017) in a variant of the New Keynesian model. These authors study a change in the volatility of an aggregate preference shock for the representative consumer, calibrated to generate a realistic spike in the forward volatility of aggregate equity returns.

Figure 5: Impulse Responses to an Uncertainty Shock



Note: This figure shows the response of macroeconomic variables to a transitory increase in uncertainty, beginning from a steady state. Each panel contrasts the responses in the New Old Keynesian model (blue solid line) with those in the New Keynesian model (orange dashed line) with the same parametrization. The shock, shown in the bottom right panel, corresponds to a temporary increase in uncertainty about the idiosyncratic demand shock θ and the idiosyncratic productivity shock A, such that $\sigma_U = \text{StdDev}[\log A - \log \theta]$ rises by 0.2 on impact, and the ratio of uncertainty regarding θ and A remains constant. “Consumption” is the consumption quantity index (Equation 64); “Average Inflation” is the growth in the ideal price index, $P_t = M_t/C_t$, where M_t is the money supply and C_t is the consumption quantity index; “Labor” is total labor hours; “Labor productivity” is GDP (Consumption) per labor hour; and the “Percentage of Slack Markets” is the percentage of markets for intermediate goods in which supply exceeds demand. All variables except the percentage of slack markets and uncertainty are expressed in percent deviations from steady state.

In the New Old Keynesian model, uncertainty shocks lead to sharp contractions in consumption and labor productivity along with a temporary spike in inflation. Echoing our findings in the static model (Theorem 2), adjusting firms increase prices and slightly *increase* labor, contributing to a decline in tightness. Nonetheless, significantly more markets end up rationed and total consumption declines on impact. This shows up as a large decline in labor productivity (GDP per worker), despite constant physical productivity and almost constant labor inputs. In essence, the economy has contracted because of price increases and widespread "stock outs" (rationing). This broad pattern of transmission resembles that of a markup shock in the standard New Keynesian model, but for the fact that transmission is largely through changes in market tightness rather than changes in physical production.

Our findings offer one possible explanation for empirical findings that increases in economic uncertainty, broadly defined, lead to economic contractions and inflation (*e.g.*, Bloom, 2009; Basu et al., 2021; Mumtaz and Ruch, 2025). This happens for a particularly stark reason in our model: even though physical output is close to constant, firms' high prices discourage purchases. Thus, it appears as if the economy has lost productivity in the aggregate, even though technology is not changed.

By the same token, our findings can also be interpreted as an alternative take on what econometricians identify as a "productivity shock" in aggregate data. Provided that standard approaches to adjust total factor productivity (Fernald, 2014) for utilization rates are imperfect, especially for measuring wasted nondurable output or idle time in services, our uncertainty shock manifests as an immediate decline in aggregate productivity. In the empirical literature, such shocks have conclusively negative effects on output and positive effects on inflation, but more ambiguous effects on labor hours and investment (Gali, 1999; Basu et al., 2006). Moreover, it remains contested whether such shocks can be reasonably linked back to underlying changes in production technology (see, *e.g.*, the discussion in Alexopoulos, 2011). Our model allows for aggregate productivity shocks to arise from the interaction of uncertainty and the malfunctioning of the invisible hand, potentially helping to resolve this tension.

6 Conclusion

To overcome the Arrow (1959) critique and its modern inversion, we have developed a general equilibrium model in which both prices and quantities are determined by rational economic actors, not the invisible hand. The resulting model generates a number of interesting economic predictions that standard theories do not.

At the microeconomic level, the presence of slack and rationing changes production and pricing decisions in qualitatively important ways. For instance, optimal monopoly pricing no longer obeys the Lerner (1934) formula and uncertainty has first-order effects on firms decisions, increasing prices and depressing labor demand. At the macroeconomic level, we illustrate that the presence of uncertainty gives rise to a well-defined distribution of slack and rationed markets. This allows us to overcome the combinatorial challenge of market clearing, which has inhibited research on previous models of Walrasian equilibrium (Bénassy, 1993) and characterize cross-market spillovers in a classic Dixit and Stiglitz (1977) framework. Our results reveal that the presence of rationed markets substantially alters the standard logic under monopolistic competition that there are positive aggregate demand externalities (Blanchard and Kiyotaki, 1987): increasing production in rationed markets induces negative demand spillovers in slack markets. Moreover, the propagation of demand and uncertainty shocks is qualitatively different from standard theories. In particular, demand shocks raise aggregate consumption by reducing economic slack, generating increases in aggregate labor productivity while microeconomic productivity remains unchanged. Moreover, uncertainty shocks matter to first-order and raise prices while depressing output, akin to a cost-push or markup shock.

Finally, we have shown how this framework can be tractably combined with sticky decisions (Calvo, 1983) in a dynamic setting: one must simply add the dynamic tightness curve and the demand spillovers curve to a standard New Keynesian Phillips curve. Indeed, as our framework is modular, it could be tractably embedded in any existing equilibrium business cycle model, including larger-scale DSGE models. The simple New Old Keynesian (NOK) model that we introduce has the feature that demand and uncertainty shocks—as suggested by the static theory—propagate very differently than in New Keynesian economies. Beyond potential conceptual appeal, the NOK model can account for three important facts that simple New Keynesian models struggle to rationalize: (i) uncertainty shocks have large contractionary effects on output while generating inflation (Bloom, 2009; Mumtaz and Ruch, 2025), (ii) labor productivity increases in response to demand shocks (Christiano et al., 2005), and (iii) the response of consumption to demand shocks is much larger than the response of labor to demand shocks (Buda et al., 2025). We therefore argue that the propagation mechanisms revealed by the NOK model are both microeconomically appealing and capable of matching important facts.

References

Alexopoulos, M. (2011). Read all about it!! what happens following a technology shock? *American Economic Review*, 101(4):1144–79.

Angeletos, G.-M. and La'O, J. (2010). Noisy business cycles. *NBER Macroeconomics Annual*, 24(1):319–378.

Arrow, K. J. (1959). *Toward a theory of price adjustment*, volume 1959. Stanford University Press Stanford.

Bai, Y., Rios-Rull, J.-V., and Storesletten, K. (2025). Demand shocks as technology shocks. *Review of Economic Studies*.

Barro, R. J. (2025). The old keynesian model. Technical report, National Bureau of Economic Research.

Barro, R. J. and Grossman, H. I. (1971). A general disequilibrium model of income and employment. *The American Economic Review*, 61(1):82–93.

Basu, S. and Bundick, B. (2017). Uncertainty shocks in a model of effective demand. *Econometrica*, 85(3):937–958.

Basu, S., Candian, G., Chahrour, R., and Valchev, R. (2021). Risky business cycles. Working Paper 28693, National Bureau of Economic Research.

Basu, S., Fernald, J. G., and Kimball, M. S. (2006). Are technology improvements contractionary? *American Economic Review*, 96(5):1418–1448.

Bénassy, J.-P. (1993). Nonclearing markets: microeconomic concepts and macroeconomic applications. *Journal of Economic Literature*, 31(2):732–761.

Berry, S., Levinsohn, J., and Pakes, A. (1995). Automobile prices in market equilibrium. *Econometrica*, 63(4):841–890.

Bianchi, J., McKay, A., and Mehrotra, N. (2024). How should monetary policy respond to housing inflation? Technical report, Federal Reserve Bank of Minneapolis.

Birnbaum, Z. W. (1942). An inequality for mill's ratio. *The Annals of Mathematical Statistics*, 13(2):245–246.

Blanchard, O. J. and Kiyotaki, N. (1987). Monopolistic competition and the effects of aggregate demand. *The american economic review*, pages 647–666.

Bloom, N. (2009). The impact of uncertainty shocks. *econometrica*, 77(3):623–685.

Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten, I., and Terry, S. J. (2018). Really uncertain business cycles. *Econometrica*, 86(3):1031–1065.

Bresnahan, T. F. (1989). Empirical studies of industries with market power. *Handbook of industrial organization*, 2:1011–1057.

Broda, C. and Weinstein, D. E. (2006). Globalization and the gains from variety. *The Quarterly journal of economics*, 121(2):541–585.

Buda, G., Carvalho, V. M., Corsetti, G., Duarte, J. B., Hansen, S., Moura, A. S., Ortiz, Á., Rodrigo, T., Rodríguez Mora, J., and Alves da Silva, G. (2025). The short lags of monetary policy. Working paper, Janeway Institute.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398.

Cavallo, A. and Kryvtsov, O. (2024). Price discounts and cheapflation during the post-pandemic inflation surge. *Journal of Monetary Economics*, 148:103644.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, 113(1):1–45.

Corrado, C. and Mattey, J. (1997). Capacity utilization. *Journal of Economic Perspectives*, 11(1):151–167.

De Loecker, J., Eeckhout, J., and Unger, G. (2020). The rise of market power and the macroeconomic implications. *The Quarterly journal of economics*, 135(2):561–644.

De Loecker, J. and Warzynski, F. (2012). Markups and firm-level export status. *American economic review*, 102(6):2437–2471.

Demirer, M. (2025). Production function estimation with factor-augmenting technology: An application to markups. Working paper, MIT Sloan.

Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *The American Economic Review*, 67(3):297–308.

Fay, J. A. and Medoff, J. L. (1985). Labor and output over the business cycle: Some direct evidence. *The American Economic Review*, 75(4):638–655.

Fernald, J. (2014). A quarterly, utilization-adjusted series on total factor productivity. Working Paper 2012-19, Federal Reserve Bank of San Francisco.

Flynn, J. P., Nikolakoudis, G., and Sastry, K. A. (2024). Prices vs. quantities: A macroeconomic analysis.

Flynn, J. P., Nikolakoudis, G., and Sastry, K. A. (2026). A theory of supply function choice and aggregate supply. *American Economic Review*, 116(2):710–748.

Foster, L., Haltiwanger, J., and Syverson, C. (2008). Reallocation, firm turnover, and efficiency: Selection on productivity or profitability? *American Economic Review*, 98(1):394–425.

Gali, J. (1999). Technology, employment, and the business cycle: do technology shocks explain aggregate fluctuations? *American Economic Review*, 89(1):249–271.

Ghassibe, M. and Zanetti, F. (2022). State dependence of fiscal multipliers: the source of fluctuations matters. *Journal of Monetary Economics*, 132:1–23.

Golosov, M. and Lucas, R. E. (2007). Menu costs and phillips curves. *Journal of Political Economy*, 115(2):171–199.

Harberger, A. (1964). Taxation, resource allocation, and welfare. In *The role of direct and indirect taxes in the Federal Reserve System*, pages 25–80. Princeton University Press.

Holden, T. D. (2025). Rationing under sticky prices. *Working Paper*.

Hottman, C. J., Redding, S. J., and Weinstein, D. E. (2016). Quantifying the sources of firm heterogeneity. *The Quarterly Journal of Economics*, 131(3):1291–1364.

Huo, Z. and Ríos-Rull, J.-V. (2020). Sticky wage models and labor supply constraints. *American Economic Journal: Macroeconomics*, 12(3):284–318.

Jaffé, W. (1967). Walras' theory of tatonnement: a critique of recent interpretations. *Journal of Political Economy*, 75(1):1–19.

Lerner, A. P. (1934). The concept of monopoly and the measurement of monopoly power. *The review of economic studies*, 1(3):157–175.

Liu, Y. (2026). Inflation and competition in an old keynesian model. Working paper, SSRN.

Lorenzoni, G. and Werning, I. (2026). Tâtonnement and price setting in general equilibrium. *Working Paper*.

Michaillat, P. and Saez, E. (2015). Aggregate demand, idle time, and unemployment. *The Quarterly Journal of Economics*, 130(2):507–569.

Michaillat, P. and Saez, E. (2022). An economical business-cycle model. *Oxford Economic Papers*, 74(2):382–411.

Mumtaz, H. and Ruch, F. U. (2025). Policy uncertainty and aggregate fluctuations: Evidence from emerging and developed economies. *The World Bank Economic Review*.

Petrongolo, B. and Pissarides, C. A. (2001). Looking into the black box: A survey of the matching function. *Journal of Economic literature*, 39(2):390–431.

Reis, R. (2006). Inattentive producers. *The Review of Economic Studies*, 73(3):793–821.

Schmitt-Grohé, S. and Uribe, M. (2004). Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of economic dynamics and control*, 28(4):755–775.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American economic review*, 97(3):586–606.

USDA Economic Research Service (2026). Estimates of food loss at the retail and consumer levels. Technical report, US Department of Agriculture Economic Research Service. Available at: `https://www.ers.usda.gov/sites/default/files/images/foodloss_infographicaa.pdf`.

Woodford, M. (2003a). Imperfect common knowledge and the effects of monetary policy. In *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*. Princeton University Press, Princeton.

Woodford, M. (2003b). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

Appendices

A Omitted Derivations and Proofs

A.1 Proof of Theorem 1

Proof. We begin by establishing that an optimum exists. Define the firm's payoff from an arbitrary pair of price and labor input (p, L) as:

$$J(p, L) = \mathbb{E}[\Lambda \min\{zp^{1-\eta}, ApL\} - \Lambda wL] \tag{77}$$

Now consider a sequence of policies $\{p_n, L_n\}_{n\in\mathbb{N}}$ such that either $p_n \to \infty$ or $L_n \to \infty$. We observe that $\min\{x, y\} \leq x$ and so we may write:

$$J(p_n, L_n) \leq \mathbb{E}[\Lambda z]p_n^{1-\eta} - \mathbb{E}[\Lambda w]L_n \tag{78}$$

From this inequality, if $p_n \to \infty$, then $\limsup_n J(p_n, L_n) \leq 0$. Moreover, if $L_n \to \infty$ and $p_n \not\to 0$, then $\limsup_n J(p_n, L_n) \leq 0$. If $L_n \to \infty$ and $p_n \to 0$, it is immediate that $\limsup_n J(p_n, L_n) \leq 0$ (as it is $-\infty$). Writing J in integral form, we have that:

$$J(p, L) = \int_0^\infty \int_0^\infty \left[\int_0^{Ap^\eta L} \Lambda zp^{1-\eta} g(z, A, \Lambda)\, \mathrm{d}z + \int_{Ap^\eta L}^\infty \Lambda pALg(z, A, \Lambda)\, \mathrm{d}z\right] \mathrm{d}A\, \mathrm{d}\Lambda - \mathbb{E}[\Lambda w]L \tag{79}$$

It is immediate that this is a continuous function of (p, L) over $\mathbb{R}^2_{++}$ (while the integrand in the first integral is not well-behaved, observe that it is bounded above by $\Lambda ApL \sup_z g(z, A, \Lambda)$).

We now show that there is a $(\bar{p}, \bar{L})$ such that $f(\bar{p}, \bar{L}) > 0$. We observe that:

$$J(p, L) = \mathbb{E}[\Lambda \min\{zt, A\}p - \Lambda w]L \tag{80}$$

Thus, the firm's payoff is strictly positive at (p, L) if and only if:

$$p\mathbb{E}[\Lambda \min\{zt, A\}] > \mathbb{E}[\Lambda w] \tag{81}$$

To derive the existence of such a $(\bar{p}, \bar{L})$, consider a sequence $\{p_n, L_n\}_{n\in\mathbb{N}}$ such that $L_n p_n^\eta \to 0$ and $p_n > \frac{\mathbb{E}[\Lambda w]}{\mathbb{E}[\Lambda A]} + \varepsilon$ for all $n \in \mathbb{N}$ and some $\varepsilon > 0$. Under this construction, we have that $t_n \to \infty$ and so $\mathbb{E}[\Lambda \min\{zt_n, A\}] \to \mathbb{E}[\Lambda A]$. Thus, there exists an N such that for all $n > N$:

$$p_n > \frac{\mathbb{E}[\Lambda w]}{\mathbb{E}[\Lambda A]} + \varepsilon > \frac{\mathbb{E}[\Lambda w]}{\mathbb{E}[\Lambda \min\{zt_n, A\}]} \tag{82}$$

For any such n, we then have that $J(p_n, L_n) > 0$. Setting $\bar{p} = p_n$ and $\bar{L} = L_n$ for any such n suffices.

Pick $\varepsilon \in (0, J(\bar{p}, \bar{L}))$. We have shown that $\limsup_n J(p_n, L_n) \leq 0$ for all sequences $\{p_n, L_n\}_{n\in\mathbb{N}}$ such that $\max\{p_n, L_n\} \to \infty$. Thus, there exists an R such that $J(p, L) \leq \varepsilon$ whenever $\max\{p, L\} \geq R$. Hence, we have that:

$$\sup_{(p,L)\in\mathbb{R}^2_+} J(p, L) = \sup_{(p,L)\in[0,R]^2} J(p, L) \tag{83}$$

An optimum therefore exists as $[0, R]^2$ is compact and J is continuous, by Weierstrass' theorem.

We now show that $(0, 0)$ is not optimal. First, observe that if $p = 0$, then it must be the case that $L = 0$ (otherwise reducing L would yield a strict improvement). Second, if $L = 0$, then the firm's payoff is zero. We therefore have that $0 = J(0, 0) < J(\bar{p}, \bar{L})$.

Thus, an optimum exists (p^*, L^*) and all optima must lie in $\mathbb{R}^2_{++}$. As J is continuously differentiable, the first-order conditions $J_p(p^*, L^*) = 0$ and $J_L(p^*, L^*) = 0$ must hold at an optimum. By applying Leibniz's rule, we compute the derivative in prices:

$$\begin{aligned} J_p(p, L) &= \int_0^\infty \int_0^\infty \Bigg[\int_0^{Ap^\eta L} \Lambda(1-\eta) z p^{-\eta} g(z, A, \Lambda)\, \mathrm{d}z + \Lambda \left(\eta A p^{\eta-1} L\right) \left(ApLg(Ap^\eta L, A, \Lambda)\right) \\ &\quad + \int_{Ap^\eta L}^\infty \Lambda A L g(z, A, \Lambda)\, \mathrm{d}z - \Lambda \left(\eta A p^{\eta-1} L\right) \left(pALg(Ap^\eta L, A, \Lambda)\right) \Bigg] \mathrm{d}A\, \mathrm{d}\Lambda \\ &= \int_0^\infty \int_0^\infty \left[\int_0^{Ap^\eta L} \Lambda(1-\eta) z p^{-\eta} g(z, A, \Lambda)\, \mathrm{d}z + \int_{Ap^\eta L}^\infty \Lambda A L g(z, A, \Lambda)\, \mathrm{d}z \right] \mathrm{d}A\, \mathrm{d}\Lambda \\ &= (1-\eta)\mathbb{E}\left[\Lambda z p^{-\eta} \mathbb{I}[z \leq Ap^\eta L]\right] + \mathbb{E}\left[\Lambda A L \mathbb{I}[z \geq Ap^\eta L]\right] \end{aligned} \tag{84}$$

Thus, dividing all terms by L, we can re-write the first-order condition for prices as:

$$0 = (1-\eta)\mathbb{E}[\Lambda z \mathbb{I}[t \leq A/z]]t + \mathbb{E}[\Lambda A \mathbb{I}[t \geq A/z]] \tag{85}$$

Which implies that:

$$t = \frac{1}{\eta - 1} \frac{\mathbb{E}[\Lambda A \mathbb{I}[t \geq A/z]]}{\mathbb{E}[\Lambda z \mathbb{I}[t \leq A/z]]} = \frac{1}{\eta - 1} \frac{\mathbb{P}[A/z \leq t]\mathbb{E}[\Lambda A | A/z \leq t]}{(1 - \mathbb{P}[A/z \leq t])\, \mathbb{E}[\Lambda z | A/z \geq t]} \tag{86}$$

We now compute the derivative in labor:

$$\begin{aligned} J_L(p, L) = \int_0^\infty \int_0^\infty \Bigg[& \Lambda \left(\eta A p^{\eta-1} L\right) \left(ApLg(Ap^\eta L, A, \Lambda)\right) + \int_{Ap^\eta L}^\infty \Lambda A p g(z, A, \Lambda) \,\mathrm{d}z \\ & - \Lambda \left(\eta A p^{\eta-1} L\right) \left(pALg(Ap^\eta L, A, \Lambda)\right) \Bigg] \mathrm{d}A \,\mathrm{d}\Lambda - \mathbb{E}[w\Lambda] \\ = {} & \mathbb{E}[\Lambda A \mathbb{I}[z \geq Ap^\eta L]]p - \mathbb{E}[\Lambda w] \end{aligned} \tag{87}$$

Thus, the first-order condition for labor can be expressed as:

$$p = \frac{\mathbb{E}[\Lambda w]}{\mathbb{E}[\Lambda A \mathbb{I}[t \geq A/z]]} \tag{88}$$

We have now shown that any optimal (p, L) must solve Equations 86 and 88. Moreover, we observe that Equation 86 is a one-dimensional fixed-point equation. Moreover, for any solution t^* of Equation 86, we can find p^* by substitution into Equation 88 and labor is then given by $L^* = \frac{1}{t^* p^{*\eta}}$. Thus, the set of solutions to the pair of first-order conditions is pinned down by the set of solutions to Equation 86. We analyze this equation.

To do so, we define $x = \ln t$, and we write the Equation 86 as $x = f(x)$, where $f : \mathbb{R} \to \mathbb{R}$ is given by:

$$\begin{aligned} f(x) = \ln\left(\frac{1}{\eta - 1}\right) & + \ln\left(\mathbb{P}[\ln A - \ln z \leq x] \mathbb{E}[\exp\{\ln \Lambda + \ln A\} | \ln A - \ln z \leq x]\right) \\ & - \ln\left(\left(1 - \mathbb{P}[\ln A - \ln z \leq x]\right) \mathbb{E}[\exp\{\ln \Lambda + \ln z\} | \ln A - \ln z \geq x]\right) \end{aligned} \tag{89}$$

To analyze this equation, we first state and prove a lemma for the conditional expectations of linear combinations of Gaussian random variables when they are conditioned on lying in a half-space.

Lemma 1. *Consider an n-dimensional normal random variable $X \sim N(\mu, \Sigma)$ and for two vectors $a, b \in \mathbb{R}^n$, define $S = a'X$ and $T = b'X$. The following hold:*

$$\begin{aligned} \mathbb{P}[S \leq x] \mathbb{E}[\exp\{T\} | S \leq x] &= \exp\left\{\mu_T + \frac{1}{2}\sigma_T^2\right\} \Phi\left(\frac{x - \mu_S}{\sigma_S} - \frac{\sigma_{S,T}}{\sigma_S}\right) \\ \mathbb{P}[S \geq x] \mathbb{E}[\exp\{T\} | S \geq x] &= \exp\left\{\mu_T + \frac{1}{2}\sigma_T^2\right\} \left(1 - \Phi\left(\frac{x - \mu_S}{\sigma_S} - \frac{\sigma_{S,T}}{\sigma_S}\right)\right) \end{aligned} \tag{90}$$

where μ_Z, σ_Z, $\sigma_{W,Z}$ denote the mean, standard deviation, and covariance for generic random variables Z and W.

Proof. We derive the first equality, the second follows by an entirely analogous argument.

First, we apply the standard multivariate normal conditional expectation formula to derive:

$$T|S = s \sim N\left(\mu_T + \frac{\sigma_{S,T}}{\sigma_S^2}(s - \mu_S), \sigma_T^2 - \frac{\sigma_{S,T}^2}{\sigma_S^2}\right) \tag{91}$$

Applying the formula for the moment generating function of a Gaussian random variable, we have that:

$$\mathbb{E}[\exp\{T\}|S = s] = \exp\left\{\mu_T + \frac{1}{2}\sigma_T^2 - \frac{1}{2}\frac{\sigma_{S,T}^2}{\sigma_S^2}\right\}\exp\left\{\frac{\sigma_{S,T}}{\sigma_S}\frac{s - \mu_S}{\sigma_S}\right\} \tag{92}$$

We then have that:

$$\mathbb{E}[\exp\{T\}|S \leq x] = \exp\left\{\mu_T + \frac{1}{2}\sigma_T^2 - \frac{1}{2}\frac{\sigma_{S,T}^2}{\sigma_S^2}\right\}\mathbb{E}\left[\exp\left\{\frac{\sigma_{S,T}}{\sigma_S}\frac{S - \mu_S}{\sigma_S}\right\}|S \leq x\right] \tag{93}$$

We now compute:

$$\begin{aligned}
&\mathbb{E}\left[\exp\left\{\frac{\sigma_{S,T}}{\sigma_S}\frac{S - \mu_S}{\sigma_S}\right\}|S \leq x\right] \\
&\quad= \frac{1}{\mathbb{P}[S \leq x]}\int_{-\infty}^{x}\exp\left\{\frac{\sigma_{S,T}}{\sigma_S}\frac{z - \mu_S}{\sigma_S}\right\}\frac{1}{\sigma_S\sqrt{2\pi}}\exp\left\{-\frac{(z - \mu_S)^2}{2\sigma_S^2}\right\}\mathrm{d}z \\
&\quad= \frac{1}{\mathbb{P}[S \leq x]}\int_{-\infty}^{x}\frac{1}{\sigma_S\sqrt{2\pi}}\exp\left\{\frac{\sigma_{S,T}}{\sigma_S}\frac{z - \mu_S}{\sigma_S} - \frac{(z - \mu_S)^2}{2\sigma_S^2}\right\}\mathrm{d}z \\
&\quad= \frac{1}{\mathbb{P}[S \leq x]}\int_{-\infty}^{x}\frac{1}{\sigma_S\sqrt{2\pi}}\exp\left\{-\frac{(z - \mu_S - \sigma_{S,T})^2}{2\sigma_S^2} + \frac{1}{2}\frac{\sigma_{S,T}^2}{\sigma_S^2}\right\}\mathrm{d}z \\
&= \frac{1}{\mathbb{P}[S \leq x]}\exp\left\{\frac{1}{2}\frac{\sigma_{S,T}^2}{\sigma_S^2}\right\}\int_{-\infty}^{x}\frac{1}{\sigma_S\sqrt{2\pi}}\exp\left\{-\frac{(z - \mu_S - \sigma_{S,T})^2}{2\sigma_S^2}\right\}\mathrm{d}z \\
&= \frac{1}{\mathbb{P}[S \leq x]}\exp\left\{\frac{1}{2}\frac{\sigma_{S,T}^2}{\sigma_S^2}\right\}\int_{-\infty}^{x}\frac{1}{\sigma_S\sqrt{2\pi}}\exp\left\{-\frac{(z - \mu_S - \sigma_{S,T})^2}{2\sigma_S^2}\right\}\mathrm{d}z \\
&= \frac{1}{\mathbb{P}[S \leq x]}\exp\left\{\frac{1}{2}\frac{\sigma_{S,T}^2}{\sigma_S^2}\right\}\int_{-\infty}^{\frac{x-\mu_S}{\sigma_S} - \frac{\sigma_{S,T}}{\sigma_S}}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}y^2\right\}\mathrm{d}y \\
&= \frac{1}{\mathbb{P}[S \leq x]}\exp\left\{\frac{1}{2}\frac{\sigma_{S,T}^2}{\sigma_S^2}\right\}\Phi\left(\frac{x - \mu_S}{\sigma_S} - \frac{\sigma_{S,T}}{\sigma_S}\right)
\end{aligned} \tag{94}$$

Combining these steps, we have that:

$$\mathbb{P}[S \leq x]\mathbb{E}[\exp\{T\}|S \leq x] = \exp\left\{\mu_T + \frac{1}{2}\sigma_T^2\right\}\Phi\left(\frac{x - \mu_S}{\sigma_S} - \frac{\sigma_{S,T}}{\sigma_S}\right) \tag{95}$$

As we claimed. □

We now apply Lemma 1 to re-express f as:

$$\begin{aligned} f(x) &= \ln\left(\frac{1}{\eta-1}\right) + \mu_\Lambda + \mu_A + \frac{1}{2}\left(\sigma_\Lambda^2 + \sigma_A^2 + 2\sigma_{\Lambda,A}^2\right) + \ln\Phi\left(\frac{x-\mu_U}{\sigma_U} - \beta^S\sigma_U\right) \\ &\quad - \left(\mu_\Lambda + \mu_z + \frac{1}{2}\left(\sigma_\Lambda^2 + \sigma_z^2 + 2\sigma_{\Lambda,z}^2\right)\right) - \ln\left(1 - \Phi\left(\frac{x-\mu_U}{\sigma_U} - \beta^D\sigma_U\right)\right) \\ &= \ln\left(\frac{1}{\eta-1}\right) + \mu_A - \mu_z + \frac{1}{2}\left(\sigma_A^2 - \sigma_z^2 + 2\sigma_{\Lambda,A}^2 - 2\sigma_{\Lambda,z}^2\right) + \ln\frac{\Phi\left(\frac{x-\mu_U}{\sigma_U} - \beta^S\sigma_U\right)}{1-\Phi\left(\frac{x-\mu_U}{\sigma_U} - \beta^D\sigma_U\right)} \end{aligned} \tag{96}$$

where $\mu_U = \mathbb{E}[\ln A - \ln z]$, $\sigma_U^2 = \mathbb{V}[\ln A - \ln z]$, $\beta^S = \mathbb{C}[\ln\Lambda + \ln A, \ln A - \ln z]/\sigma_U^2$, $\beta^D = \mathbb{C}[\ln\Lambda + \ln z, \ln A - \ln z]/\sigma_U^2$. We can further define the following transformations:

$$y = \frac{x-\mu_U}{\sigma_U}, \alpha = \beta^S\sigma_U, \beta = \beta^D\sigma_U, K = \frac{1}{\sigma_U}\left(\ln\left(\frac{1}{\eta-1}\right) + \frac{1}{2}\left(\sigma_A^2 - \sigma_z^2 + 2\sigma_{\Lambda,A}^2 - 2\sigma_{\Lambda,z}^2\right)\right) \tag{97}$$

and then we can write the fixed-point equation as $y = g(y)$, where:

$$g(y) = K + \frac{1}{\sigma_U}\ln\frac{\Phi(y-\alpha)}{1-\Phi(y-\beta)} \tag{98}$$

We observe that $g(y)$ is strictly increasing and it has derivative:

$$g'(y) = \frac{1}{\sigma_U}\left(r(y-\alpha) + m(y-\beta)\right) \tag{99}$$

where $r(z) = \phi(z)/\Phi(z)$ and $m(z) = \phi(z)/(1-\Phi(z))$ are the left and right inverse Mills ratios, respectively. To find sufficient conditions for a unique fixed point, we use the following fact about Mills ratios: $r(z) > \max\{0, -z\}$ and $m(z) > \max\{0, z\}$. Thus, we have that:

$$\begin{aligned} \inf_y g'(y) &= \inf_y \frac{1}{\sigma_U}\left(r(y-\alpha) + m(y-\beta)\right) > \inf_y \frac{1}{\sigma_U}\left(\max\{0, \alpha - y\} + \max\{0, y-\beta\}\right) \\ &\geq \frac{\max\{0, \alpha-\beta\}}{\sigma_U} \end{aligned} \tag{100}$$

Thus, if $\alpha - \beta \geq \sigma_U$, then $\inf_y g'(y) > 1$ and there is a unique fixed point. This condition reduces to:

$$1 \leq \frac{\alpha-\beta}{\sigma_U} = \beta^S - \beta^D = 1 \tag{101}$$

Thus, there is always a unique fixed point.

We have thus shown that: (i) an optimum exists, (ii) any optimum must lie in $\mathbb{R}_{++}^2$, (iii) any optimum in $\mathbb{R}^2_{++}$ must satisfy Equations 86 and 88, and (iv) there is a unique solution to Equations 86 and 88. This completes the proof. $\square$

A.2 Proof of Proposition 1

Proof. We prove the three claims in reverse order.

Claim 3. Defining $x(\delta) = \ln t(\delta)$, we first prove that $\lim_{\delta \to 0} x(\delta) = \mu_A - \mu_z = x(0)$. Consider the fixed-point equation (Equation 6) and let the numerator and denominator on the right-hand side be:

$$\begin{aligned} N_\delta(t) &= \mathbb{E}[\Lambda_\delta A_\delta \mathbb{I}[t \geq A_\delta / z_\delta]] = \mathbb{P}[t \geq A_\delta / z_\delta] \mathbb{E}[\Lambda_\delta A_\delta | t \geq A_\delta / z_\delta] \\ D_\delta(t) &= \mathbb{E}[\Lambda_\delta z_\delta \mathbb{I}[t \leq A_\delta / z_\delta]] = (1 - \mathbb{P}[t \geq A_\delta / z_\delta]) \mathbb{E}[\Lambda_\delta z_\delta | t \leq A_\delta / z_\delta] \end{aligned} \tag{102}$$

By Theorem 1, we have that $t(\delta) = \exp\{x(\delta)\}$ is unique for all $\delta > 0$ and moreover that $t(\delta)$ solves:

$$(\eta - 1) t(\delta) D_\delta(t(\delta)) = N_\delta(t(\delta)) \tag{103}$$

We also have that $t(0) = \exp\{\mu_A - \mu_z\}$. Because $\frac{A_\delta}{z_\delta} = \exp\{\ln A_\delta - \ln z_\delta\} \to^{a.s.} \exp\{\mu_A - \mu_z\}$ as $\delta \to 0$, we have that:

$$\lim_{\delta \to 0} \mathbb{P}[t \geq A_\delta / z_\delta] = \begin{cases} 1, t > t(0), \\ 0, t < t(0). \end{cases} \tag{104}$$

Thus, if $\lim_\delta t(\delta) < t(0)$, then $\lim_{\delta \to 0} N_\delta(t(\delta)) = 0$ and $\lim_{\delta \to 0} D_\delta(t(\delta)) = \exp\{\mu_\Lambda + \mu_z\} > 0$, so Equation 103 cannot hold, which is a contradiction. Conversely, if $\lim_{\delta \to 0} t(\delta) > t(0)$, then $\lim_{\delta \to 0} N_\delta(t(\delta)) = \exp\{\mu_\Lambda + \mu_A\} > 0$ and $\lim_{\delta \to 0} D_\delta(t(\delta)) = 0$, so Equation 103 cannot hold, which is a contradiction. Thus, we have proven that $\lim_{\delta \to 0} t(\delta) = t(0)$.

Claim 2. Rewriting the fixed-point equation in Equation 103, we have that:

$$(\eta - 1) t(\delta) (1 - \mathbb{P}[t(\delta) \geq A_\delta / z_\delta]) \mathbb{E}[\Lambda_\delta z_\delta | t(\delta) \leq A_\delta / z_\delta] = \mathbb{P}[t(\delta) \geq A_\delta / z_\delta] \mathbb{E}[\Lambda_\delta A_\delta | t(\delta) \geq A_\delta / z_\delta] \tag{105}$$

which implies that:

$$\frac{\mathbb{P}[t(\delta) \geq A_\delta / z_\delta]}{1 - \mathbb{P}[t(\delta) \geq A_\delta / z_\delta]} = (\eta - 1) t(\delta) \frac{\mathbb{E}[\Lambda_\delta z_\delta | t(\delta) \leq A_\delta / z_\delta]}{\mathbb{E}[\Lambda_\delta A_\delta | t(\delta) \geq A_\delta / z_\delta]} \tag{106}$$

Taking the $\delta \to 0$ limit of both sides and using Claim 3, we have that:

$$\lim_{\delta \to 0} \frac{\mathbb{P}[t(\delta) \geq A_\delta / z_\delta]}{1 - \mathbb{P}[t(\delta) \geq A_\delta / z_\delta]} = (\eta - 1) \exp\{\mu_A - \mu_z\} \exp\{\mu_z - \mu_A\} = \eta - 1 \tag{107}$$

Thus, we have that:

$$\lim_{\delta \to 0} \mathbb{P}[t(\delta) \geq A_\delta / z_\delta] = \frac{\eta - 1}{\eta} \tag{108}$$

as claimed.

Claim 1. By Theorem 1, we have that for all $\delta > 0$, the optimal price is uniquely given by:

$$p(\delta) = \frac{\mathbb{E}[\Lambda_\delta w_\delta]}{\mathbb{P}[t(\delta) \geq A_\delta / z_\delta] \mathbb{E}[\Lambda_\delta A_\delta | t(\delta) \geq A_\delta / z_\delta]} \tag{109}$$

Taking the $\delta \to 0$ limit of both sides and applying Claims 2 and 3, we have that:

$$\lim_{\delta \to 0} p(\delta) = \frac{\eta}{\eta - 1} \exp\{\mu_w - \mu_A\} \tag{110}$$

Completing the proof. □

A.3 Proof of Proposition 2

Proof. We break the proof of this result into three lemmas. First, we characterize the first-order effects of volatility on the logarithm of the optimal supply-demand imbalance.

Lemma 2. *We have that:*

$$x(\delta) = \mu_A - \mu_z + \kappa \sigma_U \delta + O(\delta^2) \tag{111}$$

Proof. We showed in the proof of Theorem 1 that $f_\delta'(x) > 1$ for all $x \in \mathbb{R}$ and $\delta > 0$. Thus, by application of the implicit function theorem, we have that $x(\delta)$ is continuously differentiable at all points $\delta > 0$. We define its derivative at $\delta = 0$ by continuity: $x'(0) = \lim_{\delta \to 0} x'(\delta)$. We now make use of the change of variable:

$$y(\delta) = \frac{x(\delta) - \mu_U}{\delta \sigma_U} \tag{112}$$

And we observe that:

$$\lim_{\delta \to 0} y(\delta) = \lim_{\delta \to 0} \frac{x(\delta) - \mu_U}{\delta \sigma_U} = \frac{1}{\sigma_U} \lim_{\delta \to 0} \frac{x(\delta) - x(0)}{\delta} = \frac{1}{\sigma_U} x'(0) \tag{113}$$

We moreover have that:

$$\delta \sigma_U y(\delta) = \ln\left(\frac{1}{\eta - 1}\right) + \frac{1}{2}\delta^2(\sigma_A^2 - \sigma_z^2 + 2\sigma_{\Lambda,A} - 2\sigma_{\Lambda,z}) + \ln\left(\frac{\Phi(y(\delta) - \delta\beta^S\sigma_U)}{1 - \Phi(y(\delta) - \delta\beta^D\sigma_U)}\right) \tag{114}$$

Thus, taking the $\delta \to 0$ limit of both sides, we have that:

$$0 = \ln\left(\frac{1}{\eta - 1}\right) + \ln\left(\frac{\Phi(\frac{1}{\sigma_U}x'(0))}{1 - \Phi(\frac{1}{\sigma_U}x'(0))}\right) \tag{115}$$

And so we obtain that:

$$x'(0) = \sigma_U \Phi^{-1}\left(\frac{\eta - 1}{\eta}\right) \tag{116}$$

Application of Taylor's remainder theorem then yields the claimed formula. $\square$

Second, we use this Lemma to derive the effects of volatility on prices and inputs:

Lemma 3. *We have that:*

$$(\ln p)(\delta) = \ln\left(\frac{\eta}{\eta - 1}\right) + \mu_w - \mu_A + \left(\phi\left(\kappa\right) - \frac{\kappa}{\eta}\right)\sigma_U \delta + O(\delta^2) \tag{117}$$

And:

$$(\ln L)(\delta) = \mu_z + (\eta - 1)\mu_A - \eta\mu_w - \eta \ln\left(\frac{\eta}{\eta - 1}\right) - \eta\phi(\kappa)\sigma_U \delta + O(\delta^2) \tag{118}$$

Proof. By Corollary 1, we have that:

$$(\ln p)(\delta) = \mu_w - \mu_A + \frac{1}{2}\delta^2\left(\sigma_w^2 - \sigma_A^2 + 2\sigma_{\Lambda,w} - 2\sigma_{\Lambda,A}\right) - \ln \Phi\left(y(\delta) - \delta\beta^S\sigma_U\right) \tag{119}$$

Using the fact from Lemma 2 that $y(0) = \Phi^{-1}\left(\frac{\eta-1}{\eta}\right)$, we have that:

$$(\ln p)(0) = \mu_w - \mu_A - \ln \Phi(y(0)) = \ln\left(\frac{\eta}{\eta - 1}\right) + \mu_w - \mu_A \tag{120}$$

Moreover, as x is continuously differentiable for all $\delta > 0$, we also have that y is continuously differentiable for all $\delta > 0$ and, therefore, that p is continuously differentiable for all $\delta > 0$. As before, we define derivatives at zero by continuity. Differentiating both sides of Equation 114 and evaluating at $\delta = 0$, we obtain that:

$$\sigma_U y(0) = \left(y'(0) - \beta^S\sigma_U\right) r(y(0)) + \left(y'(0) - \beta^D\sigma_U\right) m(y(0)) \tag{121}$$

and hence that:

$$y'(0) = \frac{\sigma_U y(0) + \beta^S\sigma_U r(y(0)) + \beta^D\sigma_U m(y(0))}{r(y(0)) + m(y(0))} \tag{122}$$

Differentiating both sides of Equation 119 and evaluating at $\delta = 0$, we obtain:

$$(\ln p)'(0) = -\left(y'(0) - \beta^S\sigma_U\right) r(y(0)) \tag{123}$$

Combining the previous two equations, we have that:

$$(\ln p)'(0) = -\left(\frac{\sigma_U y(0) + \beta^S \sigma_U r(y(0)) + \beta^D \sigma_U m(y(0))}{r(y(0)) + m(y(0))} - \beta^S \sigma_U\right) r(y(0)) \tag{124}$$

We now make use of the following facts:

$$y(0) = \Phi^{-1}\left(\frac{\eta - 1}{\eta}\right),\ r(y(0)) = \frac{\phi(y(0))}{\frac{\eta-1}{\eta}},\ m(y(0)) = \frac{\phi(y(0))}{1 - \frac{\eta-1}{\eta}}, \tag{125}$$

After some algebra, one obtains that:

$$\begin{aligned}(\ln p)'(0) &= (\beta^S - \beta^D)\sigma_U \phi\left(\Phi^{-1}\left(\frac{\eta-1}{\eta}\right)\right) - \frac{\sigma_U}{\eta}\Phi^{-1}\left(\frac{\eta-1}{\eta}\right) \\ &= \sigma_U \phi\left(\Phi^{-1}\left(\frac{\eta-1}{\eta}\right)\right) - \frac{\sigma_U}{\eta}\Phi^{-1}\left(\frac{\eta-1}{\eta}\right)\end{aligned} \tag{126}$$

where the final equality exploits the fact that $\beta^S - \beta^D = 1$. Application of Taylor's remainder theorem then yields the claimed formula for $\ln p$. Using the fact that $\ln L = -\eta \ln p - x$ then yields the formula for labor. □

Finally, we derive the effect of uncertainty on the rationing probability:

Lemma 4. *We have that:*

$$r(\delta) = \frac{\eta - 1}{\eta} + \frac{\eta - 1}{\eta}\left(\frac{\kappa}{\eta} + \left(\beta^S \frac{1}{\eta - 1} + \beta^D\right)\phi(\kappa)\right)\sigma_U \delta + O(\delta^2) \tag{127}$$

Proof. We have that:

$$r(\delta) = \mathbb{P}[U_\delta \leq x(\delta)] = \Phi\left(y(\delta)\right) \tag{128}$$

Thus, we have that:

$$\begin{aligned}r(\delta) &= r(0) + y'(0)\phi(y(0))\delta + O(\delta^2) \\ &= \frac{\eta-1}{\eta} + \frac{\sigma_U \kappa + \beta^S \sigma_U \frac{\eta}{\eta-1}\phi(\kappa) + \beta^D \sigma_U \eta \phi(\kappa)}{\phi(\kappa)\frac{\eta}{\eta-1} + \eta\phi(\kappa)}\phi(\kappa)\delta + O(\delta^2) \\ &= \frac{\eta-1}{\eta} + \frac{\eta-1}{\eta}\left(\frac{\kappa}{\eta} + \left(\beta^S \frac{1}{\eta-1} + \beta^D\right)\phi(\kappa)\right)\sigma_U \delta + O(\delta^2)\end{aligned} \tag{129}$$

Completing the proof. □

All claims in the result have now been proven. □

A.4 Proof of Corollary 2

Proof. The claim for hiring is immediate. For prices, the claim holds if and only if $\phi(\kappa) - \kappa/\eta > 0$. Defining $\mathcal{M} = \frac{\eta}{\eta-1}$ as the full-information optimal markup and recalling that $\kappa = \Phi^{-1}(\mathcal{M}^{-1})$, this inequality reduces to:

$$\phi(\kappa) - \kappa(1 - \Phi(\kappa)) > 0 \tag{130}$$

which holds if and only if the right Mills ratio $m(\kappa) > \kappa$. This is a global property of the Mills ratio, establishing the inequality. □

A.5 Proof of Proposition 3

Proof. We define the function $F : \mathbb{R}^3_+ \to \mathbb{R}^2$ as:

$$F(\delta, \xi) = \begin{pmatrix} \tilde{J}_p(\xi; \delta) \\ \tilde{J}_L(\xi; \delta) \end{pmatrix} \tag{131}$$

where $\xi = (p, L)$. If the matching function is smooth, then F is well-defined and, moreover, we have that $\xi^*(\delta) = (p^*(\delta), L^*(\delta))$ is the unique solution to $F(\delta, \xi^*(\delta)) = 0$. Application of the implicit function theorem to ξ^* yields that:

$$\xi^{*\prime}(\delta) = -F_\xi(0, \xi^*(0))^{-1} F_\delta(0, \xi^*(0)) \tag{132}$$

and ξ^* is twice continuously differentiable. This is because we assumed that F is three times continuously differentiable and the derivative of F, which is the Hessian of $\tilde{J}$, is negative definite and therefore invertible. Moreover, we define:

$$\begin{aligned} h(\theta_\delta, \xi) &= \Lambda_\delta \left(p m(A_\delta L, z_\delta p^{-\eta}) - w_\delta L \right) \\ \text{where} \quad \theta_\delta &= (\ln \Lambda_\delta, \ln A_\delta, \ln z_\delta, \ln w_\delta) = \mu_\theta + \delta \Sigma^{\frac{1}{2}} Z \end{aligned} \tag{133}$$

for $Z \sim N(0, I)$. We then have that for $i \in \{p, L\}$:

$$\frac{\partial}{\partial \delta} F_i(\delta, \xi) = \mathbb{E}[\nabla_\theta h_i(\theta_\delta, \xi) \Sigma^{\frac{1}{2}} Z] \implies \frac{\partial}{\partial \delta} F_i(0, \xi^*(0)) = \mathbb{E}[\nabla_\theta h_i(\mu_\theta, \xi^*(0)) \Sigma^{\frac{1}{2}} Z] = 0 \tag{134}$$

Combining these facts implies that $\xi^{*'}(0) = 0$. Application of Taylor's remainder theorem to ξ^* then yields the result. □

A.6 Proof of Proposition 4

Proof. Applying similar arguments to Theorem 1, we have that any optimum must be interior. Thus, again following the same steps as Theorem 1, we now have that any optimal choices of p and L must satisfy the first-order condition with respect to labor:

$$0 = p\mathbb{E}[\Lambda A \mathbb{I}[t \geq U]] - \mathbb{E}[\Lambda w] + \mathbb{E}[\Lambda' A \mathbb{I}[t \leq U]] \tag{135}$$

and with respect to the price:

$$0 = (1-\eta)\mathbb{E}[\Lambda z \mathbb{I}[t \leq U]]p^{-\eta} + \mathbb{E}[\Lambda A \mathbb{I}[t \geq U]]L + \eta \mathbb{E}[\Lambda' z \mathbb{I}[t \leq U]]p^{-\eta-1} \tag{136}$$

Equation 135 immediately rearranges into Equation 24. After some algebra, Equation 136 reduces to:

$$t = \frac{\mathbb{E}[\Lambda A \mathbb{I}[t \geq U]]}{(\eta - 1)\mathbb{E}[\Lambda z \mathbb{I}[t \leq U]] - \eta \mathbb{E}[\Lambda' z \mathbb{I}[t \leq U]]p^{-1}} \tag{137}$$

Combining this with the formula for p from Equation 24 then yields Equation 23.

We now solve for the first-order effects of volatility in the firms' choices. To do this, observe that we may write:

$$x(\delta) = \ln\left(\frac{1}{\eta - 1}\right) + \ln N(\delta) - \ln D(\delta) - \ln(1 - K(\delta)) \tag{138}$$

where:

$$K(\delta) = \frac{\eta}{\eta - 1}\frac{\hat{D}(\delta)}{D(\delta)}\frac{N(\delta)}{\mathbb{E}[\Lambda_\delta w_\delta] - S(\delta)} \tag{139}$$

and:

$$\hat{D}(\delta) = \mathbb{E}[\Lambda'_\delta z_\delta \mathbb{I}[t(\delta) \leq U_\delta]] = \mathbb{E}[\Lambda'_\delta z_\delta]\left(1 - \Phi(y(\delta) - \delta\hat{\beta}^D\sigma_U)\right) \tag{140}$$

$$S(\delta) = \mathbb{E}[\Lambda'_\delta A_\delta \mathbb{I}[t(\delta) \leq U_\delta]] = \mathbb{E}[\Lambda'_\delta A_\delta]\left(1 - \Phi(y(\delta) - \delta\hat{\beta}^S\sigma_U)\right) \tag{141}$$

and $\hat{\beta}^D = \mathrm{Cov}[\ln \Lambda' + \ln z, U]/\mathrm{Var}[U]$, $\hat{\beta}^S = \mathrm{Cov}[\ln \Lambda' + \ln A, U]/\mathrm{Var}[U]$. Thus, exploiting the same arguments as those in Proposition 2, we have that:

$$\delta\sigma_U y(\delta) = \ln\left(\frac{1}{\eta - 1}\right) + \ln\left(\frac{\Phi(y(\delta) - \delta\beta^S\sigma_U)}{1 - \Phi(y(\delta) - \delta\beta^D\sigma_U)}\right) - \ln(1 - K(\delta)) + O(\delta^2) \tag{142}$$

Passing to the $\delta \to 0$ limit, we obtain that:

$$K(0) = 1 - \frac{1}{\eta - 1}\frac{\Phi(y(0))}{1 - \Phi(y(0))} \tag{143}$$

Moreover, we can compute that:

$$K(0) = \frac{\eta}{\eta - 1} \exp\{\mu_{\Lambda'} - \mu_\Lambda\} \frac{\exp\{\mu_\Lambda + \mu_A\}\Phi(y(0))}{\exp\{\mu_\Lambda + \mu_w\} - \exp\{\mu_{\Lambda'} + \mu_A\}(1 - \Phi(y(0)))} \tag{144}$$

We also recall that the rationing probability is given by $r(\delta) = \Phi(y(\delta))$. Thus, $r(0) = \Phi(y(0))$. We call $r(0) = r_I$ and we define $\kappa_I = \Phi^{-1}(r_I)$. We now combine the previous two equations to solve for r_I:

$$\frac{\eta}{\eta - 1} \frac{\exp\{\mu_{\Lambda'} + \mu_A\}r_I}{\exp\{\mu_\Lambda + \mu_w\} - \exp\{\mu_{\Lambda'} + \mu_A\}(1 - r_I)} = 1 - \frac{1}{\eta - 1}\frac{r_I}{1 - r_I} \tag{145}$$

This reduces to Equation 25. Moreover, we can observe that as $\mu_{\Lambda'} \to -\infty$ (*i.e.*, $\Lambda' \to^{a.s.} 0$), that $r_I \to \frac{\eta-1}{\eta}$. Moreover, we know from Proposition 2 that $x'(0) = \sigma_U y(0) = \sigma_U \kappa_I$. Thus, we have that:

$$(\ln t)(\delta) = \mu_U + \kappa_I \sigma_U \delta + O(\delta^2) \tag{146}$$

Completing the proof. □

A.7 Proof of Proposition 5

Proof. We define tightness in the product market as $t = \frac{p^{-\eta}}{L^D}$ and tightness in the labor market as $t^L = \frac{L^D}{w^\chi}$. There are four possible cases for the two markets, ignoring the zero-probability events of Walrasian equilibrium in either or both markets:

1. Slack product and labor markets. This event occurs if $t^L < v$ and $t < U$. In this case, the firm's realized objective is

$$\Lambda(pq^d - wL^d) = \Lambda(zp^{1-\eta} - wL^d) \tag{147}$$

2. Rationed product market and slack labor market. This event occurs if $t^L < v$ and $t > U$. In this case, the firm's realized objective is

$$\Lambda(pq^s - wL^d) = \Lambda(pAL^d - wL^d) \tag{148}$$

3. Slack product market and rationed labor market. This event occurs if $t^L > v$ and $p^{-\eta}/w^\chi = tt^L < Uv$. In this case, the firm's realized objective is

$$\Lambda(pq^d - wL^s) = \Lambda(zp^{1-\eta} - vw^{1+\chi}) \tag{149}$$

4. Rationed product market and labor market. This event occurs if $t^L > v$ and $p^{-\eta}/w^\chi =$

$tt^L > Uv$. In this case, the firm's realized objective is

$$\Lambda(pq^s - wL^s) = \Lambda(pAvw^\chi - vw^{1+\chi}) \tag{150}$$

We furthermore define the events

$$R^L = \{t^L > v\},\ R^P = \{t^L < v, t > U\} \cup \{t^L > v, tt^L > Uv\} \tag{151}$$

to denote the states in which the labor and product markets, respectively, are rationed.

Using these observations, we can write the firm's objective as an expectation over these four events

$$\begin{aligned} &\mathbb{E}[\Lambda(zp^{1-\eta} - wL^d)(1-R^L)(1-R^P)] + \mathbb{E}[\Lambda(pAL^d - wL^d)(1-R^L)R^P] \\ &+ \mathbb{E}[\Lambda(zp^{1-\eta} - vw^{1+\chi})R^L(1-R^P)] + \mathbb{E}[\Lambda(pAvw^\chi - vw^{1+\chi})R^L R^P] \end{aligned} \tag{152}$$

Analogous arguments to those of Theorem 1 establish that the problem of maximizing J has a solution and that any solution must be interior. Necessary conditions for optimality are that the first-order gain to changing each of p, L^D, and w is zero.

We first take the first-order condition in L^d, observing that this has first-order effects on the firms' payoff only when the labor market is slack ($t^L < v$):

$$0 = -w\mathbb{E}[\Lambda(1-R^L)] + p\mathbb{E}[\Lambda A(1-R^L)R^P] \tag{153}$$

Re-arranging,

$$p = w\frac{\mathbb{E}[\Lambda(1-R^L)]}{\mathbb{E}[\Lambda A(1-R^L)R^P]} \tag{154}$$

Next, we consider the first-order condition with respect to p

$$0 = (1-\eta)p^{-\eta}\mathbb{E}[\Lambda z(1-R^P)] + L^d\mathbb{E}[\Lambda A(1-R^L)R^P] + w^\chi\mathbb{E}[\Lambda Av R^L R^P] \tag{155}$$

Dividing each term by L^d and re-arranging yields

$$t = \frac{1}{\eta-1}\frac{\mathbb{E}[\Lambda A(1-R^L)R^P] + \frac{1}{t^L}\mathbb{E}[\Lambda Av R^L R^P]}{\mathbb{E}[\Lambda z(1-R^P)]} \tag{156}$$

We finally derive the first-order condition with respect to w:

$$0 = -L^d\mathbb{E}[\Lambda(1-R^L)] - (1+\chi)w^\chi\mathbb{E}[\Lambda v R^L] + \chi w^{\chi-1}p\mathbb{E}[\Lambda Av R^L R^P] \tag{157}$$

Dividing all terms by w^{χ}, and substituting p/w from Equation 154:

$$0 = -t^L \mathbb{E}[\Lambda(1-R^L)] - (1+\chi)\mathbb{E}[\Lambda v R^L] + \chi \mathbb{E}[\Lambda A v R^L R^P] \frac{\mathbb{E}[\Lambda(1-R^L)]}{\mathbb{E}[\Lambda A(1-R^L)R^P]} \tag{158}$$

Re-arranging to isolate t^L,

$$t^L = \chi \frac{\mathbb{E}[\Lambda A v R^L R^P]}{\mathbb{E}[\Lambda A(1-R^L)R^P]} - (1+\chi)\frac{\mathbb{E}[\Lambda v R^L]}{\mathbb{E}[\Lambda(1-R^L)]} \tag{159}$$

□

A.8 Proof of Proposition 6

Proof. To characterize the solution to Equation 35, we attach a Lagrange multiplier λ to the expenditure constraint and μ_i to each $c_i \leq \bar{c}_i$ constraint. The first-order condition for each c_i is then:

$$\theta_i^{\frac{1}{\eta}} c_i^{-\frac{1}{\eta}} C^{\frac{1}{\eta}} - \lambda p_i - \mu_i = 0 \tag{160}$$

Thus, we have that:

$$c_i = \theta_i C \left(\lambda p_i + \mu_i\right)^{-\eta} \tag{161}$$

Taking the ratio of this condition for two varieties i and j yields:

$$c_i = \frac{\theta_i}{\theta_j} \frac{\left(\lambda p_i + \mu_i\right)^{-\eta}}{\left(\lambda p_j + \mu_j\right)^{-\eta}} c_j \tag{162}$$

Substituting this into the expenditure constraint, we have that:

$$E = \frac{c_j}{\theta_j \left(\lambda p_j + \mu_j\right)^{-\eta}} \int p_i \theta_i \left(\lambda p_i + \mu_i\right)^{-\eta} \mathrm{d}i \tag{163}$$

And so we have that:

$$c_j = E \theta_j \left(\lambda p_j + \mu_j\right)^{-\eta} \left(\int p_i \theta_i \left(\lambda p_i + \mu_i\right)^{-\eta} \mathrm{d}i \right)^{-1} \tag{164}$$

Moreover, $c_j \leq \bar{c}_j$. Thus, exploiting complementary slackness ($c_j < \bar{c}_j \implies \mu_j = 0$), we have:

$$c_j = \bar{c}_j \min \left\{ 1, \frac{\theta_j}{A_j} \frac{p_j^{-\eta}}{L_j} D \right\} \tag{165}$$

where $D = E\lambda^{-\eta}\left(\int p_i\theta_i\left(\lambda p_i + \mu_i\right)^{-\eta}\mathrm{d}i\right)^{-1}$, which is independent of j. Thus, re-ordering markets according to ι, we have that there exists a unique $k \in [0,1]$ such that all markets with $\iota_j < k$ have $c_j < \bar{c}_j$ and all markets with $\iota_j > k$ have $c_j = \bar{c}_j$. Thus, we can write the expenditure contraint as:

$$E = D\int_0^k \theta_i p_i^{1-\eta}\,\mathrm{d}i + \int_k^1 p_i\bar{c}_i\,\mathrm{d}i \tag{166}$$

which implies that:

$$D = \frac{E - \int_k^1 p_i\bar{c}_i\,\mathrm{d}i}{\int_0^k \theta_i p_i^{1-\eta}\,\mathrm{d}i} \tag{167}$$

yielding Equation 37. To complete the proof, we plug these demands directly into the consumption aggregator, yielding Equation 38. ☐

A.9 Proof of Theorem 2

Proof. We begin by deriving the exact value of mean demand in equilibrium. From Proposition 6, we have that:

$$z_i = \frac{M - \int_k^1 p_i\bar{c}_i\,\mathrm{d}i}{\int_0^k \theta_i p_i^{1-\eta}\,\mathrm{d}i}\theta_i \tag{168}$$

where here we have used the fact that $E = M$ in equilibrium. Thus, we have that:

$$\exp\{\mu_z\} = \frac{M - \int_k^1 p_i\bar{c}_i\,\mathrm{d}i}{\int_0^k \theta_i p_i^{1-\eta}\,\mathrm{d}i}\exp\{\mu_\theta + \frac{1}{2}\sigma_\theta^2\} \tag{169}$$

We now solve for μ_z without any approximation. To do this, we first observe that (i) firms are *ex ante* symmetric and (ii) have a uniquely optimal solution for prices and labor. This in any equilibrium, all firms choose the same price p and the same labor input L. With this, we state the following formula for mean demand.

Lemma 5. *We have that:*

$$\exp\{\mu_z\} = \frac{M}{p}\frac{1}{p^{-\eta}\left(1 - \Phi\left(\Phi^{-1}(r) - \frac{\sigma_{\theta,U}}{\sigma_U}\right)\right) + \bar{L}\exp\{\mu_A + \frac{1}{2}\sigma_A^2\}\Phi\left(\Phi^{-1}(r) - \frac{\sigma_{A,U}}{\sigma_U}\right)} \tag{170}$$

where $\bar{L} = L\exp\{-\mu_z\}$ is invariant to μ_z.

Proof. To solve Equation 169, we first solve for the integrals in the numerator and denominator:

$$\begin{aligned}\int_k^1 p_i\bar{c}_i \mathrm{d}i &= pL\mathbb{E}\left[\exp\{\ln A\}\mathbb{I}\left[\ln A - \ln\theta \le \ln\frac{A_k}{\theta_k}\right]\right] \\ &= pL\exp\{\mu_A + \frac{1}{2}\sigma_A^2\}\Phi\left(\frac{\ln\frac{A_k}{\theta_k} - \mathbb{E}[\ln A - \ln\theta]}{\sigma_U} - \frac{\sigma_{A,U}}{\sigma_U}\right)\end{aligned} \tag{171}$$

$$\begin{aligned}\int_0^k \theta_i p_i^{1-\eta}\,\mathrm{d}i &= p^{1-\eta}\mathbb{E}\left[\exp\{\ln\theta\}\mathbb{I}\left[\ln A_i - \ln\theta_i \ge \ln\frac{A_k}{\theta_k}\right]\right] \\ &= p^{1-\eta}\exp\{\mu_\theta + \frac{1}{2}\sigma_\theta^2\}\left(1 - \Phi\left(\frac{\ln\frac{A_k}{\theta_k} - \mathbb{E}[\ln A - \ln\theta]}{\sigma_U} - \frac{\sigma_{\theta,U}}{\sigma_U}\right)\right)\end{aligned} \tag{172}$$

Combining these expressions with Equation 169, we obtain that:

$$\begin{aligned}&\exp\{\mu_z\}p^{1-\eta}\left(1 - \Phi\left(\frac{\ln\frac{A_k}{\theta_k} - \mathbb{E}[\ln A - \ln\theta]}{\sigma_U} - \frac{\sigma_{\theta,U}}{\sigma_U}\right)\right) \\ &\qquad = M - pL\exp\{\mu_A + \frac{1}{2}\sigma_A^2\}\Phi\left(\frac{\ln\frac{A_k}{\theta_k} - \mathbb{E}[\ln A - \ln\theta]}{\sigma_U} - \frac{\sigma_{A,U}}{\sigma_U}\right)\end{aligned} \tag{173}$$

We now simplify to remove the term $\ln\frac{A_k}{\theta_k}$. To do this, we observe that k and $\ln\frac{A_k}{\theta_k}$ satisfy the following relationship:

$$\begin{aligned}k &= \mathbb{P}\left[\ln\theta_i - \ln A_i \le \ln\frac{\theta_k}{A_k}\right] = 1 - \mathbb{P}\left[\ln A_i - \ln\theta_i \le \ln\frac{A_k}{\theta_k}\right] \\ &= 1 - \mathbb{P}\left[\frac{\ln A_i - \ln\theta_i - \mathbb{E}[\ln A_i - \ln\theta_i]}{\sigma_U} \le \frac{\ln\frac{A_k}{\theta_k} - \mathbb{E}[\ln A_i - \ln\theta_i]}{\sigma_U}\right] \\ &= 1 - \Phi\left(\frac{\ln\frac{A_k}{\theta_k} - \mathbb{E}[\ln A_i - \ln\theta_i]}{\sigma_U}\right)\end{aligned} \tag{174}$$

We can now also note that k is related to the rationing probability by the identity $k = 1 - r$. From this, we can solve that $\ln\frac{A_k}{\theta_k}$ in terms of the rationing probability:

$$r = \Phi\left(\frac{\ln\frac{A_k}{\theta_k} - \mathbb{E}[\ln A_i - \ln\theta_i]}{\sigma_U}\right) \implies \ln\frac{A_k}{\theta_k} = \sigma_U\Phi^{-1}(r) + \mathbb{E}[\ln A_i - \ln\theta_i] \tag{175}$$

Combining this with Equation 173 yields the relationship:

$$\exp\{\mu_z\}p^{1-\eta}\left(1-\Phi\left(\Phi^{-1}(r)-\frac{\sigma_{\theta,U}}{\sigma_U}\right)\right)=M-pL\exp\{\mu_A+\frac{1}{2}\sigma_A^2\}\Phi\left(\Phi^{-1}(r)-\frac{\sigma_{A,U}}{\sigma_U}\right) \tag{176}$$

We now return to the firm's problem to establish the general relationship between p, L, and r with μ_z, noting that all other terms in this expression are parameters. From the expressions in Theorem 1, it is immediate to verify that $\exp\{\mu_z\}t$ is invariant to μ_z and p is invariant to μ_z. These facts imply that L is log-linear in μ_z: $L=\bar{L}\exp\{\mu_z\}$ and r is invariant to μ_z. Exploiting these facts and rearranging, we obtain the claimed expression. □

We now compute the first-order expansion of μ_z in volatility.

Lemma 6. *We have that:*

$$\mu_z=(\eta-1)\ln\frac{\eta}{\eta-1}+(\eta-1)(\mu_w-\mu_A)+\ln M+\eta\phi(\kappa)\sigma_U\delta+O(\delta^2) \tag{177}$$

Proof. We first compute μ_z in the $\delta\to 0$ limit. In this limit, by Equation 170, we have that:

$$\begin{aligned}\exp\{\mu_z(0)\}&=\lim_{\delta\to 0}\exp\{\mu_z(\delta)\}=\frac{M}{p(0)}\frac{1}{p(0)^{-\eta}\frac{1}{\eta}+\bar{L}(0)\exp\{\mu_A\}\frac{\eta-1}{\eta}}\\&=\frac{M}{\frac{\eta}{\eta-1}\exp\{\mu_w-\mu_A\}}\\&\times\frac{1}{\left(\frac{\eta}{\eta-1}\right)^{-\eta}\exp\{-\eta\mu_w+\eta\mu_A\}\frac{1}{\eta}+\exp\{(\eta-1)\mu_A-\eta\mu_w\}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\exp\{\mu_A\}\frac{\eta-1}{\eta}}\\&=\frac{M}{\left(\frac{1}{\eta-1}+1\right)\left(\frac{\eta}{\eta-1}\right)^{-\eta}\exp\{(\eta-1)(\mu_A-\mu_w)\}}\\&=\left(\frac{\eta}{\eta-1}\right)^{\eta-1}\frac{M}{\exp\{(\eta-1)(\mu_A-\ln\alpha-\ln M)\}}\end{aligned} \tag{178}$$

where the final equality uses the fact that $w=\alpha M$.

We now proceed to find the slope of μ_z in δ at $\delta=0$. We have that:

$$\begin{aligned}\mu_z=\ln M-\ln p-\ln\bar{L}-\ln\Bigg(&\exp\{\mu_z\}t\left(1-\Phi\left(\Phi^{-1}(r)-\frac{\sigma_{\theta,U}}{\sigma_U}\right)\right)\\&+\exp\{\mu_A+\frac{1}{2}\sigma_A^2\}\Phi\left(\Phi^{-1}(r)-\frac{\sigma_{A,U}}{\sigma_U}\right)\Bigg)\end{aligned} \tag{179}$$

We now define:

$$\begin{aligned} A(\delta) &= (\exp\{\mu_z\}t)(\delta)\left(1 - \Phi\left(\Phi^{-1}(r(\delta)) - \delta\frac{\sigma_{\theta,U}}{\sigma_U}\right)\right) \\ B(\delta) &= \exp\{\mu_A + \frac{1}{2}\delta^2\sigma_A^2\}\Phi\left(\Phi^{-1}(r(\delta)) - \delta\frac{\sigma_{A,U}}{\sigma_U}\right) \end{aligned} \tag{180}$$

And so we have that:

$$\mu_z'(0) = -(\ln p)'(0) - (\ln \bar{L})'(0) - \frac{A'(0) + B'(0)}{A(0) + B(0)} \tag{181}$$

From Proposition 2, we have that $(\ln p)'(0) = \left(\phi(\kappa) - \frac{\kappa}{\eta}\right)\sigma_U$ and $(\ln \bar{L})'(0) = -\eta\phi(\kappa)\sigma_U$. We also have that:

$$A(0) = \exp\{\mu_A\}\frac{1}{\eta}, \quad B(0) = \exp\{\mu_A\}\frac{\eta - 1}{\eta} \tag{182}$$

and so $A(0) + B(0) = \exp\{\mu_A\}$. Moreover, we calculate:

$$\begin{aligned} A'(0) &= (\exp\{\mu_z\}t)(\delta)'(0)\frac{1}{\eta} - \exp\{\mu_A\}\left(r'(0)\Phi^{-1\prime}(r(0)) - \frac{\sigma_{\theta,U}}{\sigma_U}\right)\phi(\Phi^{-1}(r(0))) \\ &= \exp\{\mu_A\}\frac{\kappa}{\eta}\sigma_U - \exp\{\mu_A\}\left(r'(0)\frac{1}{\phi(\kappa)} - \frac{\sigma_{\theta,U}}{\sigma_U}\right)\phi(\kappa) \\ B'(0) &= \exp\{\mu_A\}\left(r'(0)\frac{1}{\phi(\kappa)} - \frac{\sigma_{A,U}}{\sigma_U}\right)\phi(\kappa) \end{aligned} \tag{183}$$

Combining this information we have that:

$$\begin{aligned} \frac{A'(0) + B'(0)}{A(0) + B(0)} &= \frac{\kappa}{\eta}\sigma_U - \left(r'(0)\frac{1}{\phi(\kappa)} - \frac{\sigma_{\theta,U}}{\sigma_U}\right)\phi(\kappa) + \left(r'(0)\frac{1}{\phi(\kappa)} - \frac{\sigma_{A,U}}{\sigma_U}\right)\phi(\kappa) \\ &= \frac{\kappa}{\eta}\sigma_U + \frac{\sigma_{\theta,U} - \sigma_{A,U}}{\sigma_U}\phi(\kappa) \end{aligned} \tag{184}$$

And so we obtain:

$$\begin{aligned} \mu_z'(0) &= -\left(\phi(\kappa) - \frac{\kappa}{\eta}\right)\sigma_U + \eta\phi(\kappa)\sigma_U - \frac{\kappa}{\eta}\sigma_U + \frac{\sigma_{A,U} - \sigma_{\theta,U}}{\sigma_U}\phi(\kappa) \\ &= \left((\eta - 1) + \frac{\sigma_{A,U} - \sigma_{\theta,U}}{\sigma_U^2}\right)\phi(\kappa)\sigma_U \\ &= \eta\phi(\kappa)\sigma_U \end{aligned} \tag{185}$$

As claimed. □

The formula for equilibrium prices follows immediately by combining Proposition 2 with the fact that $\mu_w = \ln \alpha + \ln M$. To obtain the formula for equilibrium labor inputs, we combine the formula for μ_z with the formula for $\ln L$ from Proposition 2:

$$\begin{aligned}\ln L &= \mu_z + (\eta-1)\mu_A - \eta\mu_w - \eta \ln\left(\frac{\eta}{\eta-1}\right) - \eta\phi(\kappa)\sigma_U \\ &= (\eta-1)\ln\frac{\eta}{\eta-1} + (\eta-1)(\mu_w - \mu_A) + \ln M + \eta\phi(\kappa)\sigma_U \\ &\quad + (\eta-1)\mu_A - \eta\mu_w - \eta\ln\left(\frac{\eta}{\eta-1}\right) - \eta\phi(\kappa)\sigma_U \\ &= -\ln\frac{\eta}{\eta-1} - \mu_w + \ln M \\ &= -\ln\frac{\eta}{\eta-1} - \ln\alpha\end{aligned} \tag{186}$$

To solve for aggregate consumption, we use Equation 38 from Proposition 6 along with the fact that $E = M$:

$$C = \left(\left(M - \int_k^1 p_i \bar{c}_i \,\mathrm{d}i\right)^{\frac{\eta-1}{\eta}} \left(\int_0^k \theta_i p_i^{1-\eta}\right)^{\frac{1}{\eta}} + \int_k^1 \theta_i^{\frac{1}{\eta}} \bar{c}_i^{\frac{\eta-1}{\eta}} \,\mathrm{d}i\right)^{\frac{\eta}{\eta-1}} \tag{187}$$

We moreover re-arrange Equation 169 to obtain that:

$$M - \int_k^1 p_i \bar{c}_i \,\mathrm{d}i = \exp\{\mu_z\} \exp\{-\mu_\theta - \frac{1}{2}\sigma_\theta^2\} \left(\int_0^k \theta_i p_i^{1-\eta} \,\mathrm{d}i\right) \tag{188}$$

Thus, we have that:

$$\ln C = \frac{\eta}{\eta-1} \ln\left(\exp\left\{\frac{\eta-1}{\eta}\left(\mu_z - \mu_\theta - \frac{1}{2}\sigma_\theta^2\right)\right\}\left(\int_0^k \theta_i p_i^{1-\eta} \,\mathrm{d}i\right) + \int_k^1 \theta_i^{\frac{1}{\eta}} \bar{c}_i^{\frac{\eta-1}{\eta}} \,\mathrm{d}i\right) \tag{189}$$

We already computed the first integral on the right-hand side in the proof of Lemma 5:

$$\int_0^k \theta_i p_i^{1-\eta} \,\mathrm{d}i = \exp\{\mu_\theta + \frac{1}{2}\sigma_\theta^2\} p^{1-\eta} \left(1 - \Phi\left(\Phi^{-1}(r) - \frac{\sigma_{\theta,U}}{\sigma_U}\right)\right) \tag{190}$$

We can further compute the second integral by defining the random variable $\zeta_i = \ln\left(\theta_i^{\frac{1}{\eta}} A_i^{\frac{\eta-1}{\eta}}\right)$

and following similar steps to those in Lemma 5:

$$\begin{aligned}\int_k^1 \theta_i^{\frac{1}{\eta}} \bar{c}_i^{\frac{\eta-1}{\eta}} \, \mathrm{d}i &= L^{\frac{\eta-1}{\eta}} \int_k^1 \theta_i^{\frac{1}{\eta}} A_i^{\frac{\eta-1}{\eta}} \, \mathrm{d}i \\ &= L^{\frac{\eta-1}{\eta}} \mathbb{E}\left[\exp\{\ln\zeta\}\mathbb{I}\left[\ln A - \ln\theta \leq \ln A_k - \ln\theta_k\right]\right] \\ &= L^{\frac{\eta-1}{\eta}} \exp\{\mu_\zeta + \frac{1}{2}\sigma_\zeta^2\}\Phi\left(\Phi^{-1}\left(r\right)) - \frac{\sigma_{\zeta,U}}{\sigma_U}\right)\end{aligned} \tag{191}$$

Putting all of this together, we have that:

$$\begin{aligned}\ln C = \frac{\eta}{\eta-1}\ln\Bigg(&\exp\left\{\frac{\eta-1}{\eta}\mu_z + \frac{1}{\eta}\left(\mu_\theta + \frac{1}{2}\sigma_\theta^2\right)\right\} p^{1-\eta}\left(1 - \Phi\left(\Phi^{-1}(r) - \frac{\sigma_{\theta,U}}{\sigma_U}\right)\right) \\ &+ L^{\frac{\eta-1}{\eta}} \exp\{\mu_\zeta + \frac{1}{2}\sigma_\zeta^2\}\Phi\left(\Phi^{-1}\left(r\right) - \frac{\sigma_{\zeta,U}}{\sigma_U}\right)\Bigg)\end{aligned} \tag{192}$$

and we note that we have already solved for all quantities on the right-hand side. We now parameterize volatility by δ and compute consumption to first-order in δ. That is, we wish to compute:

$$(\ln C)(\delta) = (\ln C)(0) + (\ln C)'(0)\delta + O(\delta^2) \tag{193}$$

We can first compute:

$$\begin{aligned}(\ln C)(0) &= \frac{\eta}{\eta-1}\ln\left(\exp\left\{\frac{\eta-1}{\eta}\mu_z(0) + \frac{1}{\eta}\mu_\theta\right\} p(0)^{1-\eta}\left(1 - r(0)\right) + L(0)^{\frac{\eta-1}{\eta}} \exp\{\mu_\xi\} r(0)\right) \\ &= \frac{\eta}{\eta-1}\ln\left(\frac{1}{\eta}\exp\left\{\frac{\eta-1}{\eta}\mu_z(0) + \frac{1}{\eta}\mu_\theta + (1-\eta)\ln p(0)\right\} + \frac{\eta-1}{\eta}\exp\left\{\mu_\xi + \frac{\eta-1}{\eta}\ln L(0)\right\}\right) \\ &= \frac{\eta}{\eta-1}\ln\Bigg(\frac{1}{\eta}\exp\Bigg\{\frac{\eta-1}{\eta}\left((\eta-1)\ln\frac{\eta}{\eta-1} + (\eta-1)(\ln\alpha - \mu_A) + \eta\ln M\right) + \frac{1}{\eta}\mu_\theta \\ &\quad + (1-\eta)\left(\ln\left(\frac{\eta}{\eta-1}\right) + \ln\alpha + \ln M - \mu_A\right)\Bigg\} \\ &\quad + \frac{\eta-1}{\eta}\exp\left\{\frac{1}{\eta}\mu_\theta + \frac{\eta-1}{\eta}\mu_A - \frac{\eta-1}{\eta}\left(\ln\left(\frac{\eta}{\eta-1}\right) + \ln\alpha\right)\right\}\Bigg) \\ &= \mu_A + \frac{\mu_\theta}{\eta-1} - \ln\alpha - \ln\left(\frac{\eta}{\eta-1}\right)\end{aligned} \tag{194}$$

We now compute the derivative, first noting that:

$$(\ln C)'(0) = \frac{\eta}{\eta-1} C(0)^{\frac{1}{\eta}-1} (A+B) \tag{195}$$

where:

$$\begin{aligned} A = & \left(\frac{\eta-1}{\eta}\mu_z'(0) + (1-\eta)(\ln p)'(0)\right)\frac{1}{\eta}\exp\left\{\frac{\eta-1}{\eta}\mu_z(0) + \frac{1}{\eta}\mu_\theta + (1-\eta)\ln p(0)\right\} \\ & - \phi(\Phi^{-1}(r(0)))\left(\frac{r'(0)}{\phi(\Phi^{-1}(r(0)))} - \frac{\sigma_{\theta,U}}{\sigma_U}\right)\exp\left\{\frac{\eta-1}{\eta}\mu_z(0) + \frac{1}{\eta}\mu_\theta + (1-\eta)\ln p(0)\right\} \\ = & \, C(0)^{\frac{\eta-1}{\eta}}\left(\frac{1}{\eta}\frac{\eta-1}{\eta}\kappa\sigma_U - \phi(\kappa)\left(\frac{r'(0)}{\phi(\kappa)} - \frac{\sigma_{\theta,U}}{\sigma_U}\right)\right) \end{aligned} \tag{196}$$

and:

$$\begin{aligned} B = & \frac{\eta-1}{\eta}(\ln L)'(0)\exp\left\{\mu_\xi + \frac{1}{2}\sigma_\xi^2 + \frac{\eta-1}{\eta}(\ln L)(0)\right\}\frac{\eta-1}{\eta} \\ & \phi(\Phi^{-1}(r(0)))\left(\frac{r'(0)}{\phi(\Phi^{-1}(r(0)))} - \frac{\sigma_{\xi,U}}{\sigma_U}\right)\exp\left\{\mu_\xi + \frac{1}{2}\sigma_\xi^2 + \frac{\eta-1}{\eta}(\ln L)(0)\right\} \\ = & \, C(0)^{\frac{\eta-1}{\eta}}\phi(\kappa)\left(\frac{r'(0)}{\phi(\kappa)} - \frac{1}{\eta}\frac{\sigma_{\theta,U}}{\sigma_U} - \frac{\eta-1}{\eta}\frac{\sigma_{A,U}}{\sigma_U}\right) \end{aligned} \tag{197}$$

Combining all of this, we have that:

$$\begin{aligned} (\ln C)'(0) &= \frac{\eta}{\eta-1}\left(\frac{1}{\eta}\frac{\eta-1}{\eta}\kappa\sigma_U - \frac{\eta-1}{\eta}\phi(\kappa)\left(\frac{\sigma_{A,U}}{\sigma_U} - \frac{\sigma_{\theta,U}}{\sigma_U}\right)\right) \\ &= -\left(\phi(\kappa) - \frac{1}{\eta}\kappa\right)\sigma_U \end{aligned} \tag{198}$$

Completing the proof. □

A.10 Proof of Theorem 3

Proof. We now exploit the fact from Equation 42 that aggregate consumption is given by:

$$\ln C = \frac{\eta}{\eta-1}\ln\left(\left(\exp\{\ln M\} - \int_k^1 p_i \bar{c}_i \, \mathrm{d}i\right)^{\frac{\eta-1}{\eta}}\left(\int_0^k \theta_i p_i^{1-\eta}\,\mathrm{d}i\right)^{\frac{1}{\eta}} + \int_k^1 \theta_i^{\frac{1}{\eta}}\bar{c}_i^{\frac{\eta-1}{\eta}}\,\mathrm{d}i\right) \tag{199}$$

where $\ln M = \ln \bar{M} + \ln \Delta$, and $p_i = p$ and $L_i = L$ are invariant to Δ. Differentiating this expression with respect to Δ, we have that:

$$\frac{\partial \ln C}{\partial \ln \Delta} = \frac{\partial \ln C}{\partial \ln M}\frac{\partial \ln M}{\partial \ln \Delta} = \frac{\partial \ln C}{\partial \ln M} \tag{200}$$

Moreover, we can compute:

$$\begin{aligned}
\frac{\partial \ln C}{\partial \ln M} &= \frac{\eta}{\eta-1} C^{-\frac{\eta-1}{\eta}} \frac{\partial}{\partial \ln M}\left(\left(\exp\{\ln M\} - \int_k^1 p_i \bar{c}_i \,\mathrm{d}i\right)^{\frac{\eta-1}{\eta}} \left(\int_0^k \theta_i p_i^{1-\eta}\,\mathrm{d}i\right)^{\frac{1}{\eta}} + \int_k^1 \theta_i^{\frac{1}{\eta}} \bar{c}_i^{\frac{\eta-1}{\eta}}\,\mathrm{d}i\right)\\
&= \frac{\eta}{\eta-1} C^{-\frac{\eta-1}{\eta}} \frac{\partial}{\partial \ln M}\left(\left(\exp\{\ln M\} - pL\int_k^1 A_i \,\mathrm{d}i\right)^{\frac{\eta-1}{\eta}} \left(p^{1-\eta}\int_0^k \theta_i\,\mathrm{d}i\right)^{\frac{1}{\eta}} + L^{\frac{\eta-1}{\eta}}\int_k^1 \theta_i^{\frac{1}{\eta}} A_i^{\frac{\eta-1}{\eta}}\,\mathrm{d}i\right)\\
&= \frac{\eta}{\eta-1} C^{-\frac{\eta-1}{\eta}} \left[\frac{\eta-1}{\eta}\left(M + pLA_k\frac{\partial k}{\partial M}\right)\left(\frac{\exp\{\ln M\} - pL\int_k^1 A_i\,\mathrm{d}i}{p^{1-\eta}\int_0^k \theta_i\,\mathrm{d}i}\right)^{-\frac{1}{\eta}}\right.\\
&\quad \left. + \frac{1}{\eta}p^{1-\eta}\theta_k \frac{\partial k}{\partial M}\left(\frac{\exp\{\ln M\} - pL\int_k^1 A_i\,\mathrm{d}i}{p^{1-\eta}\int_0^k \theta_i\,\mathrm{d}i}\right)^{\frac{\eta-1}{\eta}} + L^{\frac{\eta-1}{\eta}}\theta_k^{\frac{1}{\eta}} A_k^{\frac{\eta-1}{\eta}}\frac{\partial k}{\partial M}\right]
\end{aligned} \tag{201}$$

We now evaluate at $\ln \Delta = 0$. This allows us to make use of the fact that:

$$D = \frac{\exp\{\ln \bar{M}\} - pL\int_k^1 A_i\,\mathrm{d}i}{p^{1-\eta}\int_0^k \theta_i\,\mathrm{d}i} = \exp\{\mu_z - \mu_\theta - \frac{1}{2}\sigma_\theta^2\} \tag{202}$$

which gives us that:

$$\begin{aligned}
\left.\frac{\partial \ln C}{\partial \ln M}\right|_{M=\bar{M}} &= C^{-\frac{\eta-1}{\eta}}\bar{M}\exp\left\{-\frac{1}{\eta}\left(\mu_z - \mu_\theta - \frac{1}{2}\sigma_\theta^2\right)\right\}\\
&\quad + \frac{\eta}{\eta-1}C^{-\frac{\eta-1}{\eta}}\left.\frac{\partial k}{\partial M}\right|_{M=\bar{M}}\left[\left(\frac{\eta-1}{\eta}A_kL + \frac{1}{\eta}\theta_k D p^{-\eta}\right)pD^{-\frac{1}{\eta}} - (A_kL)^{\frac{\eta-1}{\eta}}(\theta_k D p^{-\eta})^{\frac{1}{\eta}}pD^{-\frac{1}{\eta}}\right]\\
&= C^{-\frac{\eta-1}{\eta}}\bar{M}\exp\left\{-\frac{1}{\eta}\left(\mu_z - \mu_\theta - \frac{1}{2}\sigma_\theta^2\right)\right\}\\
&\quad + \frac{\eta}{\eta-1}C^{-\frac{\eta-1}{\eta}}pD^{-\frac{1}{\eta}}\left.\frac{\partial k}{\partial M}\right|_{M=\bar{M}}\left[\frac{\eta-1}{\eta}A_kL + \frac{1}{\eta}\theta_k D p^{-\eta} - (A_kL)^{\frac{\eta-1}{\eta}}(\theta_k D p^{-\eta})^{\frac{1}{\eta}}\right]\\
&= C^{-\frac{\eta-1}{\eta}}\bar{M}\exp\left\{-\frac{1}{\eta}\left(\mu_z - \mu_\theta - \frac{1}{2}\sigma_\theta^2\right)\right\} + \frac{\eta}{\eta-1}C^{-\frac{\eta-1}{\eta}}\left.\frac{\partial k}{\partial M}\right|_{M=\bar{M}}pD^{-\frac{1}{\eta}}\left[A_kL - A_kL\right]\\
&= \bar{M}\exp\left\{-\frac{\eta-1}{\eta}\ln C - \frac{1}{\eta}\left(\mu_z - \mu_\theta - \frac{1}{2}\sigma_\theta^2\right)\right\}
\end{aligned} \tag{203}$$

where we exploited the fact that $A_k L = \theta_k D p^{-\eta}$ for the threshold market k. We now compute the small certainty expansion. We first compute:

$$\begin{aligned}\lim_{\delta\to 0}\mathscr{M} &= \lim_{\delta\to 0}\frac{\partial \ln C}{\partial \ln M}\bigg|_{M=\bar{M}} = \bar{M}\exp\left\{-\frac{\eta-1}{\eta}\ln C(0) - \frac{1}{\eta}\mu_z(0) + \frac{1}{\eta}\mu_\theta\right\} \\ &= \bar{M}\exp\{-\ln \bar{M}\} = 1\end{aligned} \tag{204}$$

where we have used the expressions from Theorem 2. We now compute:

$$\begin{aligned}\lim_{\delta\to 0}\frac{\partial}{\partial\delta}\left[\frac{\partial \ln C}{\partial \ln M}\bigg|_{M=\bar{M}}\right] &= \left(-\frac{\eta-1}{\eta}(\ln C)'(0) - \frac{1}{\eta}\mu_z'(0)\right)\bar{M}\exp\left\{-\frac{\eta-1}{\eta}\ln C(0) - \frac{1}{\eta}\mu_z(0) + \frac{1}{\eta}\mu_\theta\right\} \\ &= \left(\frac{\eta-1}{\eta}\left(\phi(\kappa) - \frac{\kappa}{\eta}\right)\sigma_U - \frac{1}{\eta}\eta\phi(\kappa)\sigma_U\right)\times 1 \\ &= -\frac{1}{\eta}\left(\phi(\kappa) + \frac{\eta-1}{\eta}\kappa\right)\sigma_U\end{aligned} \tag{205}$$

Completing the proof. ☐

A.11 Proof of Corollary 3

Proof. In Theorem 3, we proved that $\lim_{\delta\to 0}\mathscr{M} = 1$. Now consider the economy in which $\delta = 0$. In this economy, demand and supply are equal in all markets. Thus, $c_i = \bar{c}_i$ for all $i \in [0,1]$. As $\bar{c}_i$ cannot change after the unanticipated monetary shock, neither can c_i. Hence, C is invariant to the unanticipated monetary shock. ☐

A.12 Proof of Proposition 7

Proof. From Theorem 3, we have that:

$$\operatorname{sgn}\left(\frac{\partial\mathscr{M}}{\partial\eta}\right) = \operatorname{sgn}\left(-\frac{\partial}{\partial\eta}\left[\frac{1}{\eta}\left(\phi(\kappa) + \frac{\eta-1}{\eta}\kappa\right)\right]\right) \tag{206}$$

We now make use of the fact that $\eta = 1/(1-\Phi(\kappa))$ to write:

$$\frac{1}{\eta}\left(\phi(\kappa) + \frac{\eta-1}{\eta}\kappa\right) = (1-\Phi(\kappa))(\phi(\kappa) + \kappa\Phi(\kappa)) \equiv g(\kappa) \tag{207}$$

Observing that κ is a strictly increasing function of η, it suffices to sign the derivative of g, which is given by:

$$\begin{aligned} g'(\kappa) &= -\phi(\kappa)(\phi(\kappa) + \kappa\Phi(\kappa)) + (1 - \Phi(\kappa))(\phi'(\kappa) + \Phi(\kappa) + \kappa\phi(\kappa)) \\ &= \Phi(\kappa)(1 - \Phi(\kappa)) - \phi(\kappa)(\phi(\kappa) + \kappa\Phi(\kappa)) \end{aligned} \tag{208}$$

where we have used the fact that $\phi'(\kappa) = -\kappa\phi(\kappa)$. The roots of g' must therefore solve:

$$1 = m(\kappa)(\kappa + r(\kappa)) \equiv R(\kappa) \tag{209}$$

Moreover, we know that m is a strictly increasing function. Thus, if $\kappa + r(\kappa)$ is an increasing function, then $g(\kappa)$ has at most one root. This is an increasing function if:

$$0 \leq 1 + r'(\kappa) = 1 - (\kappa + r(\kappa))r(\kappa) \iff r(\kappa)(\kappa + r(\kappa)) \leq 1 \tag{210}$$

We now use some facts about Mills ratios. First, as $\Phi(-x) = 1 - \Phi(x)$ and $\phi(-x) = \phi(x)$, we have that $r(\kappa) = m(-\kappa)$. Thus, we require that $m(-\kappa)(\kappa + m(-\kappa)) \leq 1$. Equivalently, we must prove that $m(x)(-x + m(x)) \leq 1$ for all $x \in \mathbb{R}$. But this is equivalent to the inequality $m(x) \leq \frac{1}{2}(x + \sqrt{x^2 + 4})$, which holds for all $x \in \mathbb{R}$ by the theorem of Birnbaum (1942). We further have that $R(0) = m(0)(0 + r(0)) = 4\phi(0)^2 = 4(1/\sqrt{2\pi})^2 = 2/\pi < 1$. Moreover, $\lim_{\kappa\to\infty} R(\kappa) = \infty$ (as $m(\kappa) \geq \kappa$ and $r(\kappa) \geq 0$). Thus, we have that g' changes sign exactly once, at the unique solution κ^* of the equation $R(\kappa) = 1$. Moreover, when $\kappa < \kappa^*$, we have that $g'(\kappa) > 0$ and when $\kappa > \kappa^*$, we have that $g'(\kappa) < 0$. Hence, the extent of monetary non-neutrality is decreasing in η when $\eta < \eta^*$ and increasing in η when $\eta > \eta^*$, where $\eta^* = 1/(1 - \Phi(\kappa^*))$. Numerical calculations yield that $\kappa^* \approx 0.376$ and $\eta^* \approx 2.83$. □

A.13 Proof of Proposition 8

Proof. We start by log-linearizing the firm's dynamic tightness and pricing decisions.

Lemma 7. *When log-linearized around the steady state, the optimal reset tightness and reset prices follow:*

$$\hat{t}_\tau = -(1 - \beta\omega)\left(\hat{D}_\tau + \tilde{\psi}\hat{\sigma}_{U,\tau}\right) + \beta\omega\mathbb{E}_\tau[\hat{t}_{\tau+1}] \tag{211}$$

$$\hat{p}_\tau = (1 - \beta\omega)\left(\hat{M}_\tau - \tilde{\zeta}\hat{\sigma}_{U,\tau}\right) + \beta\omega\mathbb{E}_\tau[\hat{p}_{\tau+1}] \tag{212}$$

where $\tilde{\psi}$ and $\tilde{\zeta}$ are constants given in the proof. Moreover, the implied optimal reset level of *labor inputs follows:*

$$\hat{L}_\tau = (1-\beta\omega)(\hat{D}_\tau - \eta\hat{M}_\tau + (\tilde{\psi} + \eta\tilde{\zeta})\hat{\sigma}_{U,\tau}) + \beta\omega\mathbb{E}_\tau[\hat{L}_{\tau+1}] \tag{213}$$

Proof. We begin by log-linearizing the forward-looking decision rules of the firm that we derived in Equation 71:

$$\begin{aligned} t_\tau &= \frac{1}{\eta-1}\frac{\sum_{j=0}^{\infty}(\beta\omega)^j \frac{1}{M_{\tau+j}}\mathbb{E}_\tau[A_{\tau+j}\mathbb{I}[t_\tau D_{\tau+j} \geq A_{\tau+j}/\theta_{\tau+j}]]}{\sum_{j=0}^{\infty}(\beta\omega)^j \frac{1}{M_{\tau+j}} D_{\tau+j}\mathbb{E}_\tau[\theta_{\tau+j}\mathbb{I}[t_\tau D_{\tau+j} \leq A_{\tau+j}/\theta_{\tau+j}]]} \\ p_\tau &= \frac{\frac{\alpha}{1-\beta\omega}}{\sum_{j=0}^{\infty}(\beta\omega)^j \frac{1}{M_{\tau+j}}\mathbb{E}_\tau[A_{\tau+j}\mathbb{I}[t_\tau D_{\tau+j} \geq A_{\tau+j}/\theta_{\tau+j}]]} \end{aligned} \tag{214}$$

We define $\mathcal{A}_{\tau+j} = \frac{1}{M_{\tau+j}}\mathbb{E}_\tau[A_{\tau+j}\mathbb{I}[t_\tau D_{\tau+j} \geq A_{\tau+j}/\theta_{\tau+j}]]$ and $\mathcal{B}_{\tau+j} = \frac{1}{M_{\tau+j}} D_{\tau+j}\mathbb{E}_\tau[\theta_{\tau+j}\mathbb{I}[t_\tau D_{\tau+j} \leq A_{\tau+j}/\theta_{\tau+j}]]$. We therefore have that:

$$\begin{aligned} \hat{t}_\tau &= (1-\beta\omega)\mathbb{E}_\tau\left[\sum_{j=0}^{\infty}(\beta\omega)^j\left[\hat{\mathcal{A}}_{\tau+j} - \hat{\mathcal{B}}_{\tau+j}\right]\right] \\ \hat{p}_\tau &= (1-\beta\omega)\mathbb{E}_\tau\left[\sum_{j=0}^{\infty}(\beta\omega)^j\left[-\hat{\mathcal{A}}_{\tau+j}\right]\right] \end{aligned} \tag{215}$$

We have already computed that:

$$\begin{aligned} \mathcal{A}_{\tau+j} &= \frac{1}{M_{\tau+j}}\exp\left\{\mu_{A,\tau+j} + \frac{1}{2}\sigma_{A,\tau+j}^2\right\}\Phi\left(\frac{\ln(t_\tau D_{\tau+j}) - \mu_{U,\tau+j}}{\sigma_{U,\tau+j}} - \beta^S\sigma_{U,\tau+j}\right) \\ \mathcal{B}_{\tau+j} &= \frac{D_{\tau+j}}{M_{\tau+j}}\exp\left\{\mu_{\theta,\tau+j} + \frac{1}{2}\sigma_{\theta,\tau+j}^2\right\}\left(1 - \Phi\left(\frac{\ln(t_\tau D_{\tau+j}) - \mu_{U,\tau+j}}{\sigma_{U,\tau+j}} - \beta^D\sigma_{U,\tau+j}\right)\right) \end{aligned} \tag{216}$$

and so we have that (as we have assumed that first moments of A and θ do not change):

$$\begin{aligned} \hat{\mathcal{A}}_{\tau+j} &= -\hat{M}_{\tau+j} + \lambda_a\left(\hat{t}_\tau + \hat{D}_{\tau+j} + \phi_a\hat{\sigma}_{U,\tau+j}\right) \\ \hat{\mathcal{B}}_{\tau+j} &= -\hat{M}_{\tau+j} + \hat{D}_{\tau+j} - \lambda_b\left(\hat{t}_\tau + \hat{D}_{\tau+j} + \phi_b\hat{\sigma}_{U,\tau+j}\right) \end{aligned} \tag{217}$$

where:

$$\begin{aligned}
\lambda_a &= \frac{1}{\bar{\sigma}_U} r \left(\frac{\ln\left(\bar{t}\bar{D}\right) - \mu_U}{\bar{\sigma}_U} - \beta^S \bar{\sigma}_U \right) > 0 \\
\lambda_b &= \frac{1}{\bar{\sigma}_U} m \left(\frac{\ln\left(\bar{t}\bar{D}\right) - \mu_U}{\bar{\sigma}_U} - \beta^D \bar{\sigma}_U \right) > 0 \\
\phi_a &= -\left(\left(\frac{\ln\left(\bar{t}\bar{D}\right) - \mu_U}{\bar{\sigma}_U} - \beta^S \bar{\sigma}_U \right) + 2\beta^S \bar{\sigma}_U \right) \bar{\sigma}_U \\
\phi_b &= -\left(\left(\frac{\ln\left(\bar{t}\bar{D}\right) - \mu_U}{\bar{\sigma}_U} - \beta^D \bar{\sigma}_U \right) + 2\beta^D \bar{\sigma}_U \right) \bar{\sigma}_U
\end{aligned} \tag{218}$$

Combining this information, we have that reset tightness follows (so long as $\lambda_a + \lambda_b \neq 1$, which we established in the proof of Theorem 1):

$$\begin{aligned}
\hat{t}_\tau &= (1-\beta\omega)\mathbb{E}_\tau \left[\sum_{j=0}^{\infty} (\beta\omega)^j \left[(\lambda_a + \lambda_b)\hat{t}_\tau + (\lambda_a + \lambda_b - 1)\hat{D}_{\tau+j} + (\lambda_a\phi_a + \lambda_b\phi_b)\,\hat{\sigma}_{U,\tau+j} \right] \right] \\
&= (\lambda_a + \lambda_b)\hat{t}_\tau + (\lambda_a + \lambda_b - 1)(1-\beta\omega)\mathbb{E}_\tau \left[\sum_{j=0}^{\infty} (\beta\omega)^j \left(\hat{D}_{\tau+j} + \tilde{\psi}\hat{\sigma}_{U,\tau+j} \right) \right]
\end{aligned} \tag{219}$$

where $\tilde{\psi} = (\lambda_a\phi_a + \lambda_b\phi_b)/(\lambda_a + \lambda_b - 1)$. This yields that:

$$\begin{aligned}
\hat{t}_\tau &= -(1-\beta\omega)\mathbb{E}_\tau \left[\sum_{j=0}^{\infty} (\beta\omega)^j \left(\hat{D}_{\tau+j} + \tilde{\psi}\hat{\sigma}_{U,\tau+j} \right) \right] \\
&= -(1-\beta\omega)\left(\hat{D}_\tau + \tilde{\psi}\hat{\sigma}_{U,\tau} \right) + \beta\omega\mathbb{E}_\tau[\hat{t}_{\tau+1}]
\end{aligned} \tag{220}$$

Turning to reset prices, we have that:

$$\begin{aligned}
\hat{p}_\tau &= (1-\beta\omega)\mathbb{E}_\tau\left[\sum_{j=0}^{\infty}(\beta\omega)^j\left[\hat{M}_{\tau+j}-\lambda_a\left(\hat{t}_\tau+\hat{D}_{\tau+j}+\phi_a\hat{\sigma}_{U,\tau+j}\right)\right]\right]\\
&= (1-\beta\omega)\mathbb{E}_\tau\left[\sum_{j=0}^{\infty}(\beta\omega)^j\hat{M}_{\tau+j}\right]-\lambda_a\left[\hat{t}_\tau+(1-\beta\omega)\mathbb{E}_\tau\left[\sum_{j=0}^{\infty}(\beta\omega)^j\left(\hat{D}_{\tau+j}+\phi_a\hat{\sigma}_{U,\tau+j}\right)\right]\right]\\
&= (1-\beta\omega)\mathbb{E}_\tau\left[\sum_{j=0}^{\infty}(\beta\omega)^j\hat{M}_{\tau+j}\right]\\
&\qquad -\lambda_a\left[\hat{t}_\tau+(1-\beta\omega)\mathbb{E}_\tau\left[\sum_{j=0}^{\infty}(\beta\omega)^j\left(\hat{D}_{\tau+j}+\tilde{\psi}\hat{\sigma}_{U,\tau+j}+(-\tilde{\psi}+\phi_a)\hat{\sigma}_{U,\tau+j}\right)\right]\right]\\
&= (1-\beta\omega)\mathbb{E}_\tau\left[\sum_{j=0}^{\infty}(\beta\omega)^j\left(\hat{M}_{\tau+j}-\lambda_a(-\tilde{\psi}+\phi_a)\hat{\sigma}_{U,\tau+j}\right)\right]\\
&= (1-\beta\omega)\left(\hat{M}_\tau-\lambda_a(-\tilde{\psi}+\phi_a)\hat{\sigma}_{U,\tau}\right)+\beta\omega\mathbb{E}_\tau[\hat{p}_{\tau+1}]
\end{aligned} \tag{221}$$

Letting $\tilde{\zeta}=\lambda_a(-\tilde{\psi}+\phi_a)$ then yields the claimed expression. The formula for labor inputs follows from the identity that $\hat{L}_\tau=-\eta\hat{p}_\tau-\hat{t}_\tau$. □

We next aggregate these variables and use the Calvo assumption to derive equations in terms of the average tightness, $\bar{t}_\tau$, and the average price level $\bar{p}_\tau$.

Lemma 8. *When log-linearized around the steady state, we have that price inflation π_τ and tightness inflation υ_τ obey the following equations:*

$$\pi_\tau=\Gamma(\hat{M}_\tau-\bar{p}_\tau)+\beta\mathbb{E}_\tau[\pi_{\tau+1}]+\zeta\hat{\sigma}_{U,\tau} \tag{222}$$

$$\upsilon_\tau=-\Gamma(\hat{D}_\tau+\bar{t}_\tau)+\beta\mathbb{E}_\tau[\upsilon_{\tau+1}]+\psi\hat{\sigma}_{U,\tau} \tag{223}$$

where $\Gamma=\frac{(1-\omega)(1-\beta\omega)}{\omega}$, $\zeta=-\Gamma\tilde{\zeta}$, and $\psi=-\Gamma\tilde{\psi}$.

Proof. We begin by noting that, under the Calvo assumption, $\bar{p}_\tau=\omega\bar{p}_{t-1}+(1-\omega)\hat{p}_\tau$, which implies that:

$$\hat{p}_\tau=\frac{\bar{p}_\tau-\omega\bar{p}_{\tau-1}}{1-\omega}=\frac{\bar{p}_\tau-\omega(\bar{p}_t-\Delta\bar{p}_\tau)}{1-\omega}=\bar{p}_\tau+\frac{\omega}{1-\omega}\Delta\bar{p}_\tau \tag{224}$$

Plugging this into the reset equation from Lemma 7, we obtain that:

$$\begin{aligned}\bar{p}_\tau + \frac{\omega}{1-\omega}\Delta\bar{p}_\tau &= (1-\beta\omega)(\hat{M}_\tau - \tilde{\zeta}\hat{\sigma}_{U,\tau}) + \beta\omega\mathbb{E}_\tau\left[\bar{p}_{\tau+1} + \frac{\omega}{1-\omega}\Delta\bar{p}_{\tau+1}\right] \\ &= (1-\beta\omega)(\hat{M}_\tau - \tilde{\zeta}\hat{\sigma}_{U,\tau}) + \beta\omega\mathbb{E}_\tau\left[\bar{p}_\tau + \Delta\bar{p}_{\tau+1} + \frac{\omega}{1-\omega}\Delta\bar{p}_{\tau+1}\right] \\ &= (1-\beta\omega)(\hat{M}_\tau - \tilde{\zeta}\hat{\sigma}_{U,\tau}) + \beta\omega\mathbb{E}_\tau\left[\bar{p}_\tau + \frac{1}{1-\omega}\Delta\bar{p}_{\tau+1}\right]\end{aligned} \tag{225}$$

Which we can re-arrange as:

$$\Delta\bar{p}_\tau = \Gamma(\hat{M}_\tau - \bar{p}_\tau) + \beta\mathbb{E}_\tau[\Delta\bar{p}_{\tau+1}] + \zeta\hat{\sigma}_{U,\tau} \tag{226}$$

We can now apply the same principle for tightness. In particular, we plug the fact that $\hat{t}_\tau = \bar{t}_\tau + \frac{\omega}{1-\omega}\Delta\bar{t}_\tau$ into the reset equation from Lemma 7 to obtain that:

$$\bar{t}_\tau + \frac{\omega}{1-\omega}\Delta\bar{t}_\tau = -(1-\beta\omega)(\hat{D}_\tau + \tilde{\psi}\hat{\sigma}_{U,\tau}) + \beta\omega\mathbb{E}_\tau\left[\bar{t}_{\tau+1} + \frac{\omega}{1-\omega}\Delta\bar{t}_{\tau+1}\right] \tag{227}$$

Identical algebraic steps yield the result. ☐

Finally, we log-linearize to obtain the demand shifter in slack markets.

Lemma 9. *When log-linearized around the steady state, the household's demand in slack markets follows:*

$$\hat{D}_\tau = \gamma_M\hat{M}_\tau + \gamma_p\bar{p}_\tau + \gamma_t\bar{t}_\tau + \gamma_U\hat{\sigma}_{U,\tau} \tag{228}$$

where γ_M, γ_p, and γ_t are coefficients derived in the proof.

Proof. We start from the fact that:

$$D_\tau = \frac{M_\tau - R_\tau}{S_\tau} \tag{229}$$

where $R_\tau = \int_0^1 p_{i,\tau}L_{i,\tau}A_{i,\tau}\mathbb{I}[t_{i,\tau}D_\tau \geq A_{i,\tau}/\theta_{i,\tau}]\,\mathrm{d}i$ and $S_\tau = \int_0^1 p_{i,\tau}^{1-\eta}\theta_{i,\tau}\mathbb{I}[t_{i,\tau}D_\tau \leq A_{i,\tau}/\theta_{i,\tau}]\,\mathrm{d}i$. Thus, we can log-linearize this as:

$$\hat{D}_\tau = \frac{\bar{M}}{\bar{M}-\bar{R}}\hat{M}_\tau - \frac{\bar{R}}{\bar{M}-\bar{R}}\hat{R}_\tau - \hat{S}_\tau \tag{230}$$

Or, defining the steady-state expenditure share on slack markets as $s = (\bar{M}-\bar{R})/\bar{M}$, we have that:

$$\hat{D}_\tau = \frac{1}{s}\hat{M}_\tau - \hat{S}_\tau - \frac{1-s}{s}\hat{R}_\tau \tag{231}$$

From the arguments of Lemma 5, we have that:

$$\begin{aligned} R_\tau &= \exp\left\{\mu_{A,\tau} + \frac{1}{2}\sigma_{A,\tau}^2\right\}\int_0^1 p_{i,\tau}L_{i,\tau}\Phi\left(\frac{\ln\left(t_{i,\tau}D_\tau\right) - \mu_{U,\tau}}{\sigma_{U,\tau}} - \beta^S\sigma_{U,\tau}\right)\mathrm{d}i \\ S_\tau &= \exp\left\{\mu_{\theta,\tau} + \frac{1}{2}\sigma_{\theta,\tau}^2\right\}\int_0^1 p_{i,\tau}^{1-\eta}\left(1 - \Phi\left(\frac{\ln\left(t_{i,\tau}D_\tau\right) - \mu_{U,\tau}}{\sigma_{U,\tau}} - \beta^D\sigma_{U,\tau}\right)\right)\mathrm{d}i \end{aligned} \tag{232}$$

Log-linearizing these expressions around the steady state (and noting that all firms decisions are symmetric in the steady state), we obtain that:

$$\begin{aligned} \hat{R}_\tau &= \bar{p}_\tau + \bar{L}_\tau + \lambda_a(\bar{t}_\tau + \hat{D}_\tau + \phi_a\hat{\sigma}_{U,\tau}) \\ \hat{S}_\tau &= (1-\eta)\bar{p}_\tau - \lambda_b(\bar{t}_\tau + \hat{D}_\tau + \phi_b\hat{\sigma}_{U,\tau}) \end{aligned} \tag{233}$$

Using the fact that $\bar{L}_\tau = -\eta\bar{p}_\tau - \bar{t}_\tau$, we then obtain that:

$$\begin{aligned} \hat{D}_\tau &= \frac{1}{s}\hat{M}_\tau - (1-\eta)\bar{p}_\tau + \lambda_b(\bar{t}_\tau + \hat{D}_\tau + \phi_b\hat{\sigma}_{U,\tau}) \\ &\qquad - \frac{1-s}{s}\Big((1-\eta)\bar{p}_\tau - \bar{t}_\tau + \lambda_a(\bar{t}_\tau + \hat{D}_\tau + \phi_a\hat{\sigma}_{U,\tau})\Big) \\ &= \frac{1}{s}\hat{M}_\tau - \frac{1}{s}(1-\eta)\bar{p}_\tau + \left(\lambda_b - \frac{1-s}{s}\left(\lambda_a - 1\right)\right)\bar{t}_\tau \\ &\qquad + \left(\lambda_b - \frac{1-s}{s}\lambda_a\right)\hat{D}_\tau + \left(\lambda_b\phi_b - \frac{1-s}{s}\lambda_a\phi_a\right)\hat{\sigma}_{U,\tau} \end{aligned} \tag{234}$$

Solving this equation for $\hat{D}_\tau$ yields the claim. □

This completes the proof of the claims in the result. □

B Supplemental Appendix

B.1 Calibration of σ_U

In this Appendix, we describe a strategy for separately calibrating the volatility of productivity (A) and idiosyncratic demand shocks (z) faced for firms. This allows us to calibrate the volatility of the composite parameter $U = A/z$, which in turn is critical for calibrating the likelihood of rationing and slack. This calculation is especially subtle in our model for two reasons. First, productivity and demand shocks—which often have a symmetric role in standard market-clearing models, as shifters of the value marginal product of inputs—have sharply *asymmetric* roles in our theory, because they have different effects on the state of the market (rationed vs. slack). Second, as we will soon make clear, optimizing firms' choices of tightness complicates the mapping from fundamental shocks to the randomness an econometrician would observe in "value-based" measures of productivity.

Measurement. We base our calibration strategy on the availability of two measurements in any firm-level microdata with separate observations of inputs, quantities produced, and the value of output sold.

The first measurement is "traditional TFP," or TFPT. In the data, this is defined as plant output in *physical units* (*e.g.*, widgets) divided by an index of inputs. That is, in the data,

$$\ln \text{TFPT} = \ln \text{Physical Output} - \ln f(\text{Inputs}) \tag{235}$$

where $f(\text{Inputs})$ is an identified production function (*e.g.*, under Cobb-Douglas, a sum of inputs in logarithmic units times their output elasticities). Crucially for our purposes, physical output is measured as shipments plus changes in inventories and *not* as the physical quantity of goods sold. Thus, viewed through the lens of our model, it is umambiguous that

$$\ln \text{TFPT} = \ln q^S - \ln L = \ln A \tag{236}$$

where A is the physical productivity of the firm.

The second measurement is revenue-based productivity, or TFPR. In the data, this is analogous to TFPT but with *sales* (price times transacted quantity) replacing physical output:

$$\ln \text{TFPR} = \ln \text{Sales} - \ln f(\text{Inputs}) \tag{237}$$

Through the lens of the model, we interpret TFPR thusly. Products are sold at a constant unit price p, chosen in advance. Sales equal $q^D = zp^{-\eta}$ is the market is slack, or $q^S = AL$ if the market is rationed. Thus,

$$\begin{aligned} \ln \text{TFPR} &= \ln p + \ln \min\{zp^{-\eta}, AL\} - \ln L \\ &= \ln p + \ln \min\{zt, A\} \end{aligned} \tag{238}$$

where we remind that the definition of tightness is $t = p^{-\eta}/L$. Moreover, in our model, optimal tightness depends on the variances of random variables as described in Theorem 1 and Proposition 1.

From Data to Theory. To proceed, we use measurements from two empirical studies.

First, Foster et al. (2008) conduct a detailed comparison of "traditional" and revenue-based measures of TFP using US Census of Manufacturing microdata from particular manufacturing sectors for which physical output is particularly homogeneous, such as cardboard boxes, white bread, and plywood. As the authors note, these sample restrictions are helpful to make cross-firm comparisons of physical productivity meaningful given the impracticality of constructing firm-specific measures of the quality of output. In particular, the authors report moments of the cross-sectional, plant-level distribution of TFPT and TFPR within narrow product groups. We interpret this as a measure of the ratio of volatility in physical productivity versus revenue productivity (see Table I of that paper):

$$\frac{\sigma_A}{\sigma_{\mathrm{TFPR}}} = \frac{0.21}{0.22} \tag{239}$$

Second, Bloom et al. (2018) use plant-level US Census of Manufacturing microdata *across sectors* to construct detailed measures of one-quarter-ahead firm uncertainty regarding revenue-based TFP. This incorporates three corrections that are useful for our analysis: (i) isolating the statistically unforecastable component of firm-level shocks, (ii) further estimating the fraction of that statistical uncertainty that is plausibly faced by the firm (given measurement error as well as unobservable information), and (iii) translating the estimate to a decision period of one quarter. This results in an estimated one-quarter ahead variance of $\sigma_{\mathrm{TFPR}} = 0.10$.[11] Combined with the earlier calibration from Foster et al. (2008), this results in an estimate of

$$\sigma_A = \frac{0.21}{0.22} \times 0.10 = 0.095 \tag{240}$$

What remains is to calibrate σ_z. We do this numerically to match the estimate $\sigma_{\mathrm{TFPR}} = 0.10$, simulating the distribution of $\ln \mathrm{TFPR}$ using Equation 239 and, for each possible value of σ_z, also calculating the firm's optimal tightness using Corollary 1.[12] Doing this results in an estimate of $\sigma_z = 0.24$, and thus $\sigma_U = \sqrt{\sigma_A^2 + \sigma_z^2} = 0.26$.

---

[11] These calculations are based on the calibrated values of Table V of Bloom et al. (2018). Specifically, these authors consider an environment with two volatility states that evolve via a discrete Markov process. In the authors' calibration, the "low" and "high" volatility state respectively occur with probability 0.68 and 0.32. In the former state, the standard deviation of unforecastable quarterly TFPR shocks is $0.051 = 5.1\%$, and in the latter, it is $0.209 = 20.9\%$. Averaging these, and expressing them in variance terms, yields the estimate $0.68 \times 0.051 + 0.32 \times 0.209 = 0.10$.

[12] We assume, for this calculation, that all uncertainty about A and z comes from idiosyncratic shocks and moreover that this uncertainty is orthogonal to that in the stochastic discount factor Λ.